Three-Dimensional Interpretation of Quadrilaterals

Gang Xu*f and Saburo Tsuji

Department of Control Engineering. Osaka University
Toyonaka, Osaka 560, Japan
*also AI Department, ATR Communication Systems Research Laboratories
Seika-cho, Soraku-gun, Kyoto 619-02, Japan

Abstract

Quadrilaterals are figures with which everybody becomes familiar in his/her very early stage of education. By studying these seemingly simple figures we can obtain some insights into the nature of the general problem of interpreting image contours. This paper discusses in detail how quadrilaterals are interpreted three-dimensionally, and draws feasible inferences about the general properties of the human system of processing line drawings. First the rectangularity regularity is proposed to be the prime constraint in the visual interpretation of quadrilaterals. The subjective "image center"[1] and focal length (finite and infinite) are determined together with rectangle orientation. Secondly, interpretation of quadrilaterals as faces of a rectangular polyhedron is examined at both the geometrical level and the perceptual level. Finally the gravity regularity is proposed to derive constraints on the rectangle orientation by analyzing the relation among the camera, the ground and the rectangles supported by the ground.

1. Introduction

An image is a two-dimensional projection of a three-dimensional scene, and a contour image represents significant changes in surface shape, reflectance and illumination, which are reflected directly or indirectly in the 2-D image. A contour image can be a. line drawing, drawn by human hands, which only includes topologically well-defined, semantically significant, but not necessarily positionally accurate contours; or it can be generated by computer from a real image, which may include many noise edges, if not processed very carefully.

Line drawing is probably the most abstract and efficient means of describing our 3-dimensional world in a 2-dimensional manner. Thus it is often used in human communications and is potentially very useful in human-machine interfaces. Although the contours alone do not provide sufficient constraints on the surfaces, humans seem not to have any difficulty in recovering 3-D shapes from the 2-D contours. The task is so effortless for our eyes that we rarely pause to ask ourselves how we do. As we try to answer, however, we realize that it is a difficult question.

f The work was done at Osaka University. Dr. Gang Xu's address from this autumn is Center of Information Science, Peking University, Beijing, China

A number of papers have been published that try to answer this "shape from contour" question. Among them are [Barrow &; Tenenbaum, 1981], (Kanade, 1981], [Barnard, 1983] and [Brady & Yuille, 1984]. All the papers listed above generally deal with only closed contours and assume that the image contours are, globally or locally, projections of planar space curves. Ellipses are interpreted as circles by additional assumptions of: uniformity of curvature [Barrow &; Tenenbaum, 1981], maximum entropy [Barnard, 1983] and maximum compactness [Brady & Yuille, 1984], and quadrilaterals or parallelograms are interpreted as rectangles by additional assumptions of maximum symmetry [Kanade, 1981; Barnard, 1983]. Reviewing the papers, it is not difficult to find that the approaches are based on the psycological facts that ellipses and quadrilaterals (including parallellograms) are perceived as circles and rectangles, respectively. The approaches differ only in how the facts are accounted for and in what specific criteria, they choose to achieve the predetermined aims. The psyoology of line drawing perception was first studied by the Gestalt school. They propose *prdgnanz,* or figural goodness, to be the criterion on which the human perception is based. The circle and rectangle interpretations are preferred because they are the most beautiful interpretations among the possible. Unfortunately, however, the Gestalt psycologists could not give a theoretical account of the term *prdgnanz* from a standpoint of information processing, and figural goodness remains to be judged mainly by human eyes (note that a recent progress is an attempt to characterize pragnanz by transformational invariance [Palmer, 1983].) The lack of a definition of *pragnanz* leaves room for proposal of various specific criteria. The uniformity of curvature, entropy, compactness and symmetry criteria are developed in, and thus well suited to the specific cases, but they are not surely universal and may not apply to other cases.

Under the circumstances, two general paradigms are available. The first is the quantitative paradigm, in which a universal criterion is developed and interpretations are selected by maximizing or minimizing that criterion, as done in [Barrow & Tenenbaum, 1981], [Barnard, 1983] and [Brady & Yuille, 1984], The second is the qualitative paradigm, in which specific figural configurations that have definite interpretations are searched for and interpreted. Examples are [Stevens, 19S1&1986], in which parallel (curved) contours are interpreted as lines of curvature on a cylindrical surface, [Xu & Tsuji, 1987a,b], in

which a closed boundary is segmented and interpreted as four lines of curvature if certain conditions are satisfied, and [Barnard & Pentland, 1983], in which elliptic arcs are directly interpreted as circular arcs. The causal relation between the interpreted and the interpretation is referred to as regularity. The task is thus to discover regularities, or causal relations, and then to apply them to specific interpretation processes. We consider that the qualitative paradigm is more advantageous because (1) a line drawing is generally only qualitative, not quantitative, especially when it is hand-drawn; and (2) while interpretations are qualitatively stable, they are not always quantitatively stable.

2. The Rectangularity Regularity and Rectangle Orientation

2.1 The rectangularity regularity

The human visual perception, as a part of the brain, is the product of millions of years of evolution. As a consequence, various regularities of nature have been embedded into the vision system. It is these natural regularities that are secrets of the human vision (and the human perception at large.) They fill in the blank inherent in the mapping from two-dimensionality onto three-dimensionality. Only by understanding them, can we really understand the human vision and further develop any computer vision systems. (See [Pentland, 1986] for a general discussion on the role that natural regularities play in visual perception, and [Ullman, 1979a,b; Reuman & Hoffman, 1986] for how natural regularities play in the visual perception of motion.)

On the other hand, the human visual perception is not simply a copy of external regularities; it has its own internal structure, which functions in its own right. One indication of such an internal structure is the perceptual system's prefenice of *pragnanz,* or simple, regular forms over complex, irregular ones. It is unfair to contribute this property to regularities of external world. Circles are preferred over ellipses [Barrow & Tenenbauin, 1981; Brady &. Yuille, 1984] not because we regularly see circles in our environment, but mainly because the internal structure of the perceptual system appreciate the simplicity or figural goodness of the circle interpretation. This kind of regularities   if one would also like to call them regularities—is subjective regularities, in contrast to external natural regularities. (In fact, the preference of simplicity is not the privilege of perception; all the phases of cognition show this tendency [Kanizsa, 1979, p.238].)

