Wireless Power Provision as a Public Good

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***Abstract*—Wireless power transfer (WPT) technology enables a cost-effective and sustainable energy supply in wireless networks, where energy users (EUs) can remotely harvest energy from the wireless signal transmitted by energy transmitters (ETs). However, the broadcast nature of wireless signal makes wireless power a *non-excludable public good*, which renders the traditional market mechanisms inefficient due to the possibility of the free-riders. In this study, we formulate the transmit power provision problem in a single-channel WPT network as a public good provision problem, aiming to maximize the social welfare of all the ET and EUs considering their private information and selfish behaviors. The considered problem also brings both economic and technical challenges in ensuring voluntary participation and distributed algorithm design. To this end, we propose a two-phase *all-or-none* procedure involving a low-complexity Power And Taxation (PAT) Nash mechanism, which ensures voluntary participation, incentive compatibility, and budget balance, and yields the socially optimal transmit power at all Nash equilibria. We further propose a distributed D-PAT Algorithm and prove its convergence by exploiting the connection between the structure of Nash equilibria and that of the optimal solutions to a related optimization problem. Finally, our simulation results validate the PAT Mechanism and the practical algorithm. We show that our design can significantly improve the social welfare compared to the benchmark market mechanism, especially when there are many and relatively comparable EUs.**

I. Introduction

A. Motivation

The far-field wireless power transfer (WPT) technology has emerged as a promising solution to supply energy to low-power wireless devices, where energy users (EUs) can harvest energy remotely from the radio frequency (RF) signals radiated by energy transmitters (ETs) over the air. For example, Powercast has developed energy receivers that can harvest 40 microwatts (μW) RF power from a distance of 10 meters, which is sufficient to power the activities of many low-power devices, such as wireless sensors and RF identification (RFID) tags [1]. Through flexibly adjusting the transmit power and time/frequency resources blocks, the WPT technology can meet the dynamically changing real-time energy demand of multiple EUs simultaneously and efficiently, by exploring EUs' heterogeneous characteristics. Therefore, WPT technology will become an important building block of commercial and industrial systems in the future [2].

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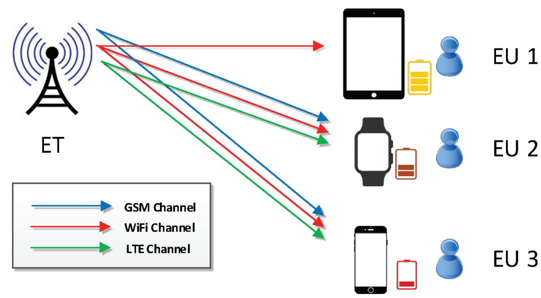

Fig. 1. An example WPT system with 1 ET, 3 EUs, and 3 channels. The broadcast power can benefit all EUs, and hence is regarded as a non-excludable public good. The EUs are heterogeneous in terms of energy consumption rates, battery status, channel conditions, and operating bands.

Fig. 1 shows an example of the WPT system, where an ET transmits power on three channels (GSM, Wi-Fi, and LTE), and each of the three EUs can harvest power on a subset of the channels. Here EUs can be heterogeneous in their channel conditions (due to different distances from the ET), energy consumption rates (due to different applications), and energy harvesting circuits (which result in different channel availabilities and energy conversion efficiencies in different channels). Due to the heterogeneous characteristics, different EUs have different energy demands and value ET's transmit power on different channels differently. For example, EU 3 is likely to have a higher energy demand than EU 2, since EU 3 is a more energy-hungry wireless device with a lower battery status.

To maximize the benefit of the WPT technology, we need to understand how an ET should choose the transmit power to balance EUs' heterogeneous power demands and the ET's operation cost. There has been much excellent prior work tackling this issue from a centralized optimization point of view (e.g., [3]–[6]), assuming that EUs are unselfish and will always truthful reveal their private information (such as the channel state information and harvested power requirements). However, in practice, EUs may have their own interests (as they may not be directly controlled by the ET) and hence may choose to misreport their private information to improve their own benefits. For example, if the ET's goal is to ensure fairness among EUs in terms of their harvested power, an EU can report a smaller channel gain in order to receive more power than he deserves. To our best knowledge, no existing work has addressed the network performance maximization problem under such a private information setting.

B. Solution Approach and Contributions

To resolve the issue of private information, it is natural to consider a decentralized market solution, where the EUs determine their demands by responding to the market price, hence indirectly reveal their private information. Such a mechanism works well in many network resource allocation problems (e.g., [7], [8]), where each user only receives benefit from the resource allocated to him. However, this may not work well in the WPT system.

More specifically, the resource in the WPT systems, the wireless power, is a *non-excludable public good* that is different from many previous considered wireless resources. Due to the broadcast nature of the wireless signal, one EU harvesting power from the wireless signal does not affect the available energy to other EUs, hence wireless power is ***non-rivalrous*** and thus a ***public good***. Furthermore, it is difficult to exclude some EUs from harvesting the energy once the wireless signal is transmitted, hence it is ***non-excludable***. Hence, some EUs may silently harvest the transmit power paid by other EUs, ending up with an inefficient wireless power provision. Such a problem does not occur in wireless communication networks with unicast transmissions (e.g. [7], [8]), because the unicast information data are *private goods*, i.e., they are excludable due to message encryption and rivalrous because the data dedicated for one user cannot benefit another.

A promising solution to efficiently provision the non-excludable public good is the *Nash mechanism implementing the Lindahl allocation*, which achieves optimum social welfare for the public good economy [9] at a *Nash equilibrium* (NE). The existing Nash implementation literature involves several mechanisms with desirable economic properties such as budget balance [10]–[15].

