# Attribute-Based Signatures for Circuits with Optimal Parameter Size from Standard Assumptions

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**Abstract.** Attribute-based signatures (ABS) allow users to simultaneously sign messages and prove their possession of some attributes while hiding the attributes and revealing only the fact that they satisfy a public policy. In this paper, we propose a generic construction of ABS for circuits of unbounded depth and size, with optimal parameter size—meaning the lengths of public parameters, keys, and signatures are all constant. Our construction can be instantiated from various standard assumptions, including LWE and DLIN. This substantially improves the state-of-the-art ABS scheme by Boyle, Goldwasser, and Ivan (PKC 2014), which, while achieving optimal parameter size, relies on succinct non-interactive arguments of knowledge that can only be constructed from non-standard assumptions. Our generic construction is based on RAM delegations. At a high level, we leverage the fact that the circuit associated with the signature can be made public and compress it using the power of RAM delegation. This allows us to achieve an overall optimal parameter size while simultaneously hiding the user's policy.

## 1 Introduction

### 1.1 Backgrounds

Attribute-based signatures (ABS), first proposed by Maji, Prabhakaran, and Rosulek [40], allow users to simultaneously sign messages and prove their possession of some attributes while hiding the attributes and revealing only that they satisfy a public policy. In the typical scenario of using ABS, we consider two entities: a key issuing authority and signers. The authority first generates a master secret key together with some public parameter and issues a user secret key associated with the user's attribute. After receiving the user secret key, each signer can generate a signature on a message with a policy. Such a signature is publicly verifiable, and anyone can verify that a signer who generates the signature has some attributes that satisfy the policy if the verification passes. The important feature is that the signature hides the attributes used to satisfy

Table 1: Comparison of efficiency among expressive ABS schemes.

| | Policy | Param. | Sig. | Key | Assumption |
|---|---|---|---|---|---|
| BGI14 [7] | Unbounded circuits* | $O(1)$ | $O(1)$ | $O(1)$ | zk-SNARKs |
| SAH16 [47] | Unbounded circuits | $O(\|x\|)$ | $O(\|C\|)$ | $O(1)$ | pairings |
| EK18 [22] | Unbounded circuits | $O(\|x\|)$ | $O(\|C\|)$ | $O(1)$ | lattices in ROM |
| SKAH18 [48] | TM† | $O(1)$ | $O(T^2)$ | $O(\|\Gamma\|)$ | pairings |
| DDK23 [16] | TM | $O(1)$ | $O(1)$ | $O(\|x\|)$ | iO |
| LNP+24 [39] | Unbounded circuits | $O(\|x\|)$ | $O(\|C\|)$ | $O(1)$ | codes in QROM |
| Ours | Unbounded circuits | $O(1)$ | $O(1)$ | $O(1)$ | pairings or lattices |

* Unbounded circuits : Circuits of unbounded depth and size
† TM : Unbounded polynomial-time deterministic Turing machines of unbounded input length and description size
$|x|$ : Length of attributes
$|C|$ : Length of policy circuits
$T$ : Computational time of Turing machines
$|\Gamma|$ : Length of policy descriptions

the policy and any information identifying the signer. ABS draws increasing attention for its applications such as anonymous credentials [49], non-transferable access controls [38], electronic medical records [30], etc.

*Expressiveness and Efficiency.* After Maji et al. [40] introduced the notion of ABS and proposed ABS schemes for monotone span programs, earlier works [3, 12, 31, 38, 43, 44, 44, 49–51] proposed ABS for limited class of policies. The work by Sakai, Attrapadung, and Hanaoka [47], who proposed an ABS for *circuits with unbounded depth and size*, significantly broadened the class of policies. After this work, several works proposed ABS schemes for quite expressive classes of policies, including unbounded circuits [22, 39] and Turing machines [16, 48]. We summarized these schemes in Table 1. As shown in the table, no existing ABS scheme dealing with circuits or more expressive class realizes optimal parameters, i.e., constant size of the public parameters, signatures, and secret keys simultaneously. Only exception is the construction proposed by [7], but it requires succinct non-interactive arguments of knowledge (SNARKs) as a building block, whose instantiation is not known from standard assumptions. Given the state of affairs, we pose the following natural question:

*Can we construct an ABS for unbounded circuits with optimal parameter size from standard assumptions?*

### 1.2 Our Contribution

The main contribution of this paper is to propose a construction of an ABS for circuits with unbounded depths and sizes that has optimal parameter size,

answering the above question in the affirmative. As shown in Table 1, no existing scheme but BGI14 [7] does not achieve the constant lengths. Moreover, our construction can be constructed from any of the following assumptions: LWE, DLIN over pairing groups, or simultaneously assuming QR and DDH over groups without pairings. Namely, we are the first to propose such an optimal ABS from standard assumptions.

### 1.3 Technical Overview

Here, we present an overview of our construction of ABS with constant-size parameters. Our construction is generic and based on several cryptographic primitives that can be instantiated by standard assumptions.

For brevity, we consider a simpler variant of ABS, known as (message-policy) constrained signature (CS) [6,51]. One can regard CS as a variant of ABS without messages. More formally, in CS, a user signing key is associated with an attribute $x$. Given the user signing key, one can generate a signature on policy $C$ if $C(x) = 1$. As for the security, we require unforgeability, which stipulates that any PPT adversary cannot forge a signature w.r.t policy $C$ if it is only given user signing keys for $x$ such that $C(x) = 0$. We also require privacy, which stipulates that any PPT adversary cannot distinguish a signature on $C$ that is generated by a signing key for $x_0$ from that generated by a signing key for $x_1$, provided $C(x_0) = C(x_1) = 1$.

*Naïve Construction.* The challenge of constructing a succinct CS for unbounded circuits lies in realizing constant lengths of user secret keys and signatures while revealing no information other than the fact that a signer has some attribute $x$ satisfying the policy $C$, i.e., $C(x) = 1$. If we do not require the succinctness, we can construct CS very easily:

- The public parameter consists of the common reference string (CRS) of an NIZK and a verification key of a (plain) digital signature scheme. The master secret key is the signing key for the signature.
- To generate a user signing key for attribute $x$, the key issuing authority generates a signature $\sigma_x$ on $x$. The user signing key is $\sigma_x$.
- A user with attribute $x$ signs on a policy $C$ by providing a NIZK proof $\Pi$ proving that (i) it has a pair $(x, \sigma_x)$ such that $\sigma_x$ is a valid signature on $x$ and (ii) the attribute $x$ satisfies the policy $C$. Then, the user publishes the proof $\Pi$ as its signature.
- To verify the signature, we simply check whether the received proof $\Pi$ is a valid NIZK proof of the above statement, which is defined by $C$ and the public parameters.

In this naïve construction, the proof length depends on both the attribute and the policy sizes, since so is the size of the verification circuit of the NIZK statement. Therefore, our first goal is to compress the verification circuit of the underlying NIZK statement. To simplify the following discussion, we will temporarily ignore the privacy requirement for CS, thereby removing the need for NIZK.

*Compression Using RAM Delegation.* The verification circuit in the naïve construction consists of two parts: one to verify the signature $\sigma_x$ on the attribute $x$, and the other to check if the attribute $x$ satisfies the policy $C$, i.e., $C(x) = 1$. Our first insight is that the latter part can be compressed using RAM delegation [9, 15, 33–35], which allows a verifier to succinctly verify the veracity of the result of heavy computation. More formally, in RAM delegation, a prover, given a CRS $\mathsf{crs}$, a RAM machine $\mathcal{R}$, and an input $x$, produces a short proof that the RAM machine $\mathcal{R}$ takes $x$ as input and outputs $y$. The key property of RAM delegations is that anyone can check the validity of the proof *with the short digest of the input to the RAM machine*. In addition, the proof length and the verifier runtime are independent of the input size and the runtime of the RAM machine $\mathcal{R}$.[3]

Using a RAM delegation for a RAM machine $\mathcal{R}$ that takes $(x, C)$ as inputs and computes $C(x) = b \in \{0, 1\}$, we can construct a more efficient CS as follows (we use the underline for the difference from the naïve construction for clarity):

- The public parameter consists of the CRS of RAM delegation and a verification key of a (plain) digital signature scheme. The master secret key is the signing key for the signature.
- To generate a user signing key for attribute $x$, the key issuing authority generates a signature $\sigma_x$ on $x$. The user signing key is $\sigma_x$.
- To signs on a policy $C$, a signer with attribute $x$ computes a proof $\pi$ of the RAM delegation proving $C(x) = 1$. The ABS signature $\Sigma$ consists of the attribute $x$, the signature $\sigma_x$, and the RAM delegation proof $\pi$. The proof $\pi$ can be verified using a digest $\mathsf{d}$ of $(x, C)$. Recall that we do not consider privacy requirement here and $x$ and $\sigma_x$ are included in $\Sigma$ in the clear.
- To verify the ABS signature $\Sigma = (x, \sigma_x, \pi)$, we compute a digest $\mathsf{d}$ of $(x, C)$ and check if the signature $\sigma_x$ on the attribute $x$ is valid and the RAM delegation proof $\pi$ is valid using the digest $\mathsf{d}$.

Due to the usage of the RAM delegation, the size of the verification circuit for the signature is now independent of the size of the circuit $C$. However, its size still depends on the attribute $x$. In the next step, we remove this dependency by introducing additional ideas.

*First Attempt for Removing the Dependency on Attribute Size.* To remove the dependency on the attribute size, a natural idea would be to compress the attribute $x$ using a hash function. More concretely, we change the above construction so that the user signing key is replaced by the signature $\sigma_{\mathsf{d}_x}$ on a digest $\mathsf{d}_x$ of the attribute $x$. We then modify the signing algorithm to output $\Sigma = (\mathsf{d}_x, \sigma_{\mathsf{d}_x}, \pi)$, where the RAM delegation proof $\pi$ is generated for the same statement as before. As intended, the ABS signature is now compact. However, this introduces a

[3] The proof length is polylogarithmically dependent on the runtime of the RAM machine. However, since we are only interested in polynomially bounded computation, the proof length is bounded by a fixed polynomial, as $\log(\mathsf{poly}(\lambda)) < \lambda$ asymptotically.

problem with the verification algorithm: specifically, we cannot compute the digest $\mathsf{d}$ of $(x, C)$, which is required to verify $\pi$, because the verifier is not provided with $x$ in the clear, but only with $\mathsf{d}_x$.