All quadrilaterals can be queued, according to the de gree of regularity, as: rectangle, parallelogram, trapezium and generic quadrilateral (Fig. 1). It is generally difficult to define degree of regularity universally, but here it is intuitive and we do not try to give a theoretical definition Palmer, 1983]. It is observed that a single quadrilateral tends to be always perceived as a rectangle[1]. To put it another way, a quadrilateral in image, whatever its degree of regularity is, is interpreted to be a rectangle in space, the most regular interpretation among the possible, and the 2-D irregularity is thought of as being caused by the projection. It is from this observation that the rect angularity regularity is generalized. This regularity, as discussed in the last paragraph, is a. subjective regularity resulting
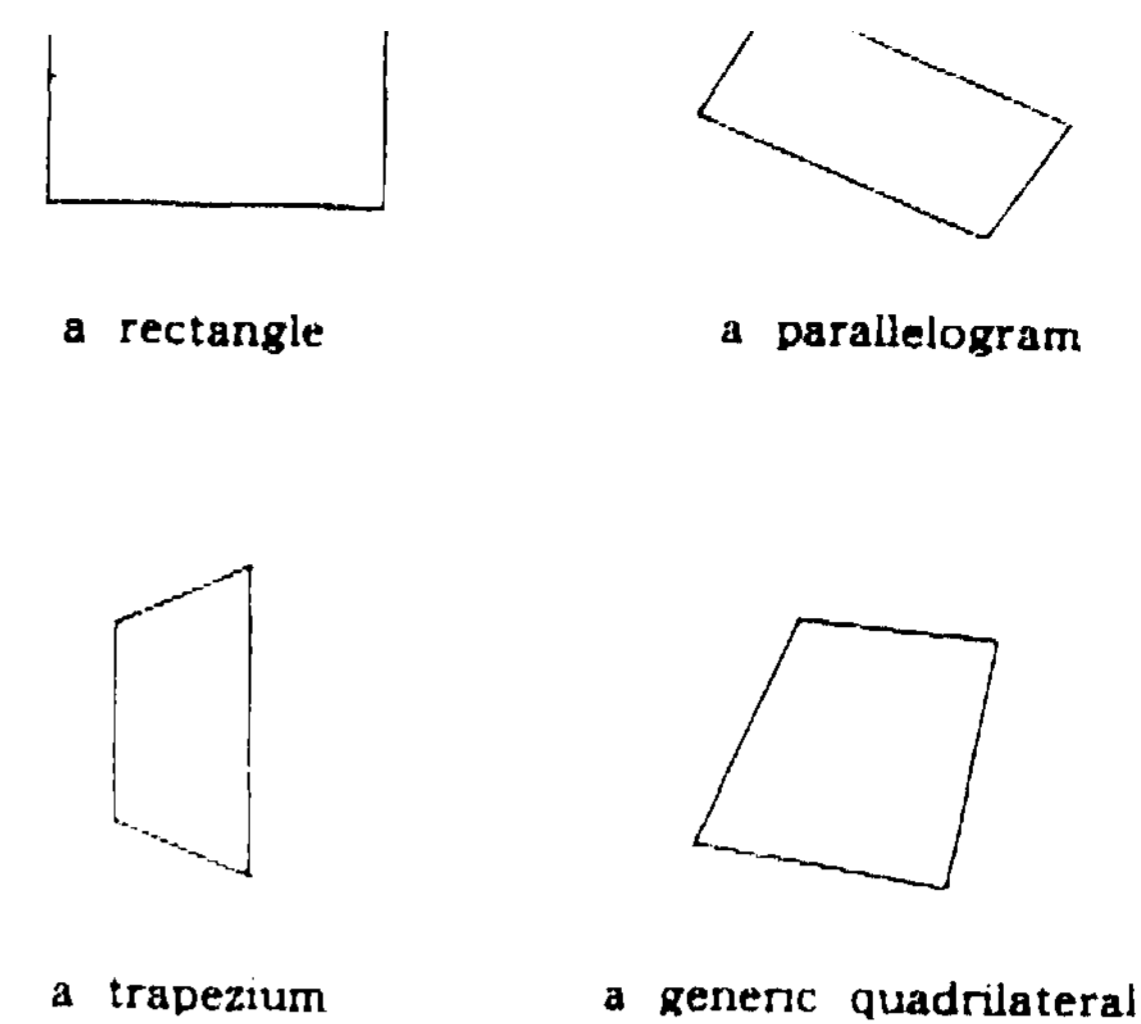
Fig. 1 Quadrilaterals are queued by the degree of regularity.

from the internal structure of the perceptual system.

The rectangularity regularity has long been recognized a;, being of importance. Barnard (1983) computes the planar orientation of a rectangle from its image, a quadrilateral, under perpspective projection. Kanatani (1986) discusses how to compute spatial orientations of the faces of a rectangular trihedral polyhedron. Xu and Tsuji (1987a,b) extends this regularity to curved surfaces and proposes the LOC (line of curvature) regularity to recover shape of curved surfaces. In this paper we give a unified and detailed account of interperting quadrilaterals in image as rectangles in space under both orthographic and perspective projections.

2.2 The focal point and rectangle orientation

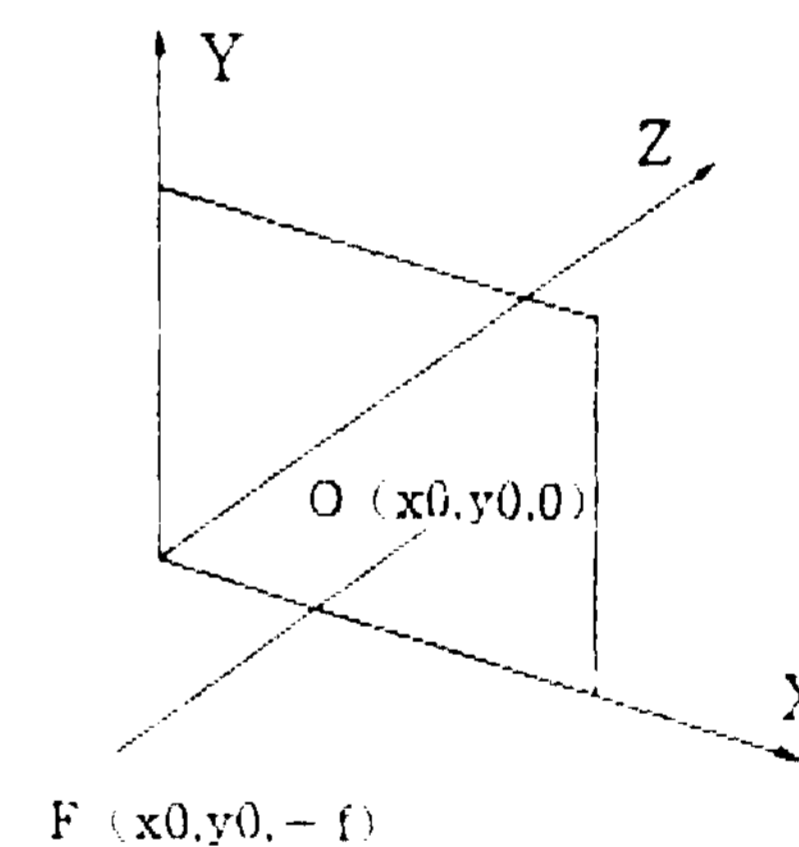In the following we discuss how and to what extent the

Fig. 2 The coordinate system

3D orientation of a quadrilateral is determined by incorporating the rectangularity regularity. The coordinate system we assume, as shown in Fig. 2, is different from those we usually use, in that, the coordinate origin is located on the image plane and in that the z-axis is independent of the focal print, which has the coordinates (:7'0, y/0, — f). Both the "image center" $(x(),y0)$ and the focal length /are then to be determined in the process of interpretation. While the camera system is an objective one if the image is a. real one, it is a. subjective one if the image is a hand drawn figure.