Nevertheless, there are still two unaddressed issues in the literature of Nash implementation for public good provision. First, the existing approaches cannot perfectly incentivize agents to voluntarily participate in the mechanism, hence cannot completely avoid the free-riding problem [17]. Second, for the constrained public good provision problem (the case we study in this paper due to the maximum transmit power constraint), there does not exist a user adaptation algorithm that is guaranteed to converge to the NE. This motivates us to propose a two-phase *all-or-none* procedure with a proper economic mechanism and a distributed algorithm to resolve these two issues.

We summarize our main contributions of this work as follows:

- *Problem Formulation:* To our best knowledge, this is the first work that addresses a wireless resource allocation problem from the perspective of non-excludable public goods. In particular, we solve the effective WPT provision problem by considering the EUs' private information and selfish behaviors.
- *Mechanism Design:* We propose a two-phase all-or-none allocation procedure and design a Power And Taxation (PAT) Mechanism. Our scheme can incentivize the EUs to voluntarily participate in the mechanism, and can achieve several desirable economic properties such as efficiency and budget balanced.
- *Distributed Algorithm Design:* We propose a distributed D-PAT Algorithm under which the decisions of ET and EUs are guaranteed to converge to the NE. We prove its convergence by mapping the NE of the induced game to the saddle point of the Lagrangian of a corresponding distributively solvable optimization problem. The proof methodology suggests a general approach of distributed algorithm design.
- *Performance Evaluation:* We show that our proposed PAT Mechanism achieves the most social welfare improvement over a benchmark mechanism when the number of EUs is large and the EUs are relatively comparable.

We organize the rest of the paper as follows. In Section II, we introduce the system model and the problem formulation. We propose the PAT Mechanism in Section III and the D-PAT Algorithm in Section IV. In Section V, we provide numerical results to validate our analysis. **In Section VI, we review the related work.** Finally, we conclude our work in Section VII.

II. System Model and Problem Formulation

In this section, we introduce the system model that captures several unique characteristics of the WPT problem. Accordingly, we formulate the public good provision problem, with an objective of social welfare maximization.

A. System Model

We consider a WPT system consisting of one ET who transmits the power to a group $\mathcal{K} = \{1, 2, ..., K\}$ of K EUs. Let $\widetilde{\mathcal{K}} = \mathcal{K} \cup \{0\}$ denote the set of both EUs and ET, or simply called *agents*, where agent 0 corresponds to the ET. For the purpose of presentation, we will refer to the ET as "she" and an EU as "he". The ET has an *omnidirectional* antenna, and broadcasts wireless energy on a narrowband channel.[1] Each EU has one energy receiver. Different EUs can experience different time-varying channel conditions due to shadowing and fading. We focus on a time period long enough such that the channel conditions are stationary to the EUs.

Cost of ET: The ET transmits at a power level of p and incurs a cost of $C(p)$, which is a positive, increasing, continuous, and strictly convex in p. The cost function can capture, for example, the energy consumption cost and the maintenance cost for the ET's operation. The transmit power p lies in the set of $\mathcal{P} = \{p : 0 \le p \le P_{\max}\}$, where $P_{\max}$ captures the limitation of the physical circuits or regulations. Both $C(p)$ and $\mathcal{P}$ are ET's private information and are not known by the EUs.

Utility of EUs: Each EU k has a utility function $U_k(h_k p)$, where h_k is EU k's long-term average channel gain and $h_k p$ is his received power. Function $U_k(h_k p)$ is strictly concave, increasing, and continuous.

[1] We leave the results on multi-channel WPT provision in the extended version of this paper. For the directional multi-antenna WPT and/or multi-ET networks, it is possible to extend our idea by further formulating a multiple public goods provisioning problem.

B. Problem Formulation

We assume that a *network regulator* operates the ET and aims to optimize the system performance.[2] Specifically, the ET is interested in choosing the transmit power p to solve the following Social Welfare Maximization (SWM) Problem

$$\text{(SWM)} \quad \max_{p} \; SW(p) \triangleq \sum_{k\in\mathcal{K}} U_k(h_k p) - C(p) \qquad (1)$$
$$\text{s.t.} \quad 0 \le p \le P_{\max}$$

The objective function of the SWM Problem is strictly concave and the constraint set is compact and convex. Hence, the SWM Problem admits a unique optimal solution.

To solve the SWM Problem in a centralized fashion, the ET needs to know the complete information of the EUs (i.e., utility functions $U_k(h_k p)$ for all k). This is difficult to achieve in practice, since EUs may not want to report their utility functions or their channel gains, as doing so may not maximize the benefits to the EUs. Hence we need to design an economic mechanism to effectively elicit such information from EUs.

C. Desirable Mechanism Properties

In this paper, we aim at designing a market mechanism that satisfies the following four desirable economic properties:

- (E1) ***Efficiency***: Maximizes the social welfare, i.e., achieves the optimal solution of the SWM Problem.
- (E2) ***Incentive Compatibility***: An EU should (directly or indirectly) *truthfully* reveal his private utility.
- (E3) ***Voluntary participation***: An EU should get a non-negative payoff by participating in the market mechanism.[3]
- (E4) (Strong) ***Budget balance***: The total payment from the EUs equals the revenue obtained by the ET. In other words, if the mechanism is administrated by a third-party, then there is no need to inject money into the system.

We will design a Nash mechanism to achieve the above properties (E1)-(E4). Note that for (E2), we focus on the indirect revelation, in the sense that the mechanism reveals EUs' marginal utility at NEs.[4]

III. A Decentralized Nash Mechanism

In this section, we first propose a two-stage *all-or-none* procedure and a PAT Nash Mechanism. We then show that the proposed scheme achieves the economic properties (E1)-(E4).

[2]Our analysis also applies to the case where the ET is a self-interested decision maker. In this case, we need to introduce a third-party network regulator to coordinate and implement the mechanism to be described in Section III.

[3]Most existing results on Nash mechanisms for public goods ignored (E3). The common assumption in these prior work is that a "government" has enough power to force all agents to participate in the mechanism, which is impractical due to the difficulty of enforcement.