*Solution Using Flexible RAM SNARGs.* We observe that the above problem can be resolved if the digest $\mathsf{d}$ of $(x, C)$ can be computed from $\mathsf{d}_x$ and $C$, since the verifier is given $C$ as an input. This property is satisfied by hash functions constructed by Merkle trees for example. The remaining question is whether a RAM delegation scheme exists that is compatible with Merkle trees. Fortunately, Kalai et al. [33] introduce the notion of RAM delegations, called *flexible RAM SNARGs*, where the associated hash function used to compute the digest can be chosen arbitrarily, provided it satisfies certain properties—which the Merkle tree does.

To formalize the idea, we introduce a new notion called a *Circuit SNARG*. There are two differences compared to RAM delegation. First, we consider the delegation of circuit computation rather than RAM computation. This change simplifies the exposition of our CS/ABS by using the same computational model for both the delegation and CS/ABS. The second difference is that it incorporates the flexibility in digest computation, as explained above, into the syntax. More precisely, in Circuit SNARGs, a prover can generate a succinct proof $\pi$ to show that $C(x) = 1$. Verifiers, given the digest values $\mathsf{d}_x$ and $\mathsf{d}_C$ for $x$ and $C$, along with the proof $\pi$, can then verify the validity of the proof. Circuit SNARGs can be easily obtained from the flexible SNARGs by instantiating the hash function with the Merkle tree.

*Our Final Construction.* Using the Circuit SNARGs, we can compress the verification circuit for our CS scheme. The key issuer computes a signature $\sigma_{\mathsf{d}_x}$ on the digest $\mathsf{d}_x$ of an attribute $x$ to generate a user signing key for $x$, and the signer generates the RAM delegation proof $\pi$ showing that $C(x) = 1$, indicating that the attribute $x$ satisfies the policy $C$. Now, the ABS signature is $\Sigma$ consisting of the digest $\mathsf{d}_x$, signature $\sigma_{\mathsf{d}_x}$ on it, and the proof $\pi$. Verifiers, given the ABS signature $\Sigma = (\mathsf{d}_x, \sigma_{\mathsf{d}_x}, \pi)$ with respect to the policy $C$ checks the validity of both the signature $\sigma_{\mathsf{d}_x}$ and the proof $\pi$. We observe that the size of the verification circuit is of fixed polynomial size because (i) verifying the signature $\sigma_{\mathsf{d}_x}$ on $\mathsf{d}_x$ can be done by a fixed-size circuit, as $\mathsf{d}_x$ has a fixed size, and (ii) verifying $\pi$ can also be done by a fixed-size circuit, since $\pi$ can be verified given $\mathsf{d}_x$ and $\mathsf{d}_C$, both of which have fixed sizes.

The last remaining step is making the scheme satisfy the privacy requirement, which is easily achieved by introducing NIZK again. The overview of our final CS scheme is as follows (we use the underline for the different parts from the simple construction for clarity):

- The public parameter consists of the CRS of circuit SNARG, CRS of NIZK, and a verification key of a (plain) digital signature scheme. The master secret key is the signing key for the signature.

- To generate a user signing key for attribute $x$, the key issuing authority, given some attribute $x$, computes the digest $\mathsf{d}_x$ of $x$, generates a signature $\sigma_{\mathsf{d}_x}$ on the digest. The user secret key consists of $(\mathsf{d}_x, \sigma_{\mathsf{d}_x})$.
- To sign on a policy $C$, the signer with attribute $x$ proceeds as follows:
  1. Computes the *Circuit SNARG* proof $\pi$ proving $C(x) = 1$ from $C$ and $x$;
  2. Generate a NIZK proof $\Pi$ for the following statement defined by $\mathsf{d}_C$:
     "There is a tuple $(\mathsf{d}_x, \sigma_{\mathsf{d}_x}, \pi)$ such that (i) $\sigma_{\mathsf{d}_x}$ is a valid signature on $\mathsf{d}_x$ and (ii) $\pi$ passes the verification of the *Circuit SNARG* w.r.t the digests $\mathsf{d}_x$ and $\mathsf{d}_C$."

  Then, it publishes the ABS signature $\Sigma := \Pi$.
- To verify the ABS signature $\Sigma = \Pi$, the verifier simply checks the above NIZK proof $\Pi$ w.r.t the statement above, which can be recovered from $\mathsf{d}_C$.

In the above construction, it is easy to see that lengths of the all parameters are fixed polynomial as desired. To extend the above construction of CS to ABS, we use a simulation-sound NIZK. Essentially, this change is for binding the message to the ABS signature. In addition, we make the NIZK proof-of-knowledge by adding a PKE encryption of the witness. This change allows us to reduce the unforgeability of ABS to that of the underlying (plain) signature. The formal description and security analysis of our scheme will be provided in Section 4. As a result, we obtain the following informal theorem.

**Theorem 1.1 (Informal).** *If the PKE scheme, the signature scheme, the circuit SNARG, and the NIZK are secure, then the above construction of ABS with constant-size parameters is secure.*

In particular, each building block of our construction is known to be constructed from assumptions either pairings or lattices, as described in Section 1.4. This gives us the following corollary.

**Corollary 1.1 (Informal).** *There is an ABS for unbounded circuits with constant-size parameters based either on the DLIN (on pairings) or LWE (on lattices) assumption.*

### 1.4 Related Works

*A Line of Work in ABS.* Maji et al. [40] first proposed an ABS for monotone span programs, in which each signature size depends on the policy size. Following their work, several works have studied ABS for various classes of computations including conjunction predicates [38,49], non-monotone span programs [3,44,50], threshold policies [12, 31], bounded circuits [51], and deterministic finite automata [43]. The work by Sakai et al. [47], who proposed an ABS for unbounded circuits, significantly broadened the class of policies. After this, several works realized ABS schemes for quite expressive classes of policies as follows. El Kaafarani and Katsumata [22], and Ling et al. [39] proposed an ABS for unbounded circuits, but its signatures size depends on the size of the circuits. Sakai et al. [48]

proposed an ABS for Turing machines, but its signatures size is quadratic to the running time of the Turing machines. Datta et al. [16] also proposed an ABS for Turing machines with better parameters, but its length of keys stll depends on the length of attributes.

*ABS with Additional Functionalities and Security Requirements.* In this paper, we focus on the standard ABS, but there are also some variants of ABS. For example, Escala et al. [23,39] proposed a *revocable* ABS that allows an external judge to break the anonymity of signatures. Okamoto and Takashima [45] proposed a *decentralized* ABS, in which there are multiple authorities to issue user secret keys and is no central authority. Zhang et al. [53] recently proposed a *registered* ABS that allows any user to generate their own key pairs and register them to the system. In addition to these, some works have considered traceability [18], hierarchical variants [20,21,24,27], and the universal composability [4].

*SNARGs and RAM Delegations.* Existing works [8,10,13–15,17,26,28,33,35–37,42,52] studied succinct non-interactive arguments (SNARGs) for efficiently verifying computations. Choudhuri, Jain, and Jin [15] and Kalai, Vaikuntanathan, and Zhang [37] proposed a generic compiler from SNARGs for Batch-NP computations (BARGs) with somewhere extractable succinct commitment schemes with local opening (SECOM) to RAM delegations for deterministic polynomial-time computations. Then, Kalai, Lombardi, Vaikuntanathan, and Wichs [33] bootstrapped the efficiency of RAM delegations with succinctness $\mathsf{poly}(\lambda, \log T)$ constructed from any BARG and SECOM, where $T$ is the computational time of the RAM machine. This result implies that succinct RAM delegations can be constructed from various computational assumptions since SECOM can be constructed from any of the following assumptions: LWE, QR, DDH, or DLIN [19], and BARGs can be constructed from any of the following assumptions: LWE [15], QR and DDH [32], or DLIN [52].

*Independent and Concurrent Works on Homomorphic Signatures.* Tsabary [51] showed that ABS can be directly constructed from homomorphic signatures [5, 11, 29]. Recently, in independent and concurrent works, Anthoine, Balbás, and Fiore [2] and Afshar, Cheng, and Goyal [1] respectively proposed constructions of homomorphic signatures with constant public-parameter, signature, and key sizes. It appears that we can easily obtain an ABS with optimal parameter sizes by combining these approaches. However, their constructions face two challenges: first, they achieve only *weakly-hiding*, meaning that a signature should not reveal the attribute of the signing key only for adversaries who do not know the corresponding secret keys; second, the signature size depends on the policy size. Nevertheless, these barriers could be overcome by employing NIZK and hash functions, suggesting that their approach could also be extended to construct an ABS for circuits with unbounded depth and size, achieving the same optimal parameter size as our solution.

## 2 Preliminaries

In this section, we review basic notations and formal definitions of primitives.

*Notation.* In this paper, we use the following notations. $x \leftarrow X$ denotes sampling an element $x$ from a finite set $X$ uniformly at random. $y \leftarrow \mathcal{A}(x;r)$ denotes that a probabilistic algorithm $\mathcal{A}$ outputs $y$ for an input $x$ using a randomness $r$, and we simply denote $y \leftarrow \mathcal{A}(x)$ when we need not write an internal randomness explicitly. For strings $x$ and $y$, $x||y$ denotes the concatenation of $x$ and $y$. Also, $x := y$ denotes that $x$ is defined by $y$, and $|x|$ denotes the length of $x$. $\lambda$ denotes a security parameter. A function $f(\lambda)$ is a negligible function in $\lambda$ if $f(\lambda)$ tends to 0 faster than $\frac{1}{\lambda^c}$ for every constant $c > 0$. $\mathsf{negl}(\lambda)$ denotes an unspecified negligible function. PPT stands for probabilistic polynomial time. $\emptyset$ denotes the empty set. If $n$ is a natural number, $[n]$ denotes the set of integers $\{1, \cdots, n\}$. If $x$ is a $n$ bits string, $x_i$ denotes the $i$-th bit of the string $x$ for any $i \in [n]$. If $\mathcal{O}$ is a function or an algorithm and $\mathcal{A}$ is an algorithm, $\mathcal{A}^{\mathcal{O}}$ denote that $\mathcal{A}$ has oracle access to $\mathcal{O}$.

### 2.1 Public-Key Encryption

We recall a definition of a public-key encryption (PKE) scheme.