What is known is a quadrilateral in the image, and what is to be known is the orientation of the rectangle that projects that quadrilateral, and the location of the focal point, but not the distance or size of the rectangle. Let the four corners of the quadrilateral be A, B, C and D, as shown in Fig. 3. Extending the segments AB and
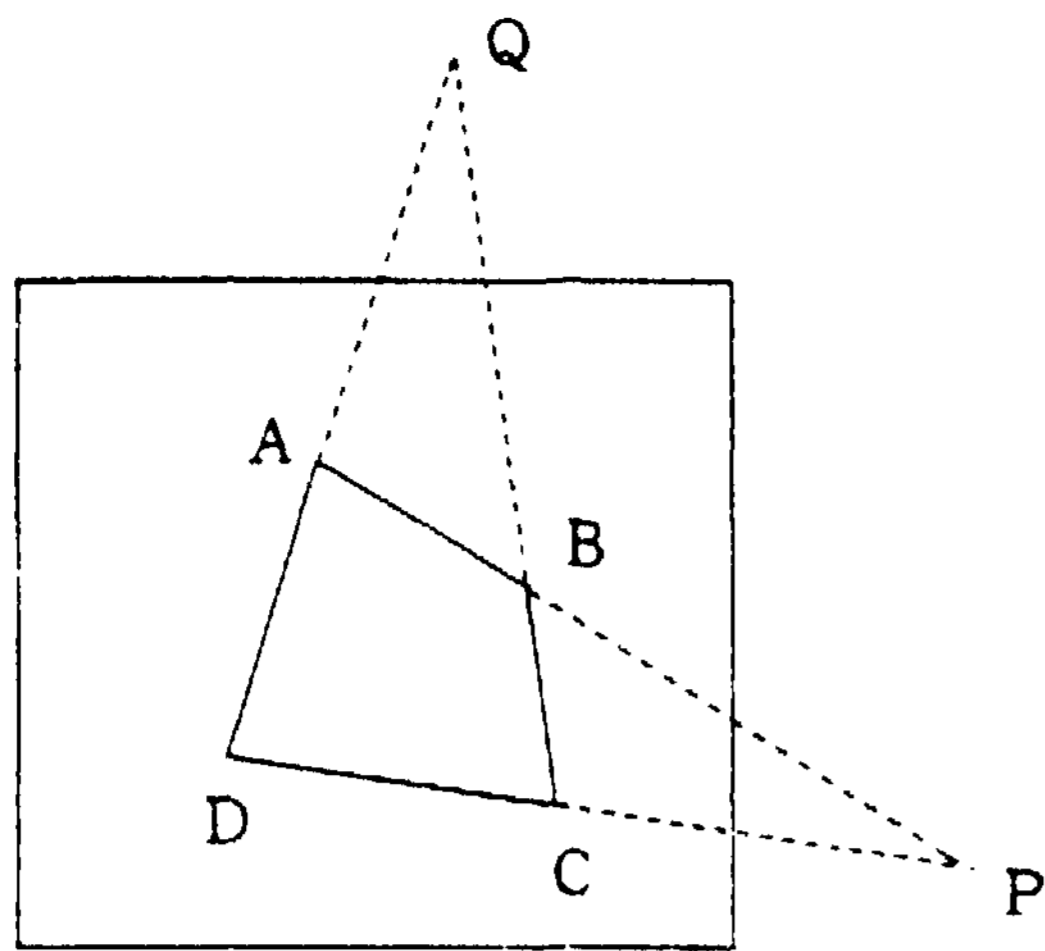
Fig. 3 Extending the laterals we have two vanishing points **P** and **Q**.

CD, we have the intersection P $(x1, y1)$. Since AB and CD are the images of two parallels, P is the vanishing point of the parallels. Similarly, extending the segments BC and DA, we have the intersection Q (x2,y2), which is also a vanishing point, of the other group of parallels. P and/or Q approach infinity if the corresponding image segments are parallel.

A straightforward demonstration of the concept of vanishing point is that the line connecting the focal point and a vanishing point is parallel to the parallel lines that give rise to that vanishing point. As shown in Fig. 4, we know that PF is parallel to A'B' and CD', and QF is parallel to B'C and D'A\ Since A'B'C'D' is a rectangle, PF is perpendicular to QF. The plane determined by PF and QF is parallel to the plane on which the rectangle lies, and thus the orientations of the two planes are identical.
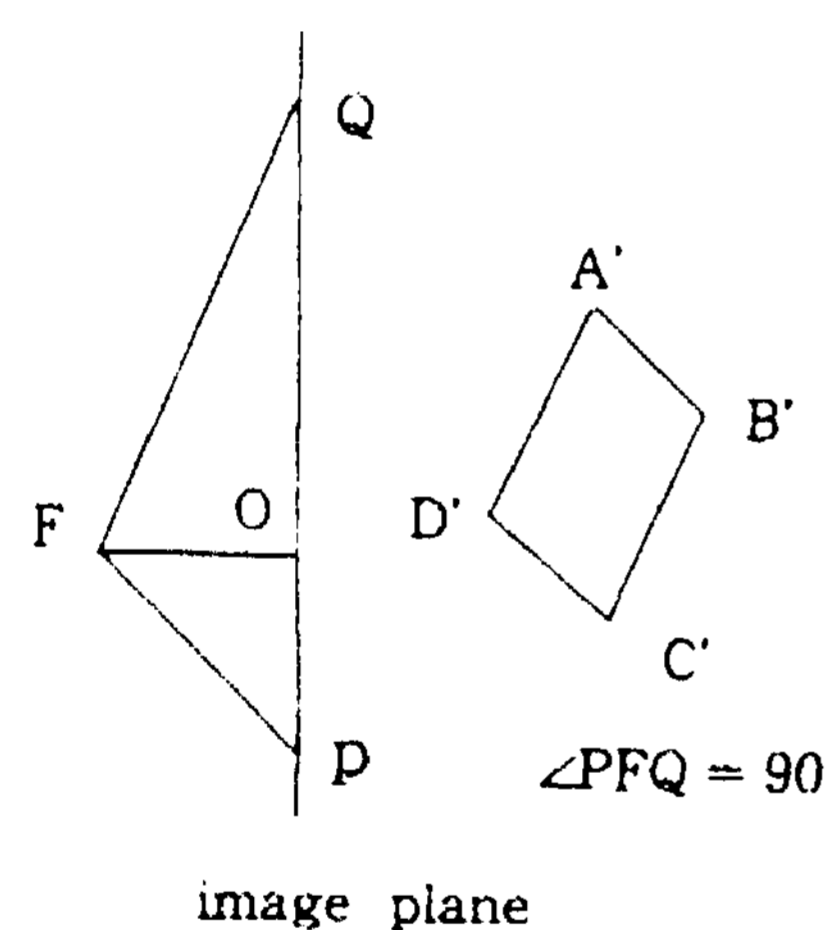

Fig. 4 **PF** is parallel to **A'B'** and **C'D'** and **QF** is parallel to **B'C'**and **D'A'**. Thus P1 is Perpendicular to **QF**.

The segments PF and QF can be expressed in vector form as $(x1-x0, y1-y0, f)$ and $(x2-:x0, y2-y0, /)$, respectively. From the perpendicularity, the inner product of the two vectors is zero. By this equation F can be either determined or at least constrained. Once F is determined, the plane normal of (let it be called n) can also be determined as the outer product of PF and QF. In the following, we discuss three cases of PF and QF: (1) neither P nor Q approaches infinity; (2) either P or Q approaches infinity; and (3) both P and Q approach infinity.

(case1) If both P and Q do not appproach infinity, then the quadrilateral is a generic one. From the perpendicularity of PF and QF we have

$$(x1 - x0)(x2 - x0) + (y1 - y0)(y2 - y0) + f^2 = 0. \quad (1)$$

Clearly, f ($f > 0$) can be determined as

$$f = \sqrt{-(x1 - x0)(x2 - x0) - (y1 - y0)(y2 - y0)}. \quad (2)$$

This equation describes a hemisphere with a diameter PQ, on which F is constrained to lie, as shown in Fig. 5. For / to have a solution, the point O (x0,y0,0) must satisfy the following inequality:

$$(x1 - x0)(x2 - x0) + (y1 - y0)(y2 - y0) < 0.$$

This means that O must lie inside the circle with PQ on the image plane. When Q reaches the midpoint of PQ, all of PO, QO and FQ become radii of the hemisphere, and / has the maximal value, half the length of PQ. To determine F completely, however, we have to first know where the attention is oriented; i.e., where O is. If there are three or more quadrilaterals in a real image, then P can be determined as the intersection point of the hemispheres corresponding to the quadrilaterals (see Section 4 for a special case.) A prerequisite to this solution is that the hemispheres do have a common point; i.e., the image is a real one. However, as in hand-drawn figures, quadrilaterals are usually prodticed by individual attentions. Consequently, they should be, and can only be, perceived separately. One reasonable choice for each quadrilateral is the intersection of the two diagonals, which is the centroid of the corresponding rectangle in space, if the centroid is within the circle with a diameter PQ. As mentioned above, the maximal value for / is half the length of PQ. Trially, the closer to a parallelogram the quadrilateral is, the greater / is. On the other hand, humans prefer long focal lengths in
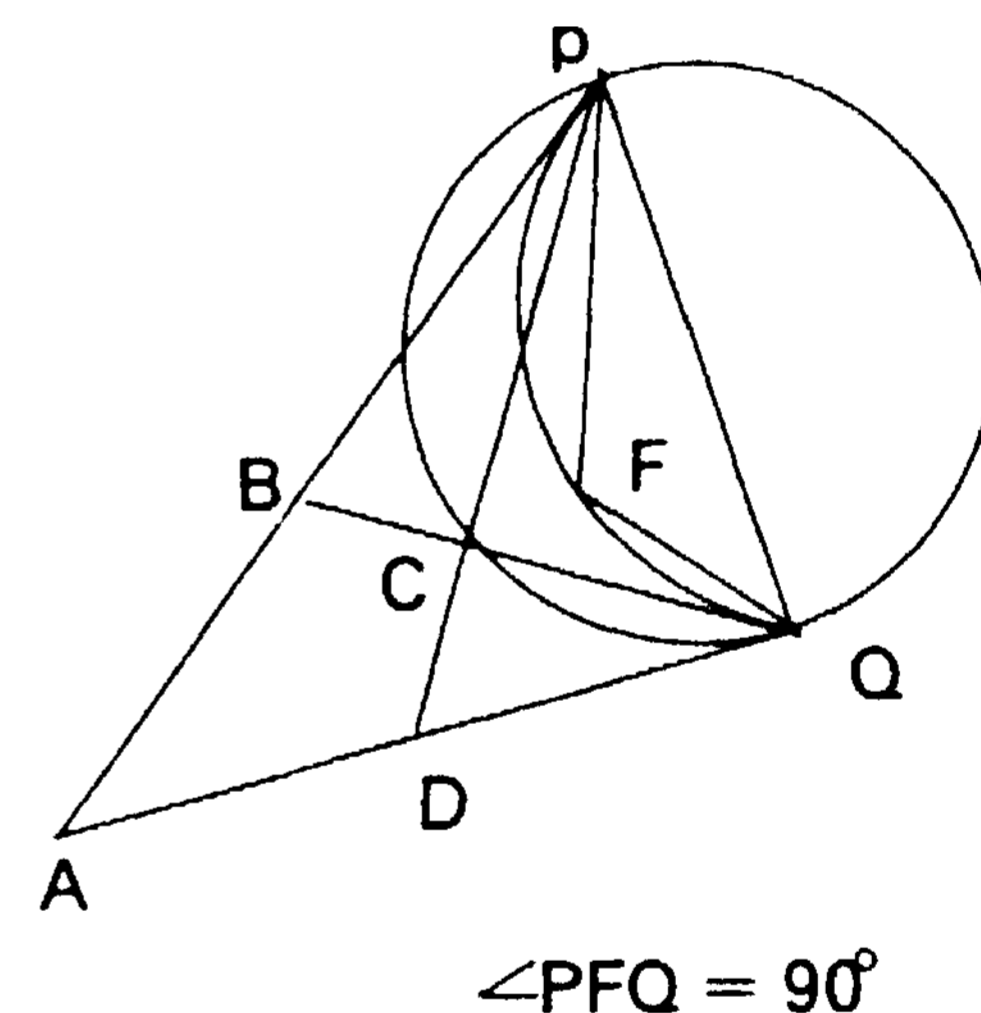

Fig. 5 F lies on the hemisphere with a diameter **PQ**.

the perception of figures. This answers why a parallelogram is more often used to represent a rectangle, and why a parallelogram is more easily perceived by our eyes as a rectangle.

(Case 2) If one of P and Q approaches infinity, then the quadrilateral is a trapezium. Without loss of generality, suppose that P approaches infinity, while Q does not. PO $(x1 \sim x0, y1-y0)$ can be expressed by (a, b) as P approaches infinity. Adding the z-component, PF can be expressed by (a,b, c), where a, b and c are all constants, c approaches 0 if f does not approach infinity, and c may still be 0 even if /does approach infinity. Can/approach infinity? Suppose that it does. Then clearly BC is parallel to DA, and their intersection Q also approaches infinity. This leads to a contradiction. Thus/cannot approach infinity and c equals zero.
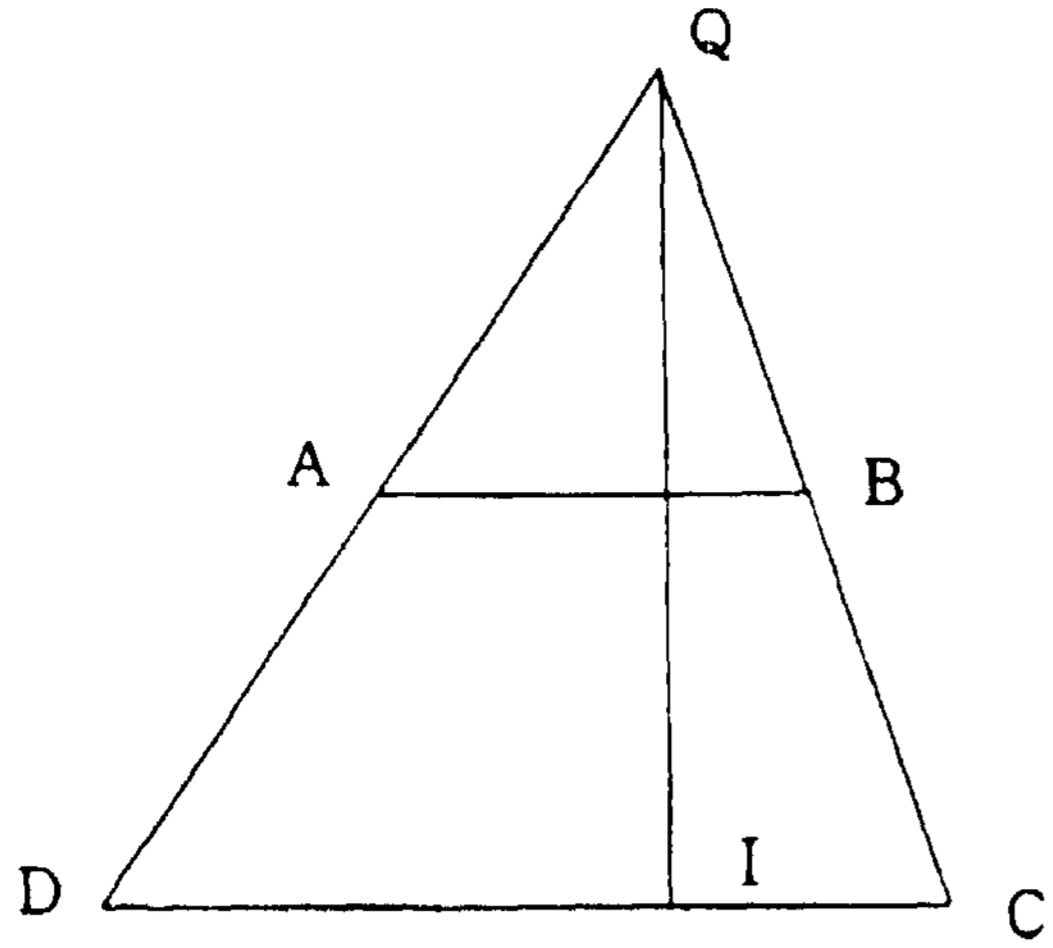
Fig. 6   **F** lies on the plane that is projected onto the image as **IQ**.

From the perpendicularity of **PF** and **QF**, we have

$$a(x2 - x0) + b(y2 - y0) = 0. \tag{3}$$

This equation constrains **OQ** to be perpendicular to **AB** and **CD**; i.e., **O** must lie on the line perpendicular to **AB** and **CD** drawn from **O**. Let us call the line **IQ** (Fig. 6). Since f is free, **F** is constrained to lie on the plane that projects onto the image as **IQ**. When **O** is located at **Q**, the orientation is completely determined by the orientations of **AB** and **CD** as $(b, -a, 0)$.