[4]This is because [16] proved that there does not exist any public goods mechanism that satisfies (E1) and induces direct and truthful revelation at the same time.

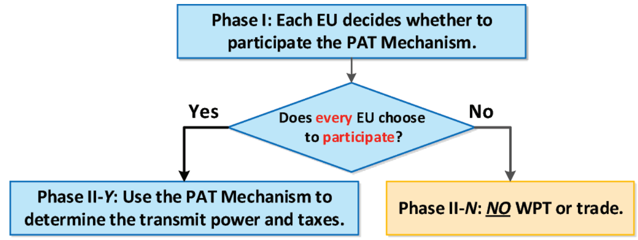

Fig. 2. The two-phase all-or-none procedure.

A. Two-Phase All-or-None Procedure

We propose a two-phase all-or-none procedure shown in Fig. 2. In Phase I, each EU sends a 1-bit message to the ET indicating whether or not to participate in the Power and Tax (PAT) mechanism (to be described later in Section III-B). In Phase II, if ***all*** agents are willing to participate, the ET and the EUs will execute the PAT Mechanism in Phase II-Y; otherwise, the ET will transmit ***no*** power and no trading occurs in Phase II-N. A key assumption of this procedure is that the ET knows the total number of EUs, K, so it knows whether some EU keeps silent without sending any indication in Phase I.[5] Hence such an all-or-none procedure can incentivize all to voluntarily participate (E3) and prevent those free-riders, i.e., those EUs who do not participate the PAT Mechanism (and pay the tax) but still benefit from wireless power contributed by others. We will prove this in Section III-C2.

B. Nash Mechanism

Next, we describe the PAT Mechanism to be executed in Phase II-Y.

Mechanism 1. *Power And Taxation (PAT) Mechanism*

- ***The message space****: Each agent $k \in \tilde{\mathcal{K}}$ sends a message $m_k \in \mathbb{R}^2$ to the ET:*

$$m_k \triangleq (\gamma_k, b_k), \qquad (2)$$

where γ_k and b_k are agent k's ***power proposal*** *and* ***price proposal****, respectively.*[6] *Note that the ET (agent 0) also needs to send a message m_0 (to herself). We denote the message profile as $\boldsymbol{m} = \{m_k\}_{k\in\tilde{\mathcal{K}}}$.*

- ***The outcome function****: The ET computes the transmit power p based on the agents' power proposals:*

$$p(\boldsymbol{m}) = \frac{1}{K+1}\sum_{k\in\tilde{\mathcal{K}}} \gamma_k. \qquad (3)$$

The ET further computes the tax rate R_k for agent $k \in \mathcal{K}$ based on the agents' price proposals:

$$R_k(\boldsymbol{m}) = b_{\omega(k+1)} - b_{\omega(k+2)}, \;\forall k \in \tilde{\mathcal{K}}, \qquad (4)$$

[5]This assumption is satisfied, for example, when all EUs also actively transmit information (e.g., in wireless sensor networks) hence can be detected by the ET [2]. Even for passive (silent) EUs, it is possible to detect their existence from the local oscillator power inadvertently leaked from their communication circuits [22].

[6]By our mechanism, it is not necessary for the ET to know the other information of EUs (e.g. battery state or the channel gain).

where $\omega(k) = \mathrm{mod}(k, K+1)$, and mod *is the modulo operator.*[7] *The ET will announce $(p(\boldsymbol{m}), R_k(\boldsymbol{m}))$ to agent k, and the agent k's* ***tax*** *(i.e., payment to the ET) is*[8]

$$t_k(\boldsymbol{m}) = R_k(\boldsymbol{m})p(\boldsymbol{m}),\ \forall k \in \tilde{\mathcal{K}}, \tag{5}$$

The key intuitions behind the PAT Mechanism are as follows. First, the determination of the transmission power p in (3) depends on every agent's power proposal. Second, agent k's tax rate in (4) does not depend on his own price proposal b_k. Finally, the agents' tax rates in (4) cancel out, i.e.,

$$\sum_{k\in\tilde{\mathcal{K}}} R_k(\boldsymbol{m}) = 0,\ \forall \boldsymbol{m} \in \mathbb{R}^{2(K+1)}, \tag{6}$$

which leads to not only the social optimal equilibrium outcome (as we will show in Theorem 1) but also the **budget balance** property (E4), i.e., $\sum_{k\in\tilde{\mathcal{K}}} t_k(\boldsymbol{m}) = 0$.

C. Properties of the PAT Mechanism

In this subsection, we will prove that our scheme achieves the economic properties of (E1)-(E4). Specifically, we will first analyze agents' decisions in Phase II, assuming that every agent chooses to participate in Phase I. Then we return to Phase I to analyze agents' participation decisions.

1) ***Phase II****:* The PAT Mechanism induces a game among agents in Phase II, which we simply refer to as the PAT Game.

Game 1. *PAT Game (Induced by the PAT Mechanism in Phase II)*

- *Players: all agents in $\tilde{\mathcal{K}}$.*
- *Strategy: $m_k \in \mathbb{R}^2$ descried in (2) for agent $k \in \tilde{\mathcal{K}}$.*
- *Payoff function $J_k(p, t_k)$: for each EU $k \in \mathcal{K}$*

$$J_k(p, t_k) = U_k(h_k p) - t_k; \tag{7}$$

for the ET (agent 0)

$$J_0(p, t_0) = \begin{cases} -C(p) - t_0, & \text{if } p \in \mathcal{P}, \\ -\infty, & \text{otherwise.} \end{cases} \tag{8}$$

Definition 1 (Nash Equilibrium (NE))**.** *An NE of the PAT Game is a message profile $\boldsymbol{m}^*$ that satisfies the following condition:*

$$J_k\left(p(\boldsymbol{m}^*), t_k\left(\boldsymbol{m}^*\right)\right) \geq J_k\left(p(m_k, \boldsymbol{m}^*_{-k}), t_k\left(m_k, \boldsymbol{m}^*_{-k}\right)\right), \quad \forall m_k \in \mathbb{R}^2,\ k \in \tilde{\mathcal{K}}, \tag{9}$$

where $\boldsymbol{m}^*_{-k} \triangleq \{m^*_l\}_{l\neq k, l\in\tilde{\mathcal{K}}}$ is the NE message profile of all other agents except agent k.