**Definition 2.1 (Public-Key Encryption).** *A PKE scheme* $\mathsf{PKE}$ *with a plaintext space* $\mathbb{M}$ *consists of the following three PPT algorithms.*

$\mathsf{KG}(1^\lambda) \to (\mathsf{ek}, \mathsf{dk})$ *: The key generation algorithm, given a security parameter* $1^\lambda$*, outputs an encryption key* $\mathsf{ek}$ *and a decryption key* $\mathsf{dk}$*.*

$\mathsf{Enc}(\mathsf{ek}, m) \to c$ *: The encryption algorithm, given an encryption key* $\mathsf{ek}$ *and a plaintext* $m$*, outputs a ciphertext* $c$*.*

$\mathsf{Dec}(\mathsf{dk}, c) \to m$ *: The (deterministic) decryption algorithm, given a decryption key* $\mathsf{dk}$*, and a ciphertext* $c$*, outputs a plaintext* $m \in \{\bot\} \cup \mathbb{M}$*.*

*Furthermore, we require a PKE scheme to satisfy the following standard properties.*

**Correctness.** *For all* $\lambda \in \mathbb{N}$ *and* $m \in \mathbb{M}$*, we have* $\Pr[(\mathsf{ek}, \mathsf{dk}) \leftarrow \mathsf{KG}(1^\lambda) : \mathsf{Dec}(\mathsf{dk}, \mathsf{Enc}(\mathsf{ek}, m)) = m] = 1$*.*

**IND-CPA Security.** *For any PPT adversary* $\mathcal{A}$*, the following advantage is negligible:*

$$\mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{PKE},\mathcal{A}}(\lambda) := \left| \Pr \left[ \begin{array}{c} b \leftarrow \{0,1\}, \\ (\mathsf{ek}, \mathsf{dk}) \leftarrow \mathsf{KG}(1^n), \\ (m_0^*, m_1^*, \mathsf{st}) \leftarrow \mathcal{A}(\mathsf{ek}), \ : b = b' \\ c^* \leftarrow \mathsf{Enc}(\mathsf{ek}, m_b^*), \\ b' \leftarrow \mathcal{A}(c^*, \mathsf{st}) \end{array} \right] - \frac{1}{2} \right|,$$

*where* $\mathcal{A}$ *is required to output* $m_0^*$ *and* $m_1^*$ *satisfying* $|m_0^*| = |m_1^*|$*.*

### 2.2 Signature Scheme

Here we recall the definition of a signature scheme.

**Definition 2.2 (Signature).** *A signature scheme* $\mathsf{SIG}$ *with a message space* $\mathbb{M}$ *consists of the following three PPT algorithms.*

$\mathsf{KG}(1^\lambda) \to (\mathsf{vk}, \mathsf{sk})$ *: The key generation algorithm, given a security parameter* $1^\lambda$*, outputs a verification key* $\mathsf{vk}$ *and a signing key* $\mathsf{sk}$*.*

$\mathsf{Sign}(\mathsf{sk}, m) \to \sigma$ *: The signing algorithm, given a signing key* $\mathsf{sk}$ *and a message* $m$*, outputs a signature* $\sigma$*.*

$\mathsf{Ver}(\mathsf{vk}, m, \sigma) \to 1/0$ *: The (deterministic) verification algorithm, given a verification key* $\mathsf{vk}$*, a message* $m$*, and a signature* $\sigma$*, outputs either* 1 *(accept) or* 0 *(reject).*

*Furthermore, we require a signature scheme to satisfy the following properties.*

**Correctness.** *For all* $\lambda \in \mathbb{N}$ *and* $m \in \mathbb{M}$*, we have* $\Pr[(\mathsf{vk}, \mathsf{sk}) \leftarrow \mathsf{KG}(1^\lambda) : \mathsf{Ver}(\mathsf{vk}, m, \mathsf{Sign}(\mathsf{sk}, m)) = 1] = 1$.

**EUF-CMA Security.** *For any PPT adversary* $\mathcal{A}$*, the advantage defined as follows is negligible:*

$$\mathsf{Adv}^{\mathsf{euf\text{-}cma}}_{\mathsf{SIG},\mathcal{A}}(\lambda) \coloneqq \Pr\left[\begin{array}{c} L_{sig} \coloneqq \emptyset, \\ (\mathsf{vk}, \mathsf{sk}) \leftarrow \mathsf{KG}(1^\lambda), \\ (m^*, \sigma^*) \leftarrow \mathcal{A}^{\mathcal{O}_{sign}}(\mathsf{vk}) \end{array} : \begin{array}{c} \mathsf{Ver}(\mathsf{vk}, m^*, \sigma^*) = 1 \\ \wedge\ m^* \notin L_{sig} \end{array}\right],$$

*where the signing oracle* $\mathcal{O}_{sign}$ *is defined as follows:*

**Signing Oracle.** *When* $\mathcal{A}$ *accesses the signing oracle* $\mathcal{O}_{sign}$ *by making a query* $m$*, it computes* $\sigma \leftarrow \mathsf{Sign}(\mathsf{sk}, m)$*, returns* $\sigma$ *to* $\mathcal{A}$*, and appends* $m$ *to* $L_{sig}$*.*

### 2.3 Non-Interactive Zero-Knowledge Proof

We define a non-interactive zero-knowledge proof (or simply NIZK). We require NIZK to satisfy the *simulation soundness*, which is known to be constructed from standard NIZK [25].

**Definition 2.3 (NIZK Proof System).** *A non-interactive zero-knowledge (NIZK) proof system* $\mathsf{NIZK}$ *for a NP relation* $\rho \subseteq \mathcal{X} \times \mathcal{W}$ *consists of the following three PPT algorithms.*

$\mathsf{Setup}(1^\lambda) \to \mathsf{crs}$ *: The setup algorithm, given a security parameter* $1^\lambda$*, outputs a common reference string* $\mathsf{crs}$*.*

$\mathsf{Prove}(\mathsf{crs}, \mathsf{X}, \mathsf{W}) \to \pi$ *: The prove algorithm, given a common reference string* $\mathsf{crs}$ *and a pair of statement and witness* $(\mathsf{X}, \mathsf{W}) \in \rho$*, outputs a proof* $\pi$*.*

$\mathsf{Ver}(\mathsf{crs}, \mathsf{X}, \pi) \to 1/0$ *: The verify algorithm, given a common reference* $\mathsf{crs}$*, a statement* $\mathsf{X}$*, and a proof* $\pi$*, outputs either* 1 *(accept) or* 0 *(reject).*

*Furthermore, we require a NIZK proof system to satisfy the following properties.*

**Correctness.** *For all* $\lambda \in \mathbb{N}$, $(\mathsf{X}, \mathsf{W}) \in \rho$, *we have*

$$\Pr\left[\begin{array}{c}\mathsf{crs} \leftarrow \mathsf{Setup}(1^\lambda), \\ \pi \leftarrow \mathsf{Prove}(\mathsf{crs}, \mathsf{X}, \mathsf{W})\end{array} : \mathsf{Ver}(\mathsf{crs}, \mathsf{X}, \pi) = 1\right] = 1.$$

**Zero-Knowledge.** *Let* $\mathsf{Sim} = (\mathsf{Sim}_0, \mathsf{Sim}_1)$ *be a zero-knowledge simulator for* NIZK. *For any PPT adversary* $\mathcal{A}$, *the advantage defined as follows is negligible:*

$$\mathsf{Adv}^{\mathsf{zk}}_{\mathsf{NIZK},\mathcal{A}}(\lambda) := \left|\begin{array}{l}\Pr[\mathsf{crs} \leftarrow \mathsf{Setup}(1^\lambda) : \mathcal{A}^{\mathcal{P}(\mathsf{crs},\cdot,\cdot)}(\mathsf{crs}) = 1] \\ \quad - \Pr[(\mathsf{crs}, \mathsf{td}) \leftarrow \mathsf{Sim}_0(1^\lambda) : \mathcal{A}^{\mathcal{S}(\mathsf{crs},\mathsf{td},\cdot,\cdot)}(\mathsf{crs}) = 1]\end{array}\right|,$$

*where* $\mathcal{P}$ *and* $\mathcal{S}$ *are oracles that on input* $(\mathsf{X}, \mathsf{W})$ *return* $\bot$ *if* $(\mathsf{X}, \mathsf{W}) \notin \rho$ *and otherwise return* $\mathsf{Prove}(\mathsf{crs}, \mathsf{X}, \mathsf{W})$ *and* $\mathsf{Sim}_1(\mathsf{crs}, \mathsf{td}, \mathsf{X})$, *respectively.*

**Simulation soundness.** *Let* $\mathsf{Sim} = (\mathsf{Sim}_0, \mathsf{Sim}_1)$ *be a zero-knowledge simulator for* NIZK. *For any PPT adversary* $\mathcal{A}$, *the advantage defined as follows is negligible:*

$$\mathsf{Adv}^{\mathsf{sim\text{-}sound}}_{\mathsf{NIZK},\mathcal{A}}(\lambda) := \Pr\left[\begin{array}{c}L_\pi := \emptyset, \\ (\mathsf{crs}, \mathsf{td}) \leftarrow \mathsf{Sim}_0(1^\lambda), \\ (\mathsf{X}, \pi) \leftarrow \mathcal{A}^{\mathcal{S}}(\mathsf{crs})\end{array} : \begin{array}{c}\mathsf{Ver}(\mathsf{crs}, \mathsf{X}, \pi) = 1 \\ \wedge\ (\mathsf{X}, \pi) \notin L_\pi \\ \wedge\ \mathsf{X} \notin L_\rho\end{array}\right],$$

*where* $\mathcal{S}$ *is a oracle that on input* $(\mathsf{X}, \mathsf{W})$ *return* $\pi \leftarrow \mathsf{Sim}_1(\mathsf{crs}, \mathsf{td}, \mathsf{X})$ *and add* $(\mathsf{X}, \pi)$ *to* $L_\pi$, *and* $L_\rho$ *is the NP language such that* $L_\rho := \{x \mid \exists w, (x, w) \in \rho\}$.

### 2.4 Hash Family with Local Opening

Here we recall the definition of a hash family with local opening [41]. The following definition refers to previous works [8, 33].

**Definition 2.4.** *A hash tree* HT *consists of the following four PPT algorithms.*

$\mathsf{Gen}(1^\lambda) \to \mathsf{hk}$ : *The key generation algorithm, given a security parameter* $1^\lambda$, *outputs a hash key* hk.