(**case 3**) If both **P** and **Q** approach infinity, then the quadrilateral is a parallelogram or a rectangle. In this case, **PO** is parallel to **AB** and **CD**, and **QO** is parallel to **BC** and **DA**. **O** is actually not constrained. Suppose that the orientations of **PO** and **QO** are expressed by (a, b) and (a', b'), respectively. Adding the z-components, **PF** and **QF** are expressed by (a, b, c) and (a', b', c'), respectively. a, b, c, a', b' and c' are all constants. Both c and c' approach 0 if f does not approach infinity, and c and c' may still be 0 even if f does approach infinity. ¿From the perpendicularity of **PF** and **QF**, we have

$$aa' + bb' + cc' = 0. \tag{4}$$

A special case is that **ABCD** is a rectangle;

$$aa' + bb' = 0; \tag{5}$$

then

$$cc' = 0. \tag{6}$$

Three cases are possible:

$$\begin{cases} c = 0, c' = 0; \\ c = 0, c' \neq 0; \\ c \neq 0, c' = 0. \end{cases}$$

If one of c and c' is not zero, then f approaches infinity; the projection can only be orthographic. If both c and c' are zero, then f is not constrained; the projection can be either orthographic or perpspective. In this case, the rectangle normal can be determined as $(0, 0, 1)$; the rectangle is parallel to the image plane.

3.   Quadrilaterals as Faces of a Rectangular Polyhedron

Any figures can be interpreted at two diferent levels: the geometrical level and the perceptual level [Sugihara, 1986]. While interpretations at the geometrical level must strictly obey mathematics, interpretations at the perceptual level show humanlike flexibility. Interpretations at two levels may be fairly different. We first deal with the geometrical level.

It can be mathematically proven that a trihedral polyhedron with quadrilateral faces is a hexahedron. If all the faces are parallelograms, the hexahedron becomes a parallelepiped. If the parallelograms are rectangles, then the parallelepiped becomes a rectangular polyhedron.

Trivially, When a parallelepiped is projected onto an image from a general position, orthographically or perspectively, at most three of its six faces are visible. All the three visible faces in the image are parallelograms if the projection is orthographic, and are not if the projection is perspective.

Now we consider the inverse problem - how to infer the original object from its non-degenerate image. We are given three quadrilaterals, every two of which have a common edge — this condition is sufficient to define their interrelations. Since there are three common edges, the total number of edges is 9 (=12-3). They can be grouped into three, each of which has three non-intersecting edges, as $e1$-$e2$-$e3$, $e4$-$e5$-$e6$, $e7$-$e8$-$e9$ shown in Fig. 7. The first condition for the figure to mean a real object is that the edges of each group meet at a common point, which may approach infinity.

In the following we try to find the conditions for the three quadrilaterals to be simultaneously interpreted as
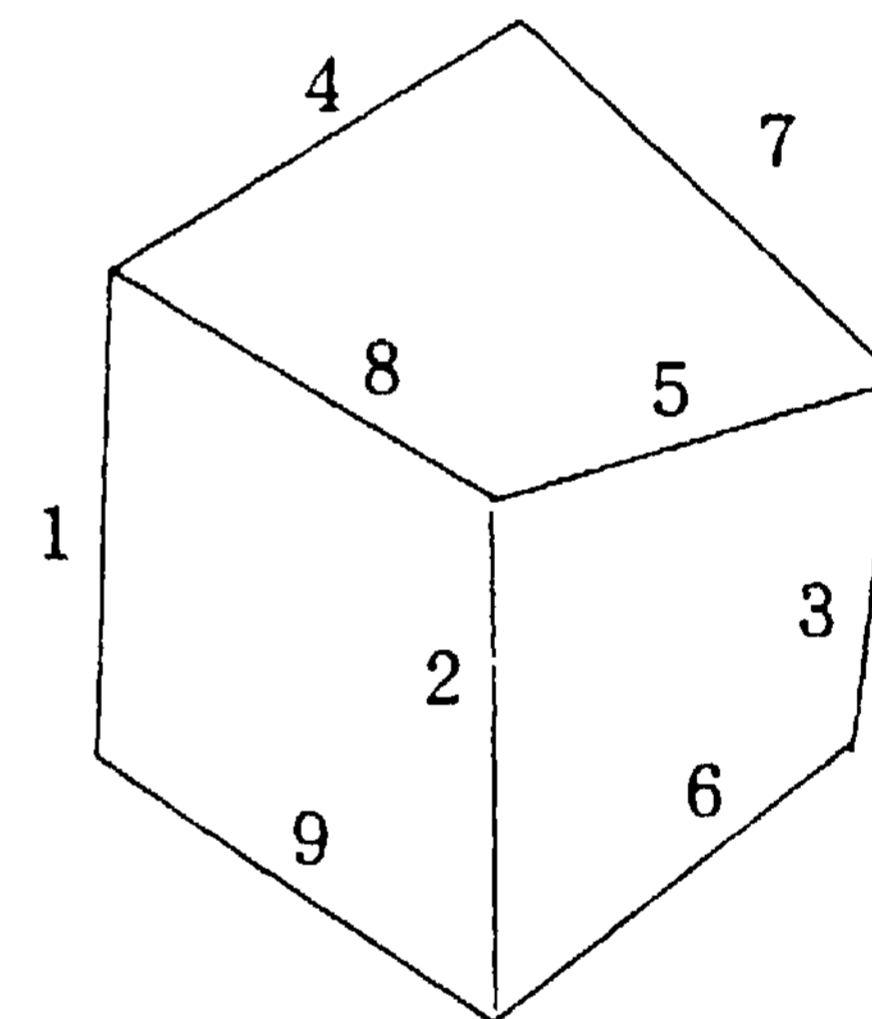

Fig. 7   Three quadrilaterals have totally 9 edges, if every two quadrilaterals have a common edge.

rectangles.

If none of the three intersection points approaches infinity, we need only to examine whether or not the three hemispheres, each of which is determined by two common intersection points as the vanishing points as described in the last section, have a common point; i.e., whether or not the following three equations have a solution for $(x0, y0, f)$.

$$(x1 - x0)(x2 - x0) + (y1 - y0)(y2 - y0) + f^2 = 0,$$
$$(x2 - x0)(x3 - x0) + (y2 - y0)(y3 - y0) + f^2 = 0,$$
$$(x3 - x0)(x1 - x0) + (y3 - y0)(y1 - y0) + f^2 = 0, \tag{7}$$

where $(x1, y1)$, $(x2, y2)$ and $(x3, y3)$ are the three intersection points. Taking the differences of every two equations,

we have

$$(x1 - x0)(x3 - x2) + (y1 - y0)(y3 - y2) = 0,$$
$$(x2 - x0)(x1 - x3) + (y2 - y0)(y1 - y3) = 0,$$
$$(x3 - x0)(x2 - x1) + (y3 - y0)(y2 - y1) = 0. \quad (8)$$

These equations mean that $(x0, y0)$ is the orthocenter of the triangle formed by the three intersection points $(x1, y1)$, $(x2, y2)$ and $(x3, y3)$. Thus, fortunately, the three hemispheres always have a common point.

If one of the three intersection points approaches infinity, then we have the following three equations,

$$a(x1 - x0) + b(y1 - y0) = 0,$$
$$a(x2 - x0) + b(y2 - y0) = 0,$$
$$(x1 - x0)(x2 - x0) + (y1 - y0)(y2 - y0) + f^2 = 0, \quad (9)$$

where (a,b) is the orientation vector, and $(x1,y1l)$ and $(x2,y2)$ are the coordinates of the other two intersection points. The difference of the first two equations means that the parallel edges are perpendicular to the line linking the other two intersection points. The focal point is not completely determined, but constrained to lie on a semicircle, whose two end points are the two intersection points, and the plane on which the semicircle lies is perpendicular to the image plane.

If two of the three intersection points approach infinity and the other one does not, then one of the three quadrilaterals must be a rectangle. As discussed in Section 2, if two vanishing points approach infinity, then the corresponding image quadrilateral is a parallelogram or a rectangle. If the parallelogram is to be interpreted as a rectangle in space, then the projection must be orthographic. If the projection is orthographic, then all rectangles in space are projected as parallelograms in image. Thus, if two of the three intersection points approach infinity and their corresponding quadrilateral is not a rectangle, then the other one must also approach infinity. As described in Section 2, if a space rectangle is parallel to the image plane, then it is projected as an image rectangle under the perspective projection. If it is a face of a rectangular polyhedron, then the other two visible faces are projected as generic quadrilaterals, with the corresponding vanishing points not approaching irifin itv.