Here, we adopt the NE interpretation of [23], i.e., we interpret NE as the "stationary" messages profile of some message exchange process (to be described later in Section IV) that possesses the equilibrium property in (9).

[7] For example, when $K = 4$, we have $\omega(13) = \mod(13, 4+1) = 3$.

[8] The PAT Mechanism is motivated by the Hurwicz mechanism [10], but is considerably simpler than the one in [10], and achieves the same desirable economic properties as explained next.

We summarize the sufficient and necessary conditions for an NE in the following lemma.

Lemma 1. *A message profile $\boldsymbol{m}^* = \{(\gamma^*_k, b^*_k)\}_{k\in\tilde{\mathcal{K}}}$ is an NE if and only if the following conditions are satisfied*

$$\gamma^*_k = (K+1)\arg\max_p J_k(p, R^*_k p) - \sum_{l\neq k, l\in\tilde{\mathcal{K}}} \gamma^*_l,\ \forall k \in \tilde{\mathcal{K}}, \tag{10}$$

where $R^*_k \triangleq b^*_{\omega(k+1)} - b^*_{\omega(k+2)}$ is the NE tax rate for agent k.

To intuitively understand Lemma 1, we first rewrite (10) as follows

$$p^* = \arg\max_p J_k(p, R^*_k p),\ \forall k \in \tilde{\mathcal{K}}, \tag{11}$$

where p^* is the NE transmit power according to (3).

Equation (11) implies that under the NE tax rates $\{R^*_k\}_{k\in\tilde{\mathcal{K}}}$, the NE transmit power p^* must maximize every agent's payoff. Otherwise, there exists at least one agent k who has the incentive to adjust γ_k to change $p(\boldsymbol{m}^*)$ and improve his payoff. Hence, the NE only occurs all agents agree on the transmit power.

We can show that there are multiple NEs for the PAT Game. To see this, given any $(\boldsymbol{\gamma}^*, \boldsymbol{b}^*)$, we can increase every b^*_k by the same constant, the new message profile $(\boldsymbol{\gamma}^*, \tilde{\boldsymbol{b}}^*)$ still satisfies the conditions described in (11) and thus is also an NE. However, we can show that the NE allocation $(p^*, \boldsymbol{t}^*) = (p(\boldsymbol{m}^*), \boldsymbol{t}(\boldsymbol{m}^*))$ is unique for all NEs, and p^* corresponds to the unique optimal solution of the SWM Problem. This leads to the following existence and efficiency result.

Theorem 1 (Efficiency and Existence)**.** *There exist multiple NEs in the PAT Game, and each NE corresponds to (i) the same transmit power p^*, which is the unique optimal solution of the SWM Problem (E1); and (ii) the same NE tax profile $\boldsymbol{t}^*$.*

The proof of the existence involves constructing an NE $\boldsymbol{m}^*$ based on the optimal solution to the SWM Problem. Moreover, the proof of statement (i) in Theorem 1 involves establishing the equivalence between the conditions (10) in Lemma 1 and the KKT conditions for the SWM Problem. Finally, the proof of statement (ii) in Theorem 1 involves showing that the NE condition in (11) can lead to a unique tax rate R^*_k for each agent k.

Together with (11), Theorem 1 also implies that the PAT Mechanism is incentive-compatible (E2), since it incentivizes the EUs to submit their power proposals so as to reveal their marginal utilities at NEs. To see this, observe that the EUs submit their optimal power proposals by maximizing their payoff functions. Hence, although the EUs do not directly submit their utility functions, the eventual average of submitted power proposals is equal to the socially optimal power. Hence, the PAT Mechanism satisfies the incentive compatibility.

*2) **Phase I:*** We now move to analyze agents' decisions in Phase I. To induce all EUs' voluntary participation, we should ensure that each agent prefers the unique NE allocation outcome induced by the PAT Mechanism $(p^*, \boldsymbol{t}^*) = (p(\boldsymbol{m}^*), \boldsymbol{t}(\boldsymbol{m}^*))$ (when everyone participates in the PAT Mechanism) to the outcome of no WPT or trade (which is equivalent to the allocation outcome of $(0, \mathbf{0}^*)$). We can derive the following theorem implying each EU's voluntarily participating behavior:

Theorem 2 (Voluntary Participation)**.** *Each EU $k \in \mathcal{K}$ will choose to participate in the PAT Mechanism in Phase I.*

The intuition is that, given any other EUs' power proposals in the PAT Game, every EU k can always choose a power proposal $\gamma_k = -\sum_{i \neq k} \gamma_i$ so that the transmit power is zero due to (3) (and so is his tax due to (5)). Such a choice is equivalent to not participating in Phase I. This means that an EU k will not be worse off by participating in the PAT Mechanism, regardless of other EUs' different utility functions or participation decisions.

To summarize, we have shown that the PAT Mechanism can achieve (E1)-(E4). In the next section, we will propose a distributed algorithm under which the agents can achieve the NE of the PAT Game through an iterative message exchange.

IV. Distributed Algorithm to Achieve the NE

In the PAT Mechanism, the ET and the EUs can directly compute the NE $\boldsymbol{m}^*$ (through solving the SWM Problem) if they know the complete network information. This is not possible when considering the private information. Hence, we will propose an iterative algorithm for the ET and EUs to exchange information and show its convergence to the NE.