$\mathsf{Hash}(\mathsf{hk}, x) \to \mathsf{d}$ : *The hash algorithm, given a hash key* hk *and a message* $x$, *outputs a hash value* d.

$\mathsf{Open}(\mathsf{hk}, x, i) \to (b, \pi)$ : *The opening algorithm, given a hash key* hk, *an input* $x$, *and an index* $i \in [N]$, *outputs a bit* $b$ *and an opening* $\pi$.

$\mathsf{Ver}(\mathsf{hk}, \mathsf{d}, i, b, \pi) \to 1/0$ : *The verification algorithm, given a hash key* hk, *a hash value* d, *an index* $i$, *a bit* $b$, *and an opening* $\pi$, *outputs either* 1 *(accept) or* 0 *(reject).*

*Furthermore, we require a hash family with local opening to satisfy the following properties.*

**Completeness.** *For all* $\lambda \in \mathbb{N}$*, all* $x \in \{0,1\}^{\mathsf{poly}(\lambda)}$*, and all* $i \in [|x|]$*, there exists a negligible function* $\mathsf{negl}$ *such that*

$$\Pr\left[\begin{array}{c}\mathsf{hk} \leftarrow \mathsf{Gen}(1^\lambda),\\ \mathsf{d} \leftarrow \mathsf{Hash}(\mathsf{hk}, x),\\ (b,\pi) \leftarrow \mathsf{Open}(\mathsf{hk}, x, i)\end{array} : \begin{array}{c}\mathsf{Ver}(\mathsf{hk}, \mathsf{d}, i, b, \pi) = 1,\\ \wedge\ b = x_i\end{array}\right] \geq 1 - \mathsf{negl}(\lambda).$$

**Efficiency.** *In the completeness experiment above, both the running times of* $\mathsf{Gen}$ *and* $\mathsf{Ver}$ *are at most* $\mathsf{poly}(\lambda)$*.*

**Collision Resistance w.r.t. Opening.** *For any PPT adversary* $\mathcal{A}$*, the advantage defined as follows is negligible:*

$$\mathsf{Adv}^{\mathsf{col}}_{\mathsf{HT},\mathcal{A}}(\lambda) := \Pr\left[\begin{array}{c}\mathsf{hk} \leftarrow \mathsf{Gen}(1^\lambda),\\ (\mathsf{d}, i, \pi_0, \pi_1) \leftarrow \mathcal{A}(\mathsf{hk})\end{array} : \begin{array}{c}\mathsf{Ver}(\mathsf{hk}, \mathsf{d}, i, 0, \pi_0) = 1,\\ \wedge\ \mathsf{Ver}(\mathsf{hk}, \mathsf{d}, i, 1, \pi_1) = 1\end{array}\right].$$

### 2.5 flexible RAM SNARGs

Here we recall the definition of a flexible RAM SNARG proposed by Kalai, Lombardi, Vaikuntanathan, and Wichs [33], which is a RAM delegation scheme [9, 15, 34, 35] in a specific model. In this scheme, we consider a read-only RAM machine that deterministically runs with random access to an external memory of arbitrary size. It allows us to verify whether the RAM machine accepts an input or not with a digest value of the initial external memory.

**Definition 2.5.** *A flexible RAM SNARG* $\mathsf{RamS}$ *for machine* $\mathcal{R}$ *corresponding to a hash family* $\mathsf{HT} = (\mathsf{HT.Gen}, \mathsf{HT.Hash}, \mathsf{HT.Open}, \mathsf{HT.Ver})$ *consists of the following four PPT algorithms.*

$\mathsf{Setup}(1^\lambda) \to \mathsf{crs}$ : *The setup algorithm, given a security parameter* $1^\lambda$*, outputs a common reference string* $\mathsf{crs}$*.*

$\mathsf{Dig}(\mathsf{hk}, x_{\mathsf{imp}}) \to \mathsf{d}$ : *The digest algorithm, given a hash key* $\mathsf{hk}$ *which is generated by* $\mathsf{HT.Gen}(1^\lambda)$ *and an implicit string* $x_{\mathsf{imp}}$*, outputs a digest* $\mathsf{d}$*.*

$\mathsf{Prove}(\mathsf{crs}, \mathsf{hk}, (x_{\mathsf{imp}}, x_{\mathsf{exp}})) \to (b, \pi)$ : *The prove algorithm, given a common reference string* $\mathsf{crs}$*, a hash key* $\mathsf{hk}$*, and a pair of implicit and explicit input* $(x_{\mathsf{imp}}, x_{\mathsf{exp}})$*, outputs a bit* $b$ *(indicating* $\mathcal{R}(x_{\mathsf{imp}}, x_{\mathsf{exp}})$*) and a proof* $\pi$*.*

$\mathsf{Ver}(\mathsf{crs}, \mathsf{hk}, \mathsf{d}, x_{\mathsf{exp}}, b, \pi) \to 1/0$ : *The verification algorithm, given a common reference string* $\mathsf{crs}$*, a hash key* $\mathsf{hk}$*, a digest* $\mathsf{d}$*, an explicit input* $x_{\mathsf{exp}}$*, a bit* $b$*, and a proof* $\pi$*, outputs either* $1$ *(accept) or* $0$ *(reject).*

*Furthermore, we require a flexible RAM SNARG to satisfy the following properties.*

**Completeness.** *For all* $\lambda \in \mathbb{N}$*, all RAM machines* $\mathcal{R}$*, all* $x = (x_{\mathsf{imp}}, x_{\mathsf{exp}})$ *such that* $\mathcal{R}(x)$ *accepts (i.e.,* $\mathcal{R}(x) = 1$*), there exists a negligible function* $\mathsf{negl}$ *such that*

$$\Pr\left[\begin{array}{c}\mathsf{crs} \leftarrow \mathsf{Setup}(1^\lambda),\\ \mathsf{d} \leftarrow \mathsf{Dig}(\mathsf{hk}, x_{\mathsf{imp}}),\\ (b,\pi) \leftarrow \mathsf{Prove}(\mathsf{crs}, \mathsf{hk}, x)\end{array} : \begin{array}{c}\mathsf{Ver}(\mathsf{crs}, \mathsf{hk}, \mathsf{d}, x_{\mathsf{exp}}, b, \pi) = 1\\ \wedge\ b = \mathcal{R}(x)\end{array}\right] \geq 1 - \mathsf{negl}(\lambda).$$

**Efficiency.** *In the completeness experiment above, the running time of* $\mathsf{Setup}$ *is at most* $\mathsf{poly}(\lambda, |x_{\mathsf{exp}}|, \log |x_{\mathsf{imp}}|)$*, and the length of a proof* $\pi$ *is at most* $\mathsf{poly}(\lambda, |x_{\mathsf{exp}}|, \log |x_{\mathsf{imp}}|)$*.*

**Soundness.** *For any PPT adversary* $\mathcal{A}$*, the advantage defined as follows is negligible:*

$$\mathsf{Adv}^{\mathsf{sound}}_{\mathsf{RamS},\mathcal{A}}(\lambda) := \Pr\left[\begin{array}{c} \mathsf{crs} \leftarrow \mathsf{Setup}(1^\lambda), \\ (x_{\mathsf{imp}}, x_{\mathsf{exp}}, b, \pi) \leftarrow \mathcal{A}(\mathsf{crs}), \\ \mathsf{d} \leftarrow \mathsf{Dig}(\mathsf{hk}, x_{\mathsf{imp}}) \end{array} : \begin{array}{c} \mathsf{Ver}(\mathsf{crs}, \mathsf{hk}, \mathsf{d}, x_{\mathsf{exp}}, b, \pi) = 1 \\ \wedge\ b \neq \mathcal{R}(x) \end{array}\right].$$

*Remark 2.1.* Kalai, Lombardi, Vaikuntanathan, and Wichs [33] proposed a RAM SNARG scheme that satisfy a stronger definition of soundness, *partial input soundness*, but a weaker definition defined as above is enough for our construction in the following. We also note that existing constructions [15,35] of a RAM delegation satisfy the weaker soundness.

*Remark 2.2.* In the above definition, we omit a time bound $T$ from an input to the setup algorithm since we can set $T$ as at most $2^\lambda$ in the existing constructions [15,33,35].

### 2.6 Attribute-Based Signature

Here we recall the definition of a attribute-based signature scheme from [40,46].

**Definition 2.6.** *An attribute-based signature* $\mathsf{ABS}$ *scheme consists of the following four PPT algorithms.*

$\mathsf{Setup}(1^\lambda, 1^\ell) \to (\mathsf{pp}, \mathsf{msk})$ : *The setup algorithm, given a security parameter* $1^\lambda$ *and an attribute length* $1^\ell$*, outputs a public parameter* $\mathsf{pp}$ *and a master secret key* $\mathsf{msk}$*.*

$\mathsf{KG}(\mathsf{msk}, x) \to \mathsf{sk}_x$ : *The key generation algorithm, given a master secret key* $\mathsf{msk}$ *and an attribute* $x \in \{0,1\}^\ell$*, outputs a user secret key* $\mathsf{sk}_x$*.*

$\mathsf{Sign}(\mathsf{pp}, \mathsf{sk}_x, x, C, m) \to \Sigma$ : *The signing algorithm, given a public parameter* $\mathsf{pp}$*, a user secret key* $\mathsf{sk}_x$*, an attribute* $x$*, a policy* $C$*, and a message* $m$*, outputs a signature* $\Sigma$*.*

$\mathsf{Ver}(\mathsf{pp}, C, m, \Sigma) \to 1/0$ : *The verification algorithm, given a public parameter* $\mathsf{pp}$*, a policy* $C$*, a message* $m$*, and a signature* $\Sigma$*, outputs either* $1$ *(accept) or* $0$ *(reject).*

*Furthermore, we require an attribute-based signature scheme to satisfy the following properties.*

**Correctness.** *For all* $\lambda \in \mathbb{N}$*, all* $\ell \in \mathsf{poly}(\lambda)$*, all attributes* $x$*, all policies* $C$ *satisfying* $C(x) = 1$*, and all messages* $m$*, we have*

$$\Pr\left[\begin{array}{l} (\mathsf{pp}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda, 1^\ell), \\ \quad \mathsf{sk}_x \leftarrow \mathsf{KG}(\mathsf{msk}, x), \\ \Sigma \leftarrow \mathsf{Sign}(\mathsf{pp}, \mathsf{sk}_x, x, C, m) \end{array} : \mathsf{Ver}(\mathsf{pp}, C, m, \Sigma) = 1\right] = 1.$$