If all the three intersection points approach infinity, then the projection is orthographic. The condition for a rectangular polyhedron interpretation is that the following three equtions have a common solution for cl,c2 and c3.

$$\mathbf{V1} \cdot \mathbf{V2} + c1c2 = 0,$$
$$\mathbf{V2} \cdot \mathbf{V3} + c2c3 = 0,$$
$$\mathbf{V3} \cdot \mathbf{V1} + c3c1 = 0, \quad (10)$$

where $\mathbf{V1} = (a1, b1)$, $\mathbf{V2} = (a2, b2)$ and $\mathbf{V3} = (a3, b3)$ are the orientation vectors of the three groups of parallel edges. Easily we have

$$c1^2 = -\frac{(\mathbf{V1} \cdot \mathbf{V2})(\mathbf{V1} \cdot \mathbf{V3})}{(\mathbf{V2} \cdot \mathbf{V3})},$$
$$c2^2 = -\frac{(\mathbf{V2} \cdot \mathbf{V3})(\mathbf{V2} \cdot \mathbf{V1})}{(\mathbf{V3} \cdot \mathbf{V1})},$$
$$c3^2 = -\frac{(\mathbf{V3} \cdot \mathbf{V1})(\mathbf{V3} \cdot \mathbf{V2})}{(\mathbf{V1} \cdot \mathbf{V2})}. \quad (11)$$

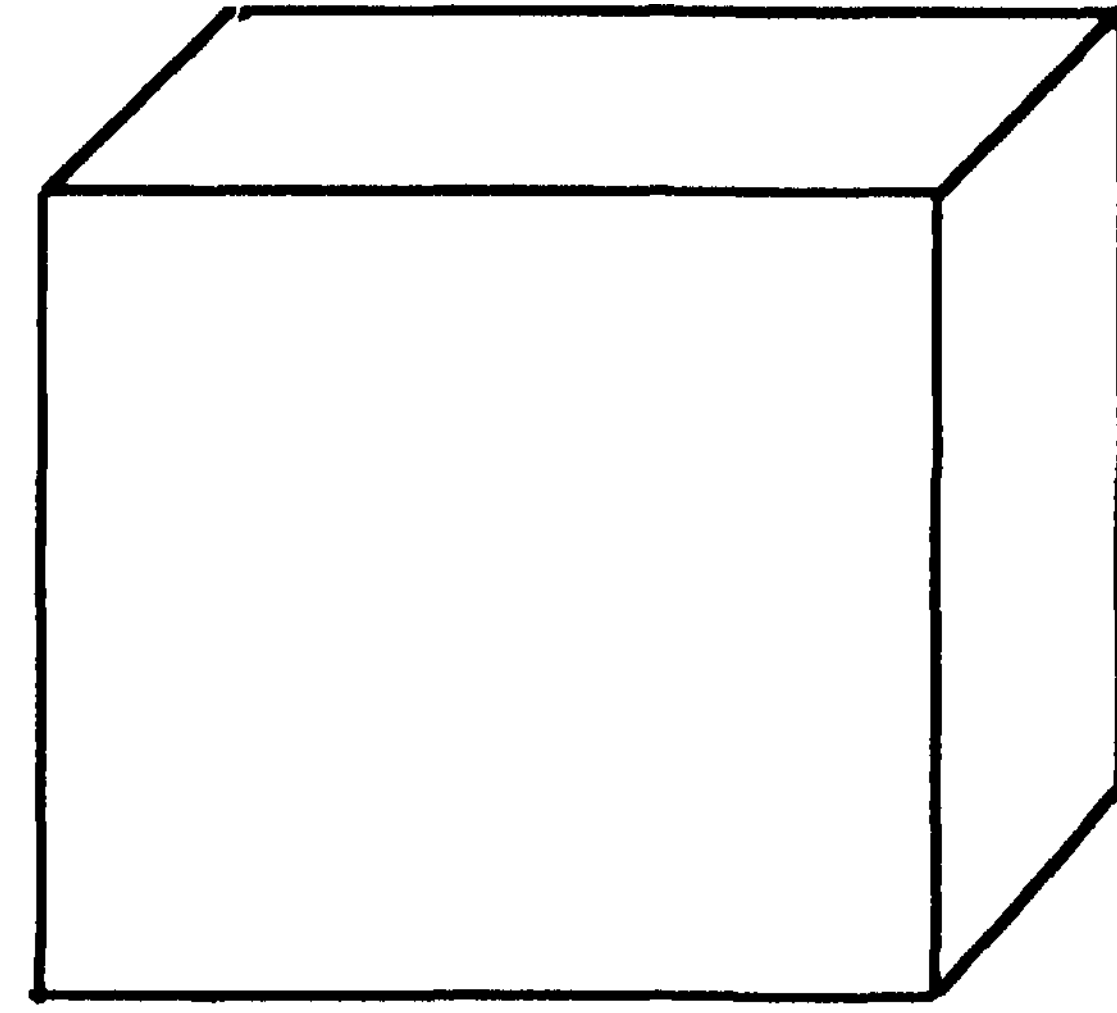

Fig. 8 Perception says, "it is a cube." Mathematics argues, "you are wrong."

The condition for a solution of cl, c2 and c3 is that all the three inner products are negative — all the three angles around the central corner are greater than 90 degrees. It is impossible that two of the three inner products are positive and the other one is negative, because otherwise one of the angles around the central corner would be greater than 180 degrees, reducing three quadrilaterals to two.

While interpreting figures at the geometrical level is strictly governed by mathematics, it must possess certain degree of flexibility at the perceptual level; otherwise it would not work on hand-drawn figures in a humanlike way [Sugihara, 198G]. Both the condition for a real object interpretation and the condition for a rectangular polyhedron interpretation derived at the geometrical level have te be changed. First, in a hand-drawn figure, it is too strict a condition that the three edges of each group, when extended, meet at a common point. Also, the parallel edges are nearly parallel. Secondly, as given in Fig. 8, even when one of the three angles around the central corner is exactly 90 degrees, the figure is still perceived as a rectangular polyhedron (the interpretation is impossible mathematically.) A feasible explanation for this fact is that the individual quadrilaterals are first interpreted separately (as rectangles) and then integrated as faces of a rectangular polyhedron in a less strict way than at the geometrical level. This kind of perception seems quite ubiquitous. When drawing a man or an animal, children usually put together a frontal view of the face and a side view of the body. But the flexibility has its limit; if one of the three angles around the central corner is more than 90 degrees, the figure is no longer perceived as a rectangular polyhedron (Fig. 9,) but an ordinary parallelepiped [Perkins, 1983; Kanade & Render, 1983].

Lastly, although the rectangular polyhedron interpretation is mathematically correct if all the three intersection points of each edge group do not approach infinity, it does not always agree with human perception, if the orthocenter of the triangle formed by the three intersection points is far away from the figure itself.