A. The Iterative D-PAT Algorithm

Algorithm 1 illustrates the proposed iterative D-PAT Algorithm, with the following key steps. Each agent k initializes his arbitrarily chosen message $m_k(0) \in \mathbb{R}^2$ (line 1). Then, the algorithm iteratively computes the messages until convergence. First, each EU k sends his message to the ET (line 4). Then the ET computes each agent k's tax rate $R_k(\tau)$, and sends $R_k(\tau)$ together with agents $\omega(k-1)$ and $\omega(k-2)$'s price proposals (lines 5-6) to EU k. Accordingly, each agent k updates his power proposal and his price proposal (line 7), where $[\cdot]_a^b = \max(\min(b, \cdot), a)$. Finally, the ET checks the termination criterion (line 9). The termination happens if the relative changes of agents' power proposals and price proposals are small, determined by the positive constants $\epsilon_1 > 0$ and $\epsilon_2 > 0$. The ET finally computes the transmit power and taxes (line 13).

The D-PAT Algorithm should be executed in a synchronous fashion, which can be achieved in a practical WPT network, since the ET and the EUs are often physically close-by. Moreover, the distributed algorithm has small communication complexity ($\mathcal{O}(K)$ per iteration) and computation complexity ($\mathcal{O}(1)$ for each EU and $\mathcal{O}(K)$ for the ET).

Algorithm 1: Distributed Algorithm to Reach the NE of the PAT Game (D-PAT Algorithm)

1: Initialize the iteration index $\tau \leftarrow 0$. Each agent $k \in \tilde{\mathcal{K}}$ randomly initializes $m_k(0)$ and the ET initializes the stopping criterion ϵ_1 and ϵ_2.
2: conv_flag $\leftarrow 0$; # *initialize the convergence flag*
3: **while** conv_flag $= 0$ **do**
4: Each EU k sends message $m_k(\tau)$ to the ET.
5: The ET computes the tax rate $R_k(\tau)$ for each agent k.
6: The ET sends $R_k(\tau)$, $\gamma_{\omega(k-1)}(\tau)$ and $\gamma_{\omega(k-2)}(\tau)$ to EU k, $\forall k \in \mathcal{K}$.
7: Each agent k computes $\gamma_k(\tau+1)$ and $b_k(\tau+1)$ by

$$\begin{aligned}&\gamma_k(\tau+1)\\ =&\begin{cases}[\arg\max_p \ J_k(p, R_k(\tau)p)]_0^{P_k^{\text{up}}}, & \text{if } k \in \mathcal{K}\\ [\arg\max_p \ J_k(p, R_k(\tau)p)]_0^{P_{\max}}, & \text{if } k = 0\end{cases}\end{aligned} \tag{12}$$

and

$$\begin{aligned}&b_k(\tau+1)\\ =&b_k(\tau) + 1/\sqrt{\tau}\left(\gamma_{\omega(k-1)}(\tau) - \gamma_{\omega(k-2)}(\tau)\right), \ \forall k \in \tilde{\mathcal{K}}.\end{aligned} \tag{13}$$

8: Set $\tau \leftarrow \tau + 1$.
9: **if** $|b_k(\tau) - b_k(\tau-1)| < \epsilon_1 |b_k(\tau-1)|$ and $|\gamma_k(\tau) - \gamma_k(\tau-1)| < \epsilon_2 |\gamma_k(\tau-1)|$, $\forall k \in \tilde{\mathcal{K}}$ **then**
10: conv_flag $\leftarrow 1$.
11: **end if**
12: **end while**
13: The ET computes $p(\boldsymbol{m}(\tau))$ and $\boldsymbol{t}(\boldsymbol{m}(\tau))$ using (3) and (5).

B. The Convergence of the D-PAT Algorithm

To prove the convergence of the D-PAT Algorithm, we will consider a reformulation of the SWM Problem with a decomposition structure, then demonstrate the connection between the saddle point of the Lagrangian of the reformulated problem and the NE of the PAT game. We will show that the D-PAT Algorithm converges to a saddle point and thus an NE of the PAT game.

1) Problem Reformulation: We reformulate the SWM Problem by introducing auxiliary variables $\boldsymbol{\pi} = \{\pi_k\}_{k \in \tilde{\mathcal{K}}}$, which will decouple agents' utility and cost functions:

$$(\mathrm{R-SWM}) \ \max_{\boldsymbol{\pi}} \ \sum_{k \in \mathcal{K}} U_k(h_k \pi_k) - C(\pi_0)$$

$$\text{s.t.} \qquad \pi_k = \pi_{\omega(k-1)}, \ \forall k \in \tilde{\mathcal{K}} \tag{14}$$

$$\pi_0 \in \mathcal{P}. \tag{15}$$

We can verify that the R-SWM Problem is equivalent to the SWM Problem and has a unique optimal solution.

2) Lagrangian: We relax the equality constraints (14) and define the Lagrangian of the R-SWM Problem as follows:

$$\mathcal{L}(\boldsymbol{\pi},\boldsymbol{\beta}) = \sum_{k\in\mathcal{K}} U_k(h_k\pi_k) - C(\pi_0) - \sum_{k\in\tilde{\mathcal{K}}} \beta_k \cdot \left(\pi_k - \pi_{\omega(k-1)}\right), \tag{16}$$

where β_k is the dual variable (or the *consistency price*) corresponding to the constraint $\pi_k = \pi_{\omega(k-1)}$.