**Experiment** $\mathsf{Expt}^{\mathsf{unf}}_{\mathsf{ABS},\mathcal{A}}(\lambda)$

$L_{corr}, L_{sig}, L_{key} \leftarrow \emptyset$
$(\mathsf{pp}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda, 1^\ell)$
$(C^*, m^*, \Sigma^*) \leftarrow \mathcal{A}^{\mathcal{O}_{sig}, \mathcal{O}_{corr}}(\mathsf{pp})$
**if** $\exists\ x \in L_{corr}$ s.t. $C^*(x) = 1$ **then return** 0
**if** $(C^*, m^*) \in L_{sig}$ **then return** 0
**if** $\mathsf{Ver}(\mathsf{pp}, C^*, m^*, \Sigma^*) = 1$ **then return** 1
**return** 0

**Oracle** $\mathcal{O}_{sig}(x, C, m)$

**if** $C(x) = 0$ **then return** $\perp$
$L_{sig} \leftarrow L_{sig} \cup \{(C, m)\}$
**if** $(x, \cdot) \notin L_{key}$ **then**
  $\mathsf{sk}_x \leftarrow \mathsf{KG}(\mathsf{msk}, x)$
  $L_{key} \leftarrow L_{key} \cup \{(x, \mathsf{sk}_x)\}$
**otherwise find** $(x, \mathsf{sk}_x) \in L_{key}$
$\Sigma \leftarrow \mathsf{Sign}(\mathsf{pp}, \mathsf{sk}_x, x, C, m)$
**return** $\Sigma$

**Oracle** $\mathcal{O}_{corr}(x)$

$L_{corr} \leftarrow L_{corr} \cup \{x\}$
**if** $(x, \mathsf{sk}_x) \in L_{key}$ **then return** $\mathsf{sk}_x$
$\mathsf{sk}_x \leftarrow \mathsf{KG}(\mathsf{msk}, x)$
$L_{key} \leftarrow L_{key} \cup \{(x, \mathsf{sk}_x)\}$
**return** $\mathsf{sk}_x$

Fig. 1: The experiment for defining *unforgeability* of ABS.

**Privacy.** *For any PPT adversary* $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$*, the advantage defined as follows is negligible:*

$$\mathsf{Adv}^{\mathsf{priv}}_{\mathsf{ABS},\mathcal{A}}(\lambda) := \left| \Pr\left[ \begin{array}{c} b \leftarrow \{0,1\}, \\ (\mathsf{pp}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda, 1^\ell), \\ (\mathsf{st}, x_0, x_1, C, m) \leftarrow \mathcal{A}_1(\mathsf{pp}, \mathsf{msk}), \\ \forall i \in \{0,1\}, \mathsf{sk}_i \leftarrow \mathsf{KG}(\mathsf{msk}, x_i)\ , \\ \Sigma \leftarrow \mathsf{Sign}(\mathsf{pp}, \mathsf{sk}_b, x_b, C, m), \\ b' \leftarrow \mathcal{A}_2(\mathsf{st}, \mathsf{sk}_0, \mathsf{sk}_1, \Sigma) \end{array} : b = b' \right] - \frac{1}{2} \right|,$$

*where* $\mathcal{A}_1$ *is required to output* $x_0$*,* $x_1$*, and* $C$ *satisfying* $C(x_0) = C(x_1) = 1$*.*

**Unforgeability.** *For any PPT adversary* $\mathcal{A}$*, the advantage* $\mathsf{Adv}^{\mathsf{unf}}_{\mathsf{ABS},\mathcal{A}}(\lambda) := \Pr[\mathsf{Expt}^{\mathsf{unf}}_{\mathsf{ABS},\mathcal{A}}(\lambda) = 1]$ *is negligible, where the experiment is defined in Figure 1.*

## 3 Circuit SNARGs

In this section, we introduce a new notion, *Circuit SNARG*, which will be a useful tool for describing our construction in Section 4. Intuitively, this primitive allows us to verify $C(x) = b \in \{0,1\}$ with digest values of an input $x$ and a circuit $C$, and a succinct proof $\pi$.

**Definition 3.1.** *A Circuit SNARG* $\mathsf{CirS}$ *for circuits* $\mathcal{C}$ *consists of the following five PPT algorithms.*

$\mathsf{Setup}(1^\lambda, 1^\ell) \to \mathsf{crs}$ : *The setup algorithm, given a security parameter* $1^\lambda$ *and an input length* $1^\ell$*, outputs a common reference string* $\mathsf{crs}$*.*

$\mathsf{DStr}(\mathsf{crs}, x) \to \mathsf{d}_x$ : *The (deterministic) string digest algorithm, given a common reference string* $\mathsf{crs}$ *and a string* $x \in \{0,1\}^\ell$*, outputs a string digest* $\mathsf{d}_x$*.*

$\mathsf{DCir}(\mathsf{crs}, C) \to \mathsf{d}_C$ : *The (deterministic) circuit digest algorithm, given a common reference string* $\mathsf{crs}$ *and a circuit* $C$*, outputs a circuit digest* $\mathsf{d}_C$*.*

$\mathsf{Prove}(\mathsf{crs}, x, C) \to (b, \pi)$ : *The prove algorithm, given a common reference string* $\mathsf{crs}$*, a string* $x$*, and a circuit* $C$*, outputs a bit* $b$ *a proof* $\pi$*.*

$\mathsf{Ver}(\mathsf{crs}, \mathsf{d}_x, \mathsf{d}_C, b, \pi) \to 1/0$ : *The verification algorithm, given a common reference string* $\mathsf{crs}$*, a string digest* $\mathsf{d}_x$*, a circuit digest* $\mathsf{d}_C$*, and a proof* $\pi$*, outputs either* $1$ *(accept) or* $0$ *(reject).*

*Furthermore, we require a Circuit SNARG to satisfy the following properties.*

**Completeness.** *For all* $\lambda \in \mathbb{N}$*, all strings* $x \in \{0,1\}^\ell$*, all circuits* $C$ *such that* $C(x) = b$*, we have*

$$\Pr\left[\begin{array}{c} \mathsf{crs} \leftarrow \mathsf{Setup}(1^\lambda, 1^\ell), \\ \mathsf{d}_x \leftarrow \mathsf{DStr}(\mathsf{crs}, x), \\ \mathsf{d}_C \leftarrow \mathsf{DCir}(\mathsf{crs}, C), \\ (b, \pi) \leftarrow \mathsf{Prove}(\mathsf{crs}, x, C) \end{array} : \mathsf{Ver}(\mathsf{crs}, \mathsf{d}_x, \mathsf{d}_C, b, \pi) = 1\right] = 1.$$

**Efficiency.** *In the completeness experiment above, the running time of* $\mathsf{Ver}$ *is at most* $\mathsf{poly}(\lambda, \log(|x| + |C|))$*, and the lengths of both digests* $\mathsf{d}_x$ *and* $\mathsf{d}_C$ *are* $O(\lambda)$*, and the length of a proof* $\pi$ *is at most* $\mathsf{poly}(\lambda, \log(|x| + |C|))$*.*

**Collision Resistance w.r.t. the String Digest.** *For any PPT adversary* $\mathcal{A}$*, the advantages defined as follows is negligible:*

$$\mathsf{Adv}^{\mathsf{col\text{-}str}}_{\mathsf{CirS},\mathcal{A}}(\lambda) \coloneqq \Pr\left[\begin{array}{c} \mathsf{crs} \leftarrow \mathsf{Setup}(1^\lambda, 1^\ell), \\ (x_0, x_1) \leftarrow \mathcal{A}(\mathsf{crs}), \\ \mathsf{d}_{x_0} \leftarrow \mathsf{DStr}(\mathsf{crs}, x_0), \\ \mathsf{d}_{x_1} \leftarrow \mathsf{DStr}(\mathsf{crs}, x_1) \end{array} : \mathsf{d}_{x_0} \neq \mathsf{d}_{x_1}\right].$$

**Collision Resistance w.r.t. the Circuit Digest.** *For any PPT adversary* $\mathcal{A}$*, the advantages defined as follows is negligible:*

$$\mathsf{Adv}^{\mathsf{col\text{-}cir}}_{\mathsf{CirS},\mathcal{A}}(\lambda) \coloneqq \Pr\left[\begin{array}{c} \mathsf{crs} \leftarrow \mathsf{Setup}(1^\lambda, 1^\ell), \\ (C_0, C_1) \leftarrow \mathcal{A}(\mathsf{crs}), \\ \mathsf{d}_{C_0} \leftarrow \mathsf{DCir}(\mathsf{crs}, C_0), \\ \mathsf{d}_{C_1} \leftarrow \mathsf{DCir}(\mathsf{crs}, C_1) \end{array} : \mathsf{d}_{C_0} \neq \mathsf{d}_{C_1}\right].$$

**Soundness.** *For any PPT adversary* $\mathcal{A}$*, the advantage defined as follows is negligible:*

$$\mathsf{Adv}^{\mathsf{sound}}_{\mathsf{CirS},\mathcal{A}}(\lambda) \coloneqq \Pr\left[\begin{array}{c} \mathsf{crs} \leftarrow \mathsf{Setup}(1^\lambda, 1^\ell), \\ (x^*, C^*, b^*, \pi^*) \leftarrow \mathcal{A}(\mathsf{crs}), \\ \mathsf{d}_{x^*} \leftarrow \mathsf{DStr}(\mathsf{crs}, x^*), \\ \mathsf{d}_{C^*} \leftarrow \mathsf{DCir}(\mathsf{crs}, C^*) \end{array} : \begin{array}{c} \mathsf{Ver}(\mathsf{crs}, \mathsf{d}_{x^*}, \mathsf{d}_{C^*}, b^*, \pi^*) = 1 \\ \wedge\ b^* \neq C^*(x^*) \end{array}\right].$$

### 3.1 Construction From flexible RAM SNARGs

We propose a construction of a Circuit SNARG for a circuit class $\mathcal{C}$ with input length $\ell$. Our construction is directly from a flexible RAM SNARG $\mathsf{RamS}$ corresponding to a hash family $\mathsf{HT}'$. Below we define a hash family $\mathsf{HT}'$ and a RAM machine $\mathcal{R}'$.