4. The Gravity Regularity

Everything, including the perceiver itself, is attracted by gravity. As a consequence, objects must be supported by something. It is usually perceived to be the ground, perpendicular to the direction of gravity, if no evidence indicates otherwise. The gravity regularity is generalized from this universal fact. Unlike the rectangularity reg-
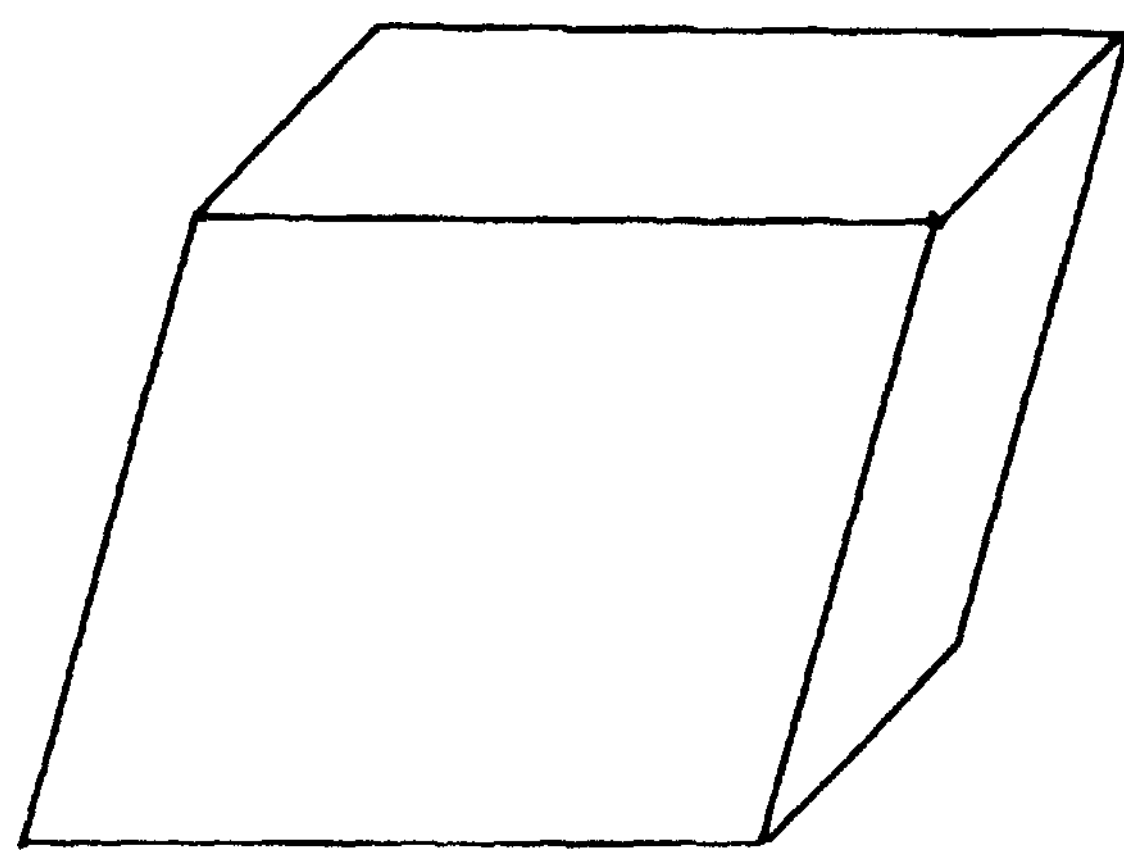
Fig. 9 If one of the three angles around the central corner is less than 90 degrees, then the figure is no longer perceived as a rectangular polyhedron.

Fig. 10 The camera-ground model

ularity, it is a natural regularity objectively existing in the external world. So far it has attracted only a little attention. Kanade *et al.* (1983) analyzes skewed symmetry under gravity. Recently, Sedgwick (1987) reports a production system that generates an interpretation of the environment based on linear perspective information and contact relations between surfaces and the ground. Tsuji *et al.* (1986) also reports a mobile robot that perceives and navigates in an indoor environment with a horizontal flat floor and objects standing vertically on the floor.

To perceive the world is, in essence, to perceive the relations among the perceiver, the ground and the objects on the ground. By introducing the ground, the relation between the perceiver and the rectangles reduces to the sum of the relation between the perceiver and the ground, and the relation between the ground and the rectangles supported by it. All these relations can be described in either a viewer-centered representation or a world-centered representation based on the ground.

Let the normal of the ground plane be expressed by n_g in the viewer-centered coordinate system. n_g actually implies the relation between the perceiver and the ground. It is not difficult to find by introspection that we usually assume the following relation to the ground. Suppose that the camera is originally so set that the optical axis of the camera and the horizontal axis of the image plane are parallel to the ground, as humans look forward while keeping two eyes horizontal. Rotate the camera around the horizontal axis of the image plane by an angle of *a* (see Fig. 10) as humans look some feet ahead on to the road. Then the normal of the ground plane is projected to be upright onto the image under orthographic projection; i.e.,

$$n_g \qquad (0,1,-\tan a), \qquad (12)$$

To keep n_g upright everywhere, which is desirable, it is necessary to assume the orthographic projection, if a / 0.

Because of the planarity of rectangle and the linearity of its sides, there exist only three kinds of contact relations between the ground and a rectangle; (1) the whole rectangle contacts the ground; (2) one of the four sides contacts the ground; and (3) only one of the four corners contacts the ground. When the whole rectangle contacts the ground, the orientation of the rectangle and that of the ground are identical. All the four sides are perpendicular to the ground. When only one side contacts the ground, that
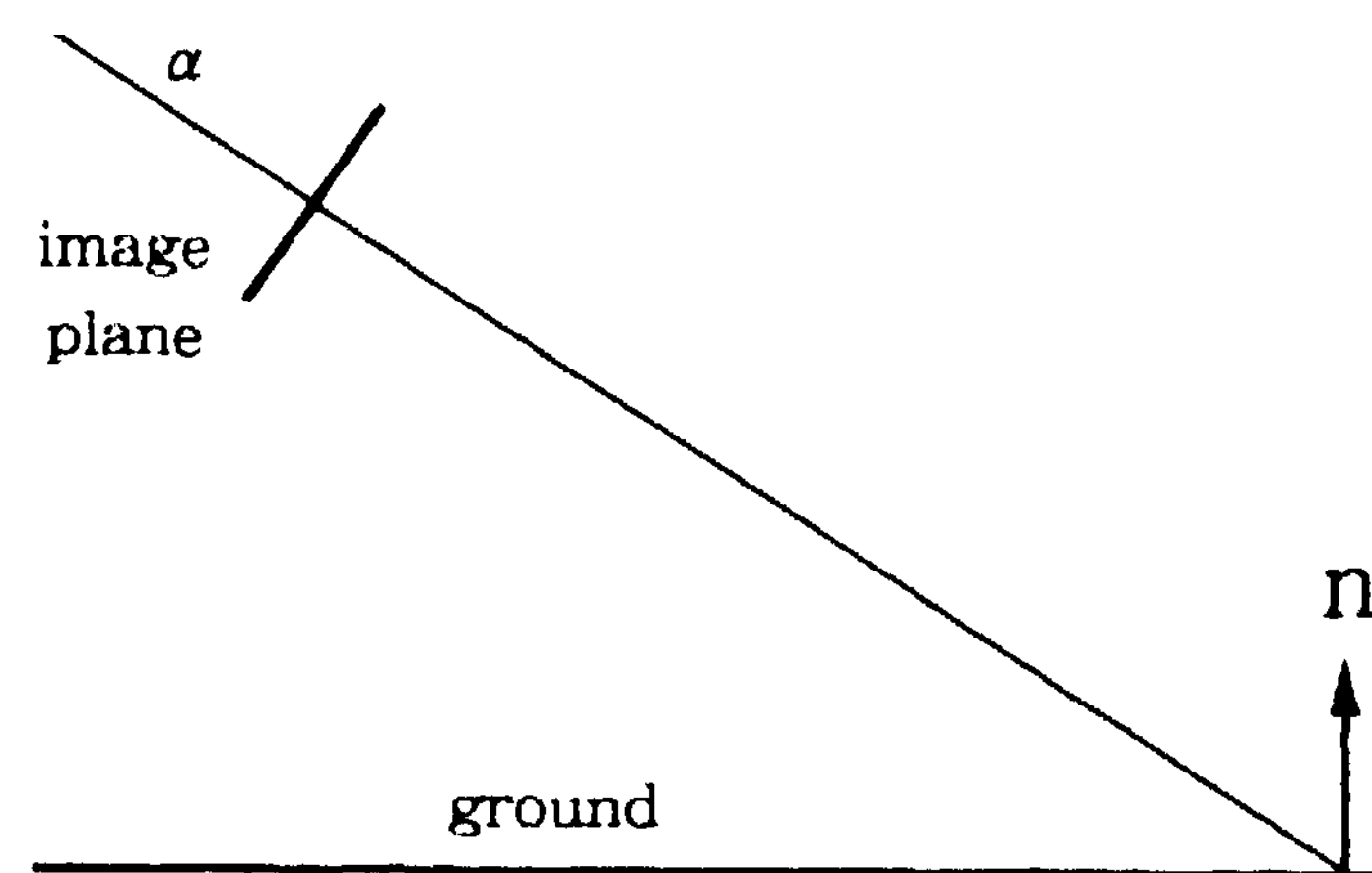
side is perpendicular to the normal of the ground. If the rectangle does not stand vertically, it is interpreted to be prevented from falling by something else behihnd it. When only one of the corners contacts the ground, it is most likely that we perceive the rectangle standing vertically.