3) Dual Decomposition: The Lagrangian in (16) has a nice dual decomposition structure, i.e., $\mathcal{L} = \sum_{k\in\tilde{\mathcal{K}}} \mathcal{L}_k$, where $\mathcal{L}_k$ is the decomposed Lagrangian for each agent k as follows,

$$\mathcal{L}_k(\pi_k,\boldsymbol{\beta}) = \begin{cases} U_k(h_k\pi_k) - \left(\beta_k - \beta_{\omega(k+1)}\right)\pi_k, & \text{if } k\in\mathcal{K},\\ -C(\pi_k) - \left(\beta_k - \beta_{\omega(k+1)}\right)\pi_k, & \text{if } k=0. \end{cases} \tag{17}$$

Hence the dual problem of the R-SWM Problem is

$$\min_{\boldsymbol{\beta}} \quad \max_{\boldsymbol{\pi}\in\Gamma} \sum_{k\in\tilde{\mathcal{K}}} \mathcal{L}_k(\pi_k,\boldsymbol{\beta}), \tag{18}$$

where $\Gamma \triangleq \{\boldsymbol{\pi} : \pi_0 \in \mathcal{P}\}$. We define the saddle point of $\mathcal{L}$ as a tuple $(\boldsymbol{\pi}^*,\boldsymbol{\beta}^*)$ that satisfies

$$\mathcal{L}(\boldsymbol{\pi},\boldsymbol{\beta}^*) \le \mathcal{L}(\boldsymbol{\pi}^*,\boldsymbol{\beta}^*) \le \mathcal{L}(\boldsymbol{\pi}^*,\boldsymbol{\beta}),\ \forall\boldsymbol{\pi}\in\Gamma, \boldsymbol{\beta}\in\mathbb{R}^{K+1}. \tag{19}$$

For such a saddle point, we can show that $\boldsymbol{\pi}^*$ is the unique optimal solution to the R-SWM Problem and $\boldsymbol{\beta}^*$ is the optimal solution to the dual problem in (18) [26, Chap. 5.4].[9]

4) Relation between the Saddle Point and the NE: If we set $p = \pi_k$ and $b_{\omega(k+1)} = \beta_k$, $\forall k \in \mathcal{K}$, then each EU's payoff in the PAT Game J_k becomes exactly the decomposed Lagrangian $\mathcal{L}_k$, i.e.,

$$J_k(\pi_k, (\beta_k - \beta_{\omega(k+1)})\pi_k) = \mathcal{L}_k(\pi_k,\boldsymbol{\beta}),\ \forall k\in\mathcal{K}. \tag{20}$$

We further exploit the relation between a saddle point for the Lagrangian in (16) and an NE of the PAT Game in the following theorem.

Theorem 3. *For any saddle point $(\boldsymbol{\pi}^*,\boldsymbol{\beta}^*)$ for the Lagrangian in* (16)*, the message profile $\hat{\boldsymbol{m}} = \{(\gamma_k = \pi_k^*, b_k = \beta_{\omega(k-1)}^*)\}$ is an NE of the PAT Game.*

The intuition of Theorem 3 is as follows. Lemma 1 asserts that an NE only occurs if all agents have the same payoff-maximizing transmit power, given the equilibrium tax rate R_k^*. On the other hand, we attain the optimal dual solution $\boldsymbol{\beta}^*$ only when the maximizer of the Lagrangian $\mathcal{L}(\boldsymbol{\pi},\boldsymbol{\beta}^*)$ satisfies the equality constraint in the constraint in (14). Together with the relation of J_k and $\mathcal{L}_k$ in (20), we can see that Theorem 3 holds.

The significance of Theorem 3 is two-fold. First, it provides a new interpretation of the messages of the PAT Mechanism. Specifically, the power proposal for each agent plays a role of the auxiliary variable, while the price proposal plays a role of the consistency price that pulls the auxiliary variables together. Second, Theorem 3 implies that for any distributed algorithm with provable convergence to a saddle point of the Lagrangian in (16), we can design a corresponding distributed algorithm that converges to an NE of the PAT Game.

We are ready to show the convergence of the D-PAT Algorithm in the following theorem.

Theorem 4. *The D-PAT Algorithm converges to a saddle point of the Lagrangian in* (16)*, hence an NE of the PAT Game.*

The proof of Theorem 4 involves showing that the D-PAT Algorithm is the subgradient method for solving the dual problem in (18). Its convergence is guaranteed [24], if we employ (i) a diminishing step size (line 7 in the D-PAT Algorithm) and (ii) the bounded subgradients, which are satisfied in the setting of Algorithm 1.

V. Numerical Results

In this section, we numerically evaluate the performance of the PAT Mechanism, focusing on the impacts of system parameters. We simulate the WPT operation in a time period of $T = 600$ seconds. The quadratic cost function for the ET is

$$C(p) = \sigma p^2 \cdot T, \tag{21}$$

where $\sigma = 0.5$. The ET transmits on a GSM band with a carrier frequency of 915 MHz.

We adopt the following strictly concave weighted α-fair utility functions [21] for the EUs

$$U_k(h_k p) = \frac{E_k}{B_k}\frac{(h_k p)^{1-\alpha}}{1-\alpha} \cdot T,\ \forall k\in\mathcal{K}, \tag{22}$$

where $\alpha = 0.15$, $E_k > 0$ represents the energy consumption rate for EU k, $B_k > 0$ indicates the battery state of EU k. Parameters B_k and E_k are uniformly and independently chosen from the intervals $[100, 200]$ and $[0.1, 0.7]$, respectively. The distance d_k between the ET and each EU k follows the independent and identically distributed (i.i.d.) uniform distribution from the interval $[1, r]$ (meter), where r is the cluster radius. The channel gain follows the long-term path-loss model: $h_k = 10^{-3} d_k^{-3}$. These parameters do not change during the time period of interest.

For performance comparison, we consider a benchmark mechanism where the wireless power is provided by means of "private" purchases by EUs [9]. Specifically, each EU only pays for the transmit power $x_k(\pi)$ he requests under a uniform market price π and selects his purchase to maximize his payoff; the ET chooses her transmit power $y(\pi)$ to maximize her profit taking the market price π as given; and a third party iteratively adjusts the market price π until the market is clear, i.e., $y(\pi) = \sum_{k\in\mathcal{K}} x_k(\pi)$. Such "private" benchmark cannot achieve the social optimum in general, because it ignores the public good nature of wireless power and incentivizes free-riders.