**Hash Family $\mathsf{HT}'$.** Let $\mathsf{HT} = (\mathsf{HT.Gen}, \mathsf{HT.Hash}, \mathsf{HT.Open}, \mathsf{HT.Ver})$ be a hash family introduced in Definition 2.4. We use a hash family $\mathsf{HT}' = (\mathsf{HT}'.\mathsf{Gen}, \mathsf{HT}'.\mathsf{Hash}, \mathsf{HT}'.\mathsf{Open}, \mathsf{HT}'.\mathsf{Ver})$ defined as follows.

$\mathsf{HT}'.\mathsf{Gen}(1^\lambda)$ : It computes $\mathsf{hk} \leftarrow \mathsf{HT.Gen}(1^\lambda)$ and outputs $\mathsf{hk}' = \mathsf{hk}$.

$\mathsf{HT}'.\mathsf{Hash}(\mathsf{hk}', x)$ : It first parses $x \in \{0,1\}^N$ as $(x_0, x_1)$, where $x_0 \in \{0,1\}^\ell$ and $x_1 \in \{0,1\}^{N-\ell}$. Then, it computes $\mathsf{d}_{x_0} \leftarrow \mathsf{HT.Hash}(\mathsf{hk}, x_0)$ and $\mathsf{d}_{x_1} \leftarrow \mathsf{HT.Hash}(\mathsf{hk}, x_1)$. Finally, it outputs $\mathsf{d} = (\mathsf{d}_{x_0}, \mathsf{d}_{x_1})$.

$\mathsf{HT}'.\mathsf{Open}(\mathsf{hk}', x, j)$ : If $j \in [\ell]$, then it sets $\rho \leftarrow \mathsf{HT.Open}(\mathsf{hk}, x_0, j)$. Otherwise, it computes $\rho \leftarrow \mathsf{HT.Open}(\mathsf{hk}, x_1, j-\ell)$. Finally, it outputs $\rho$.

$\mathsf{HT}'.\mathsf{Ver}(\mathsf{hk}', \mathsf{d}, j, b, \rho)$ : It first parses $\mathsf{d}$ as $(\mathsf{d}_{x_0}, \mathsf{d}_{x_1})$. If $j \in [\ell]$, then it outputs $\mathsf{HT.Ver}(\mathsf{hk}, \mathsf{d}_{x_0}, j, b, \rho)$. Otherwise, it outputs $\mathsf{HT.Ver}(\mathsf{hk}, \mathsf{d}_{x_1}, j, b, \rho)$.

It is easy to see that the above hash family $\mathsf{HT}'$ satisfies all properties in Definition 2.4.

**Theorem 3.1.** *If* $\mathsf{HT}$ *is a hash family with local opening, then the above* $\mathsf{HT}'$ *is a hash family with local opening.*

**flexible RAM SNARG $\mathsf{RamS}$ for $\mathcal{R}$.** We use a flexible RAM SNARG $\mathsf{RamS} = (\mathsf{RamS.Setup}, \mathsf{RamS.Dig}, \mathsf{RamS.Prove}, \mathsf{RamS.Ver})$ for a RAM machine $\mathcal{R}$, which is corresponding to the above hash family $\mathsf{HT}'$. A RAM machine $\mathcal{R}$ takes as input $(x_{\mathsf{imp}}, x_{\mathsf{exp}}) = (x||C, \perp)$ and outputs 1 if and only if $C(x) = 1$, where $x \in \{0,1\}^\ell$ and $C \in \{0,1\}^{N-\ell}$.

**Our Construction.** We show the construction of Circuit SNARG $\mathsf{CirS} = (\mathsf{CirS.Setup}, \mathsf{CirS.DStr}, \mathsf{CirS.DCir}, \mathsf{CirS.Prove}, \mathsf{CirS.Ver})$. Note that the $\mathsf{RamS.Dig}$ algorithm is deterministic and fixed by the corresponding $\mathsf{HT}'$. Let $\mathsf{hk}'$ be a hash key generated by $\mathsf{HT}'.\mathsf{Gen}(1^\lambda)$. From the construction of $\mathsf{HT}'$, two digests $(\mathsf{d}_{x_0}, \mathsf{d}_{x_1})$ are separately computable. More precisely, if a string $x_{\mathsf{imp}} \in \{0,1\}^N$ stored in the random access memory can be divided into two strings $x \in \{0,1\}^\ell$ and $C \in \{0,1\}^{N-\ell}$, then the digest value $\mathsf{d} = (\mathsf{d}_x, \mathsf{d}_C) \leftarrow \mathsf{RamS.Dig}(\mathsf{hk}', x_{\mathsf{imp}} = x||C)$ can be computed separately, i.e., there exist two algorithms $\mathsf{RamS.Dig}_1$ and $\mathsf{RamS.Dig}_2$ such that $\mathsf{d}_x \leftarrow \mathsf{RamS.Dig}_1(\mathsf{hk}', x)$ and $\mathsf{d}_C \leftarrow \mathsf{RamS.Dig}_2(\mathsf{hk}', C)$, respectively.

$\mathsf{CirS.Setup}(1^\lambda)$ : It generates a hash key $\mathsf{hk}' \leftarrow \mathsf{HT}'.\mathsf{Gen}(1^\lambda)$ and a common reference string for RAM SNARG $\mathsf{crs}_{\mathcal{R}} \leftarrow \mathsf{RamS.Setup}(1^\lambda)$ and outputs $\mathsf{crs} = (\mathsf{hk}', \mathsf{crs}_{\mathcal{R}})$.

$\mathsf{CirS.DStr}(\mathsf{crs}, x)$ : It computes $\mathsf{d}_x \leftarrow \mathsf{RamS.Dig}_1(\mathsf{hk}', x)$ and outputs $\mathsf{d}_x$.

$\mathsf{CirS.DCir}(\mathsf{crs}, C)$ : It computes $\mathsf{d}_C \leftarrow \mathsf{RamS.Dig}_2(\mathsf{hk}', C)$ and outputs $\mathsf{d}_C$.

$\mathsf{CirS.Prove}(\mathsf{crs}, x, C)$ : It generates a proof $(b, \pi) \leftarrow \mathsf{RamS.Prove}(\mathsf{crs}_{\mathcal{R}}, \mathsf{hk}', (x||C, \perp))$ using RAM SNARG and outputs $(b, \pi)$.

$\mathsf{CirS.Ver}(\mathsf{crs}, \mathsf{d}_x, \mathsf{d}_C, b, \pi) \to 1/0$ : It computes $b' \leftarrow \mathsf{RamS.Ver}(\mathsf{crs}, \mathsf{hk}', (\mathsf{d}_x, \mathsf{d}_C), \perp, b, \pi)$ and outputs $b'$.

It is easy to see that the above construction satisfies completeness and efficiency requirement if the RAM SNARG satisfies completeness and is efficient.

### 3.2 Security Analysis

In this section, we provide a security proof to show that our construction of a Circuit SNARG satisfies the collision resistance w.r.t. the circuit digest and the soundness. Although we omit a proof to show that our construction satisfies the collision resistance w.r.t. the string digest, it is easy to see that ours also satisfies it in the same way as proof of Theorem 3.2.

**Theorem 3.2.** *If the hash family* $\mathsf{HT}'$ *is collision-resistant w.r.t. opening, then the above Circuit SNARG is collision-resistant w.r.t. the circuit digest.*

*Proof.* Assume that there exists a PPT adversary $\mathcal{A}$ which breaks the collision resistance w.r.t. the circuit digest of the Circuit SNARG with non-negligible probability. Then, we can construct another PPT adversary $\mathcal{B}$ that breaks the collision-resistant w.r.t. opening of the hash family with the same probability. The description of $\mathcal{B}$ is as follows.

- $\mathcal{B}$ initially receives $\mathsf{hk}'$, computes $\mathsf{crs}_{\mathcal{R}} \leftarrow \mathsf{RamS.Setup}(1^\lambda)$, sets $\mathsf{crs} := (\mathsf{hk}', \mathsf{crs}_{\mathcal{R}})$ and runs $\mathcal{A}(\mathsf{crs})$.
- When $\mathcal{A}$ outputs $(C_0^*, C_1^*)$ and terminates, $\mathcal{B}$ finds an index $i$ such that $C_{0,i}^* \neq C_{1,i}^*$, where $C_{b,i}^*$ denotes the $i$-th bit of $C_b^*$ for $b \in \{0, 1\}$. Then, $\mathcal{B}$ computes $\mathsf{d} \leftarrow \mathsf{HT}'.\mathsf{Hash}(\mathsf{hk}', C_0^*)$, $\pi_0 \leftarrow \mathsf{HT}'.\mathsf{Open}(\mathsf{hk}', C_0^*, i)$, and $\pi_1 \leftarrow \mathsf{HT}'.\mathsf{Open}(\mathsf{hk}', C_1^*, i)$, outputs $(\mathsf{d}, i, \pi_0, \pi_1)$ if $C_{0,i}^* = 0$; otherwise, it outputs $(\mathsf{d}, i, \pi_1, \pi_0)$.

The above completes the description of $\mathcal{B}$. Since $\mathcal{A}$ breaks the collision resistance w.r.t. the circuit digest, we have $\mathsf{d} = \mathsf{HT}'.\mathsf{Hash}(\mathsf{hk}', C_0^*) = \mathsf{HT}'.\mathsf{Hash}(\mathsf{hk}', C_1^*)$. In addition, since each opening is correctly generated, we have $\mathsf{HT}'.\mathsf{Ver}(\mathsf{hk}', \mathsf{d}, i, 0, \pi_0) = 1$ and $\mathsf{HT}'.\mathsf{Ver}(\mathsf{hk}', \mathsf{d}, i, 1, \pi_1) = 1$ if $C_{0,i}^* = 0$; otherwise, we have $\mathsf{HT}'.\mathsf{Ver}(\mathsf{hk}', \mathsf{d}, i, 0, \pi_1) = 1$ and $\mathsf{HT}'.\mathsf{Ver}(\mathsf{hk}', \mathsf{d}, i, 1, \pi_0) = 1$. In both cases, $\mathcal{B}$ breaks the collision-resistant w.r.t. opening of the hash family. Therefore, we have $\mathsf{Adv}^{\mathsf{col\text{-}cir}}_{\mathsf{CirS},\mathcal{A}}(\lambda) = \mathsf{Adv}^{\mathsf{col}}_{\mathsf{HT}',\mathcal{B}}(\lambda)$. □ (**Theorem 3.2**)

**Theorem 3.3.** *If the flexible RAM SNARG* $\mathsf{RamS}$ *is sound, then the above Circuit SNARG is sound.*

*Proof.* Assume that there exists a PPT adversary $\mathcal{A}$ which breaks the soundness of the Circuit SNARG with non-negligible probability. Then, we can construct another PPT adversary $\mathcal{B}$ that breaks the soundness of the flexible RAM SNARG with the same probability. Let us fix a hash key $\mathsf{hk}'$ generated by $\mathsf{HT}'.\mathsf{Gen}(1^\lambda)$ corresponding to the flexible RAM SNARG. The description of $\mathcal{B}$ is as follows.