Which contact relation is perceived is largely dependant on the assumption of the perceiver's posture. The full contact relation is perceived only if the upper and lower corners are obtuse angles much greater than 90 degrees (Fig. 11a) — i.e., the rectangle is remarkably slanted towards the sky — because we are not used to looking downward (see [Stevens, 1981] for an analysis of the relation between the image angle of two orthogonal space vectors and the orientation of their outer product.) The corner contact relation is perceived if one of the diagonal, of which the midpoint is the centroid, is vertical in image, and the upper and lower corners are acute angles (Fig. IIb)
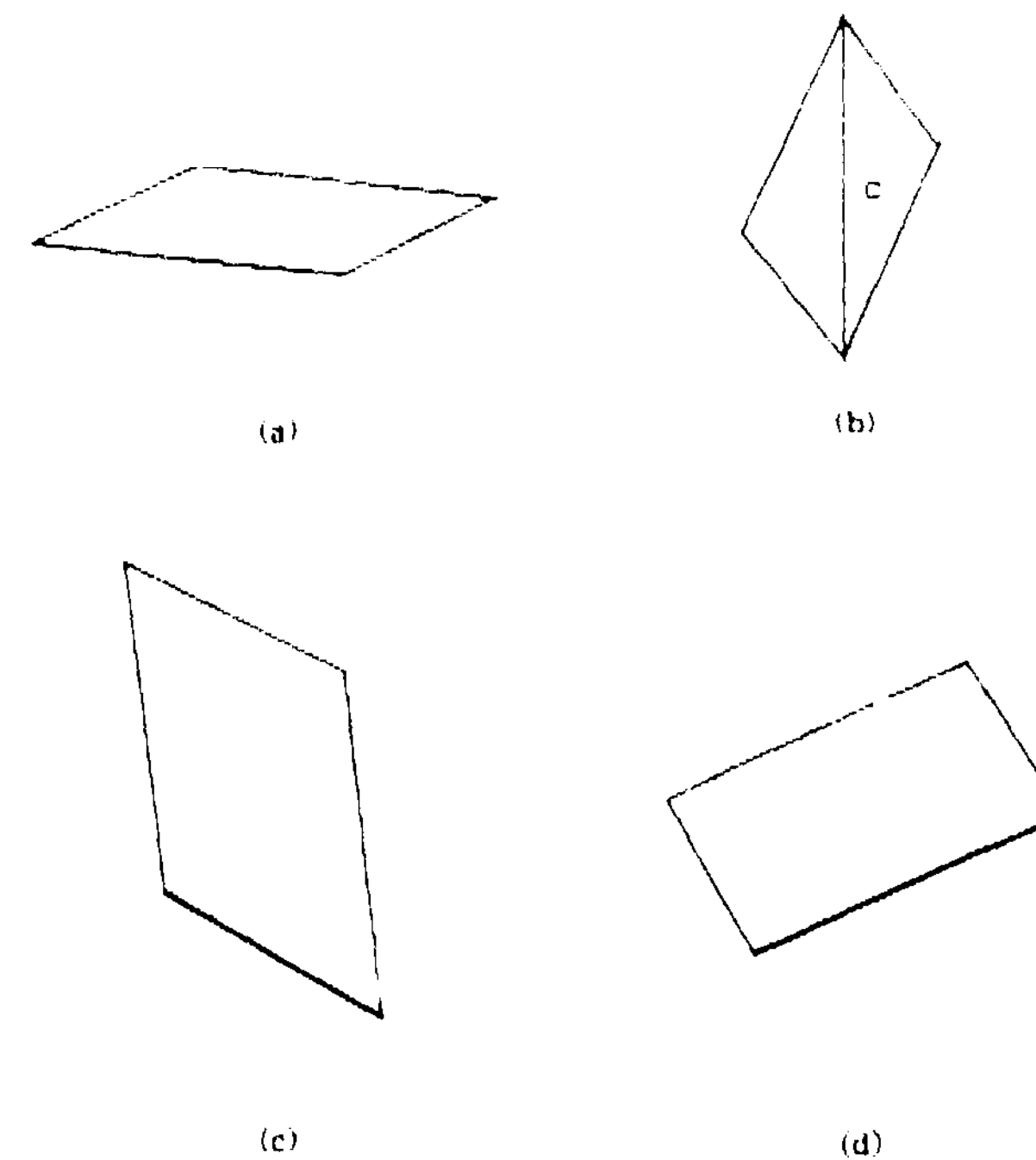

Fig. 11 (a) the full contact, (b) the corner contact, and (c) and (d) the one side contact, relations

again because of the posture assumed by the perceiver. The one side contact relation is perceived if the full contact and the one corner contact relations are not. The side that has the smaller angle to the horizontal axis is most likely perceived to contact the ground plane, because we prefer interpretations that are less slanted from the image plane (Fig. IIc,IId).

We do not intend to claim the completeness of the analysis, because perception of one contact relation is not necessarily exclusive of another and the conditions are not

completely quantified. Even so, however, if we manage somehow to quantifyt the conditions — e.g., for the full contact relation, the condition may be that the upper and lower angles are greater than 150 degrees — then we can completely determine the contact relations.

The above three contact relations can be respectively expressed as

$$\mathbf{PF} \cdot \mathbf{n_g} = 0, \quad \text{and} \quad \mathbf{QF} \cdot \mathbf{n_g} = 0; \qquad (13)$$

$$(\mathbf{PF} \times \mathbf{QF}) \cdot \mathbf{n_g} = 0; \qquad (14)$$

$$\mathbf{PF} \cdot \mathbf{n_g} = 0, \quad \text{or} \quad \mathbf{QF} \cdot \mathbf{n_g} = 0. \qquad (15)$$

Before concluding this section, we have one point to note. The orthographic projection is a necessity of the vertical-to-vertical mapping, but it at the same time does not exclude perspective projection for other purposes. The rectangles can still be projected as non-parallelograms. To perceive local shape of an object, the perspective information is required; whereas to perceive the more global relation between the perceiver and the ground, the orthography is required. It is really of interest to observe human's this flexibilty to swing between orthographic and perspective projections.

5.   Conclusions

We have proposed the rectangularity regularity to be employed in the visual interpretation of quadrilaterals. By this subjective regualrity resulting from the internal structure of human perception, quadrilaterals in image axe interpreted as rectangles in space, with the rectangle orientation and the focal point being completely determined or partially constrained. Projection is perceived altogether. As the second part, we have examined interpreting quadrilaterals as faces of a rectanuglar polyhedron at both the geometrical level and the perceptual level. Interpretations at the two levels may be fairly different. Finally we have analyzed the relations among the perceiver (camera,) the ground and the rectangles supported by the ground, and proposed the gravity regularity to derive constraints on the rectangle orientation. Through studying these seemingly simple quadrilaterals we have obtained some insights into the nature of the general problem of interpreting image contours: the rectangularity regularity is just a specific example of the perceptual system's preference of regular forms over irregular forms; while interpreting figures at the geometrical level strictly obey mathematics, interpreting figures at the perceptual level is much more flexible; and the gravity regularity can be widely applied in different forms.

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