[9] There are multiple optimal dual solutions $\boldsymbol{\beta}^*$. To see this, given any saddle point of $(\boldsymbol{\pi}^*,\boldsymbol{\beta}^*)$, we can increase every β_k^* by the same constant, and the new tuple $(\boldsymbol{\pi}^*,\tilde{\boldsymbol{\beta}}^*)$ still satisfies the conditions described in (19) and thus is also a saddle point.

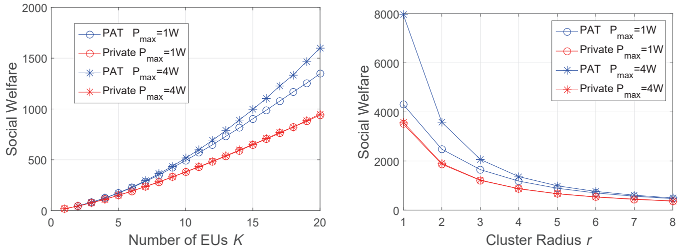

Fig. 3. (a) Impact of K on the social welfare; (b) impact of K on the EUs' average payoff; (c) impact of the cluster radius r on social welfare. The results are the average of 50000 realizations.

We first study the performance of the proposed PAT Mechanism and the benchmark mechanism, with $r = 5$m. In Fig. 3(a), we can see that the social welfare of both schemes increases in the number of EUs K. Moreover, when K becomes larger, the performance gap between $P_{\max} = 1$W and $P_{\max} = 4$W also becomes larger for the PAT Mechanism. On the other hand, the maximum power constraint has almost no impact on the benchmark mechanism. This is because as K becomes larger, a larger $P_{\max}$ can allow a larger transmit power that provides more benefits to more EUs in the PAT Mechanism. However, in the benchmark mechanism only one EU with the largest marginal utility will purchase power, hence a larger K does not significantly increase the demand. Fig. 3(a) also shows that the social welfare gap between the proposed PAT Mechanism and the benchmark private good mechanism increases in K, and the gain is around 60% when $K = 20$ and $P_{\max} = 4$W.

Fig. 3(b) shows the performance comparison of the two schemes with different cluster radius r, with $K = 15$ EUs. For both schemes, the achievable social welfare decreases when r becomes larger, since EUs experience more channel attenuation due to the larger distance. Moreover, the performance gaps between the PAT Mechanism and the benchmark mechanism also decrease in r. This is because as the cluster radius becomes larger, the diversity (difference) of EUs' utility also increases. When a single EU has a much better channel gain than the other EUs, then this single EU purchasing power alone (as in the benchmark mechanism) can achieve a social welfare close to the optimal. Hence the benefit of the PAT is the most significant when EUs are comparable.

VI. Related Work

A. Wireless Power Transfer

Most of the early studies on WPT networks focused on system optimization with unselfish users (e.g., [3]–[6]), where ETs and EUs are willing to obey the optimization outcomes and truthfully report their private information. Specifically, [3]–[6] considered the real-time wireless resource allocation in WPT networks to optimize the communication performance. To our best knowledge, there is only one recent work considering the game-theoretical analysis of the power provision problem in WPT neworks with selfish EUs [25]. Our work differs from [25] that we aim to achieve socially optimal system performance through mechanism design.

B. Mechanisms Design for Public Goods

There are several related work on Nash implementation for public goods (e.g., [10]–[15]). Specifically, in [10], [12]–[15], Hurwicz presented a Nash implementation mechanism that yields the social optimums for a public good economy, which is also individually rational and budget balanced. In [11], Sharma *et al.* studied a more general *local public goods* scenario, where the public goods may only benefit a subset of all agents.

Among these papers, only few proposed the updating processes associated with the mechanisms that converge to the NE [12]–[15], where the best response dynamics [12]–[14] and the gradient-based dynamics [15] can provably converge to the NE under some technical conditions. However, we cannot directly apply these prior algorithms to our model, since they focused on the mechanisms for unconstrained public goods provision problem.

VII. Conclusion

In this paper, we proposed a new non-excludable public good provision framework for a WPT system. We proposed a simple Nash PAT Mechanism considering agents' selfish behaviors and private information, with the desirable economic properties including efficiency, incentive compatibility, voluntary participation, and budget balance. In addition, we proposed a distributed D-PAT Algorithm that is guaranteed to converge to the NE of the PAT Mechanism. There are several directions for extending this work. One possibility is to consider multi-antenna and/or multi-ET system. Moreover, it is meaningful to study how to incentivize the voluntary participation by relaxing the assumption that the ET knows the total number of EUs. Last but not least, it is also interesting to consider hybrid information and energy transfer system.

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Appendix

A. Proof of Lemma 1

To prove the necessity of (10), suppose there exists an agent k such that $\hat{\gamma}_k = (K+1)\arg\max_p J_k(p, R_k^* p) - \sum_{l\neq k, l\in\tilde{\mathcal{K}}} \gamma_l^*$ and $\hat{\gamma}_k \neq \gamma_k^*$. In this case, agent k can always deviate and submit the power proposal $\hat{\gamma}_k$, which leads to a strictly larger payoff for agent k. This contradicts to the definition of the NE in (9). Hence the necessity is proved.

Next, we prove the sufficiency of (10). Due to (10), given other agents' NE message profile $\boldsymbol{m}_{-k}^*$, agent k cannot find another γ_k different from γ_k^* to achieve a no smaller payoff, due to the strict concavity of his payoff function. In addition, each agent's price proposal b_k does not influence his payoff.