- $\mathcal{B}$ initially receives $\mathsf{crs}_{\mathcal{R}}$, sets $\mathsf{crs} := (\mathsf{hk}', \mathsf{crs}_{\mathcal{R}})$ and runs $\mathcal{A}(\mathsf{crs})$.
- When $\mathcal{A}$ outputs $(x^*, C^*, b^*, \pi^*)$ and terminates, $\mathcal{B}$ sets $x^*_{\mathsf{imp}} = x^*||C^*$ and $x^*_{\mathsf{exp}} = \bot$, outputs $((x^*_{\mathsf{imp}}, x^*_{\mathsf{exp}}), b^*, \pi^*)$, and terminates.

The above completes the description of $\mathcal{B}$. Let $\mathsf{d}_{x^*} = \mathsf{RamS.Dig}_1(\mathsf{hk}', x^*)$ and $\mathsf{d}_{C^*} = \mathsf{RamS.Dig}_2(\mathsf{hk}', C^*)$. Since $\mathcal{A}$ breaks the soundness of the Circuit SNARG, we have $\mathsf{CirS.Ver}(\mathsf{crs}, \mathsf{d}_{x^*}, \mathsf{d}_{C^*}, b^*, \pi^*) = 1$ and $b^* \neq C^*(x^*)$. Thus, we now have $b^* = \mathsf{RamS.Ver}(\mathsf{crs}_{\mathcal{R}}, \mathsf{hk}', (\mathsf{d}_{x^*}, \mathsf{d}_{C^*}), \bot, b^*, \pi^*)$ while $b^* \neq \mathcal{R}(x^*||C^*, \bot)$ due to the definition of the RAM machine $\mathcal{R}$. Therefore, we have $\mathsf{Adv}^{\mathsf{sound}}_{\mathsf{CirS},\mathcal{A}}(\lambda) = \mathsf{Adv}^{\mathsf{sound}}_{\mathsf{RamS},\mathcal{B}}(\lambda)$. □ (**Theorem 3.3**)

## 4 Attribute-Based Signatures for General Circuits from Circuit Delegations

In this section, we provide a construction of an attribute-based signature scheme for every polynomial-size circuits with input length $\ell$. Before showing our detailed construction, we provide an intuition of the construction. To issue a user signing key, the key issuer computes a digest $\mathsf{d}_x$ of the user's attribute $x$ and signs the digest. Each user receives a signing key that consists of an attribute digest $\mathsf{d}_x$ and its signature $\sigma_{\mathsf{d}_x}$ and generates a proof $\pi$ to show that his attribute satisfies some policy $C$. In addition, for completing security proof, we require each user to encrypt a witness consisting of a message $m$ to be signed, the digest $\mathsf{d}_x$, its signature $\sigma_{\mathsf{d}_x}$, and the proof $\pi$, and include a calculated ciphertext to a signature. It also computes a NIZK proof to show that it has (i) an attribute $x$ such that $C(x) = 1$, (ii) a valid signature of its digest value, and (iii) a ciphertext of the witness.

### 4.1 Our Construction

Here we provide our construction based on following building blocks:

- a PKE scheme $\mathsf{PKE} = (\mathsf{PKE.KG}, \mathsf{PKE.Enc}, \mathsf{PKE.Dec})$;
- a signature scheme $\mathsf{SIG} = (\mathsf{SIG.KG}, \mathsf{SIG.Sign}, \mathsf{SIG.Ver})$;
- a Circuit SNARG $\mathsf{CirS} = (\mathsf{CirS.Setup}, \mathsf{CirS.DStr}, \mathsf{CirS.DCir}, \mathsf{CirS.Prove}, \mathsf{CirS.Ver})$;
- a NIZK $\mathsf{NIZK} = (\mathsf{NIZK.Setup}, \mathsf{NIZK.Prove}, \mathsf{NIZK.Ver})$ where the NIZK relation $\rho$ is defined as follows:

$$\rho := \{((\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_C, m, \mathsf{ctx}), (\mathsf{d}_x, \sigma_{\mathsf{d}_x}, \pi, \mathsf{rand})) \mid$$

$$\begin{aligned}&\mathsf{CirS.Ver}(\mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_x, \mathsf{d}_C, 1, \pi) = 1\\ &\wedge\ \mathsf{SIG.Ver}(\mathsf{vk}, \mathsf{d}_x, \sigma_{\mathsf{d}_x}) = 1\\ &\wedge\ \mathsf{PKE.Enc}(\mathsf{ek}, (m, \mathsf{d}_x, \sigma_{\mathsf{d}_x}, \pi); \mathsf{rand}) = \mathsf{ctx}\}.\end{aligned}$$

**Our Construction.** Our construction of $\mathsf{ABS} = (\mathsf{ABS.Setup}, \mathsf{ABS.KG}, \mathsf{ABS.Sign}, \mathsf{ABS.Ver})$ is as follows.

$\mathsf{ABS.Setup}(1^\lambda, 1^\ell)$ : It computes the followings:

- a key pair for PKE $(\mathsf{ek}, \mathsf{dk}) \leftarrow \mathsf{PKE}(1^\lambda)$,
- a key pair for signature $(\mathsf{sk}, \mathsf{vk}) \leftarrow \mathsf{SIG.KG}(1^\lambda)$,
- a common reference string for NIZK $\mathsf{crs}_{\mathsf{NIZK}} \leftarrow \mathsf{NIZK.Setup}(1^\lambda)$, and
- a common reference string for Circuit SNARG $\mathsf{crs}_{\mathsf{CirS}} \leftarrow \mathsf{CirS.Setup}(1^\lambda, 1^\ell)$.

Then, it outputs $\mathsf{pp} := (\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{NIZK}}, \mathsf{crs}_{\mathsf{CirS}})$ and $\mathsf{msk} := (\mathsf{sk}, \mathsf{crs}_{\mathsf{CirS}})$.

$\mathsf{ABS.KG}(\mathsf{msk}, x)$ : It computes in the following steps:

1. Parse $\mathsf{msk} = (\mathsf{sk}, \mathsf{crs}_{\mathsf{CirS}})$;
2. Compute $\mathsf{d}_x \leftarrow \mathsf{CirS.DStr}(\mathsf{crs}_{\mathsf{CirS}}, x)$ and $\sigma_{\mathsf{d}_x} \leftarrow \mathsf{SIG.Sign}(\mathsf{sk}, \mathsf{d}_x)$.

Then, it outputs $\mathsf{sk}_x := (\mathsf{d}_x, \sigma_{\mathsf{d}_x})$.

$\mathsf{ABS.Sign}(\mathsf{pp}, \mathsf{sk}_x, x, C, m)$ : It computes in the following steps:

1. Parse $\mathsf{pp} = (\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{NIZK}}, \mathsf{crs}_{\mathsf{CirS}})$ and $\mathsf{sk}_x = (\mathsf{d}_x, \sigma_{\mathsf{d}_x})$;
2. Compute $\mathsf{d}_C \leftarrow \mathsf{CirS.DCir}(\mathsf{crs}_{\mathsf{CirS}}, C)$ and $(b, \pi) \leftarrow \mathsf{CirS.Prove}(\mathsf{crs}_{\mathsf{CirS}}, x, C)$;
3. Randomly choose $\mathsf{rand} \leftarrow \{0,1\}^{\mathsf{poly}(\lambda)}$ and compute $\mathsf{ctx} \leftarrow \mathsf{PKE.Enc}(\mathsf{ek}, (m, \mathsf{d}_x, \sigma_{\mathsf{d}_x}, \pi); \mathsf{rand})$;
4. Compute $\Pi \leftarrow \mathsf{NIZK.Prove}(\mathsf{crs}_{\mathsf{NIZK}}, (\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_C, m, \mathsf{ctx}), (\mathsf{d}_x, \sigma_{\mathsf{d}_x}, \pi, \mathsf{rand}))$.

Finally, it outputs $\Sigma := (\mathsf{ctx}, \Pi)$.

$\mathsf{ABS.Ver}(\mathsf{pp}, C, m, \Sigma) \to 1/0$ : It computes in the following steps:

1. Parse $\mathsf{pp} = (\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{NIZK}}, \mathsf{crs}_{\mathsf{CirS}})$ and $\Sigma = (\mathsf{ctx}, \Pi)$;
2. Compute $\mathsf{d}_C \leftarrow \mathsf{CirS.DCir}(\mathsf{crs}_{\mathsf{CirS}}, C)$ and $b \leftarrow \mathsf{NIZK.Ver}(\mathsf{crs}_{\mathsf{NIZK}}, (\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_C, m, \mathsf{ctx}), \Pi)$.

Finally, it outputs 1 if $b = 1$; otherwise, it outputs 0.

It is easy to see that the above construction satisfies the correctness if each building block is correct.

**Efficiency.** We show that the above construction achieves optimal parameter sizes as follows: if the underlying Circuit SNARG is efficient,

- the length of the public parameter is $\mathsf{poly}(\lambda, \log(|x| + |C|))$;
- the length of the key is $\mathsf{poly}(\lambda)$ since $|\mathsf{d}_x| = O(\lambda)$;
- the length of the signature is $\mathsf{poly}(\lambda, \log(|x| + |C|))$ since we have $|\mathsf{d}_x|, |\mathsf{d}_C| = O(\lambda)$ and $|\pi| = \mathsf{poly}(\lambda, \log(|x| + |C|))$, so the verification circuit of NIZK and its proof are of lengths $\mathsf{poly}(\lambda, \log(|x| + |C|))$.