B. Proof of Theorem 1

We first rewrite the necessary and sufficient conditions for (10) in Lemma 1 in the following:

$$\frac{1}{K+1}\sum_{l\in\tilde{\mathcal{K}}} \gamma_l^* = p^* \tag{23}$$

$$U_k'(p^*) - b_{\omega(k+1)}^* + b_{\omega(k+2)}^* = 0,\ k\in\mathcal{K} \tag{24}$$

$$-C'(p^*) - b_1^* + b_2^* - \tilde{\lambda} + \tilde{\mu} = 0, \tag{25}$$

$$\tilde{\lambda}(p^* - P_{\max}) = 0,\quad \tilde{\mu}p^* \geq 0,\quad \tilde{\mu}, p^*, \tilde{\lambda} \geq 0, \tag{26}$$

where (24) is the first-order condition for each EU's payoff maximization problem and (25)-(26) are the KKT conditions of the ET's payoff maximization problem.

We are ready to prove Theorem 1.

1) Existence: Let (p^*, λ^*, μ^*) be the solution to the KKT conditions for the SWM Problem. There always exists a message profile $(\boldsymbol{m}^* = \{(\gamma_k^*, b_k^*)\}_{k\in\tilde{\mathcal{K}}}, \tilde{\mu}^*, \tilde{\lambda}^*)$ such that $\tilde{\mu}^* = \mu^*$, $\tilde{\lambda}^* = \lambda^*$, $\gamma_k^* = p^*,\ \forall k\in\tilde{\mathcal{K}}$, and

$$b_k^* = \begin{cases} -C'(p^*) - \tilde{\lambda}^* + \tilde{\mu}^* & \text{if } k=1 \\ 0 & \text{if } k=2 \\ -\sum_{l=1}^{k-2} U_l'(p^*) & \text{if } k\geq 3 \text{ or } k=0 \end{cases}. \tag{27}$$

We observe that the message profile $(\boldsymbol{m}^* = \{(\gamma_k^*, b_k^*)\}_{k\in\tilde{\mathcal{K}}}, \tilde{\mu}^*, \tilde{\lambda}^*)$ satisfies (23)-(26), indicating that it is an NE. The existence is proved.

2) Efficiency: Combining (24)-(25), we have

$$\sum_{k\in\mathcal{K}} U_k'(p^*) - C'(p^*) - \tilde{\lambda} + \tilde{\mu} = 0. \tag{28}$$

We can find that (28) and (25)-(26) are exactly equivalent to the KKT conditions for the SWM Problem. Hence, every NE leads to the unique optimal transmit power.

3) Uniqueness of Tax Rate: The tax rate for each EU is given by $R_k^* = b_{\omega(k+1)}^* - b_{\omega(k+2)}^* = U_k'(p^*)$, which is unique due to the uniqueness of optimal transmit power. Hence, the tax rate for the ET $R_0^* = -\sum_{k\in\mathcal{K}} R_k^*$ is also unique.

C. Proof of Theorem 2

By the definition of NE and the tax in (5), we have

$$J_k(\bar{p}, R_k^*\bar{p}) \leq J_k(p^*, t_k^*),\ \forall \bar{p}\in\mathbb{R},\ \forall k\in\tilde{\mathcal{K}}, \tag{29}$$

where $\bar{p} = \frac{1}{K+1}(\gamma_k + \sum_{l\in\tilde{\mathcal{K}}} \gamma_l^*) =$ for arbitrary γ_k. Let $\bar{p}=0$, (29) further implies that

$$J_k(0,0) \leq J_k(p^*, t_k^*),\ \forall k\in\tilde{\mathcal{K}}, \tag{30}$$

which means that each agent weakly prefers the allocation (p^*, t_k^*) when everyone participates than the allocation when someone chooses not to participate $(0,0)$. Hence, each EU k always weakly prefers to participating in the PAT Mechanism, regardless of other agents' decisions.

D. Proof of Theorem 3

The KKT conditions of the R-SWM Problem are given by

$$\frac{\partial U_k}{\partial \pi_k} - \beta_k + \beta_{\omega(k+1)} = 0,\ \forall k\in\mathcal{K} \tag{31}$$

$$\frac{\partial C}{\partial \pi_0} - \beta_0 + \beta_1 - \hat{\lambda} + \hat{\mu} = 0, \tag{32}$$

$$\hat{\lambda}(\pi_0 - P_{\max}) = 0,\quad \hat{\mu}\pi_0 = 0,\quad \pi_k, \hat{\mu}, \hat{\lambda} \geq 0,\ \forall k\in\tilde{\mathcal{K}}, \tag{33}$$

Letting $\gamma_k = \pi_k^* = p^o,\ \forall k\in\tilde{\mathcal{K}}$, we show that $\{\gamma_k\}_{k\in\tilde{\mathcal{K}}}$ satisfies (23). In addition, substituting $b_{\omega(k+1)}$ into β_k^*, we have that (32)-(33) have exactly the same structure as (24)-(26), which implies that $\hat{\boldsymbol{m}} = \{(\gamma_k = \pi_k^*, b_k = \beta_{\omega(k-1)}^*)\}$ satisfies the NE conditions in SWM Problem and thus is an NE.

E. Proof of Theorem 4

Let $\gamma_k = \pi_k$ and $b_k = \beta_{\omega(k-1)}$. We observe that the D-PAT Algorithm is a dual-based subgradient method for solving R-SWM Problem. According to [30], the subgradient method converges to the optimal solution if we employ (i) a diminishing step size and (ii) the bounded subgradients. First, the step size selection in line 7 in the D-PAT Algorithm satisfies the Condition (i). Second, the updated γ_k is bounded due to (12) and hence the subgradient is bounded and satisfies the Condition (ii). Thus, we have shown the D-PAT Algorithm converges to the optimal solution to the R-SWM Problem and thus to an NE of the PAT Game by Theorem 3.