### 4.2 Security Analysis

Here we provide security proofs to show that our construction satisfies the privacy and unforgeability in Theorem 4.1 and Theorem 4.2, respectively. In the following proofs of theorems and lemmata, we will use the underline to explicitly show the parts where each reduction accesses to its challenge oracle for clarity.

**Theorem 4.1.** *If the PKE scheme* $\mathsf{PKE}$ *is IND-CPA secure and the NIZK proof system* $\mathsf{NIZK}$ *is zero-knowledge, then the above ABS scheme is private.*

*Proof.* Let us fix a PPT adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ attacking the privacy of the $\mathsf{ABS}$, the security parameter $\lambda$, and the attribute length $\ell$. The attack game used to define the privacy is in Definition 2.6. We define two games as follows:

$\mathsf{Game}_0$ : This game is the original attack game.

$\mathsf{Game}_1$ : This game is the game identical with $\mathsf{Game}_0$ except that we use modified algorithms, in which some steps are replaced as follows:

- In the $\mathsf{ABS.Setup}(1^\lambda, 1^\ell)$ algorithm, $\mathsf{crs}_{\mathsf{NIZK}} \leftarrow \mathsf{NIZK.Setup}(1^\lambda)$ is replaced by $(\widetilde{\mathsf{crs}}, \mathsf{td}) \leftarrow \mathsf{Sim}_0(1^\lambda)$.
- In the $\mathsf{ABS.Sign}(\mathsf{pp}, \mathsf{sk} = (\mathsf{d}_x, \sigma), x, C, m)$ algorithm, a NIZK proof $\Pi \leftarrow \mathsf{NIZK.Prove}(\mathsf{crs}_{\mathsf{NIZK}}, (\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_C, m, \mathsf{ctx}), (\mathsf{d}_x, \sigma, \pi, \mathsf{rand}))$ is replaced by $\widetilde{\Pi} \leftarrow \mathsf{Sim}_1(\widetilde{\mathsf{crs}}, \mathsf{td}, (\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_C, m, \mathsf{ctx}))$.

For $i = 0, 1$, let $T_i$ be the event that $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ wins the privacy experiment in the game $\mathsf{Game}_i$. We will show that the probability $|\Pr[T_0] - \Pr[T_1]| \leq \mathsf{negl}(\lambda)$ and $\Pr[T_1] \leq \mathsf{negl}(\lambda)$ in the following lemmas.

We first show to have $|\Pr[T_0] - \Pr[T_1]| \leq \mathsf{negl}(\lambda)$. Intuitively, any difference between these two games $\mathsf{Game}_0$ and $\mathsf{Game}_1$ yields a PPT algorithm that distinguishes the real proof from the simulated proof.

**Lemma 4.1.** *There exists a PPT algorithm* $\mathcal{B}$ *such that*

$$|\Pr[T_0] - \Pr[T_1]| = \mathsf{Adv}^{\mathsf{zk}}_{\mathsf{NIZK},\mathcal{B}}(\lambda).$$

*Proof.* Assume that there exists a PPT adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ which makes the probability $|\Pr[T_0] - \Pr[T_1]|$ non-negligible. Then, we can construct another PPT adversary $\mathcal{B}$ that breaks the zero-knowledge property of $\Pi$ with non-negligible probability, which implies the lemma. The description of $\mathcal{B}$ is as follows.

1. $\mathcal{B}$ initially receives $\mathsf{crs}$, computes $(\mathsf{ek}, \mathsf{dk}) \leftarrow \mathsf{PKE}(1^\lambda)$, $(\mathsf{sk}, \mathsf{vk}) \leftarrow \mathsf{SIG.KG}(1^\lambda)$, and $\mathsf{crs}_{\mathsf{CirS}} \leftarrow \mathsf{CirS.Setup}(1^\lambda, 1^\ell)$, sets $\mathsf{pp} := (\mathsf{ek}, \mathsf{vk}, \mathsf{crs}, \mathsf{crs}_{\mathsf{CirS}})$ and $\mathsf{msk} := (\mathsf{sk}, \mathsf{crs}_{\mathsf{CirS}})$, and runs $\mathcal{A}_1(\mathsf{pp}, \mathsf{msk})$.
2. When $\mathcal{A}_1$ outputs $(\mathsf{st}, x_0, x_1, C, m)$ and terminates, $\mathcal{B}$ randomly chooses $b \leftarrow \{0, 1\}$ and proceeds as follows, where $\star$ denotes that some value exists but is being ignored:
   (i) Compute $\mathsf{d}_{x_i} \leftarrow \mathsf{CirS.DStr}(\mathsf{crs}_{\mathsf{CirS}}, x_i)$ for both $i \in \{0, 1\}$, $\mathsf{d}_C \leftarrow \mathsf{CirS.DCir}(\mathsf{crs}_{\mathsf{CirS}}, C)$, and $(\star, \pi_b) \leftarrow \mathsf{CirS.Prove}(\mathsf{crs}_{\mathsf{CirS}}, x_b, C)$;

(ii) Compute $\sigma_{\mathsf{d}_{x_i}} \leftarrow \mathsf{SIG.Sign}(\mathsf{sk}, \mathsf{d}_{x_i})$ for both $i \in \{0,1\}$;

(iii) Randomly choose $\mathsf{rand} \leftarrow \{0,1\}^{\mathsf{poly}(\lambda)}$ and compute $\mathsf{ctx} \leftarrow \mathsf{PKE.Enc}(\mathsf{ek}, (m, \mathsf{d}_{x_b}, \sigma_{\mathsf{d}_{x_b}}, \pi); \mathsf{rand})$;

(iv) Query $((\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_C, m, \mathsf{ctx}), (\mathsf{d}_{x_b}, \sigma_{\mathsf{d}_{x_b}}, \pi, \mathsf{rand}))$ to the challenge oracle, and receive $\Pi$;

3. $\mathcal{B}$ sets $\Sigma := (\mathsf{ctx}, \Pi)$ and runs $\mathcal{A}_2(\mathsf{st}, (\mathsf{d}_{x_0}, \sigma_{\mathsf{d}_{x_0}}), (\mathsf{d}_{x_1}, \sigma_{\mathsf{d}_{x_1}}), \Sigma)$;
4. When $\mathcal{A}_2$ outputs $b'$ and terminates, $\mathcal{B}$ outputs 1 if and only if $b' = b$; otherwise, it outputs 0.

The above completes the description of $\mathcal{B}$. If $\mathsf{crs}$ is generated by the $\mathsf{NIZK.Setup}$ (resp., $\mathsf{Sim}_0$) algorithm and $\mathcal{B}$ accesses the $\mathsf{NIZK.Prove}$ (resp., $\mathsf{Sim}_1$) oracle, then $\mathcal{B}$ perfectly simulates $\mathsf{Game}_0$ (resp., $\mathsf{Game}_1$) for $\mathcal{A}$. Therefore, we have $|\Pr[T_0] - \Pr[T_1]| = |\Pr[b = b' \text{ in } \mathsf{Game}_0] - \Pr[b = b' \text{ in } \mathsf{Game}_1]| = \mathsf{Adv}^{\mathsf{zk}}_{\mathsf{NIZK},\mathcal{B}}(\lambda)$.

□ (**Lemma 4.1**)

Next, we show to have $\Pr[T_1] \leq \mathsf{negl}(\lambda)$. Intuitively, any algorithm that makes the probability $\Pr[T_1]$ non-negligible can distinguish two ciphertexts with different messages.

**Lemma 4.2.** *There exists a PPT algorithm $\mathcal{B}$ such that*

$$\Pr[T_1] = \mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{PKE},\mathcal{B}}(\lambda).$$

*Proof.* Assume that there exists a PPT adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ which makes the probability $\Pr[T_1]$ non-negligible. Then, we can construct another PPT adversary $\mathcal{B}$ that breaks the IND-CPA security of the $\mathsf{PKE}$ with the same probability, which implies the lemma. The description of $\mathcal{B}$ is as follows.

1. $\mathcal{B}$ initially receives $\mathsf{ek}$, computes $(\mathsf{sk}, \mathsf{vk}) \leftarrow \mathsf{SIG.KG}(1^\lambda)$, $(\widetilde{\mathsf{crs}}, \mathsf{td}) \leftarrow \mathsf{Sim}_0(1^\lambda)$, and $\mathsf{crs}_{\mathsf{CirS}} \leftarrow \mathsf{CirS.Setup}(1^\lambda, 1^\ell)$, sets $\mathsf{pp} := (\mathsf{ek}, \mathsf{vk}, \widetilde{\mathsf{crs}}, \mathsf{crs}_{\mathsf{CirS}})$ and $\mathsf{msk} := (\mathsf{sk}, \mathsf{crs}_{\mathsf{CirS}})$, and runs $\mathcal{A}_1(\mathsf{pp}, \mathsf{msk})$.
2. When $\mathcal{A}_1$ outputs $(\mathsf{st}, x_0, x_1, C, m)$ and terminates, $\mathcal{B}$ proceeds as follows:
   (i) Compute $\mathsf{d}_C \leftarrow \mathsf{CirS.DCir}(\mathsf{crs}_{\mathsf{CirS}}, C)$, $(\cdot, \pi_i) \leftarrow \mathsf{CirS.Prove}(\mathsf{crs}_{\mathsf{CirS}}, x_i, C)$, and $\mathsf{d}_{x_i} \leftarrow \mathsf{CirS.DStr}(\mathsf{crs}_{\mathsf{CirS}}, x_i)$ for both $i \in \{0,1\}$;
   (ii) Compute $\sigma_{\mathsf{d}_{x_i}} \leftarrow \mathsf{SIG.Sign}(\mathsf{sk}, \mathsf{d}_{x_i})$ for both $i \in \{0,1\}$;
   (iii) Query $((m, \mathsf{d}_{x_0}, \sigma_{\mathsf{d}_{x_0}}, \pi_0), (m, \mathsf{d}_{x_1}, \sigma_{\mathsf{d}_{x_1}}, \pi_1))$ to the challenge oracle, and receive $\mathsf{ctx}$;
   (iv) Compute $\Pi \leftarrow \mathsf{Sim}_1(\widetilde{\mathsf{crs}}, \mathsf{td}, (\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_C, m, \mathsf{ctx})$;
3. $\mathcal{B}$ sets $\Sigma := (\mathsf{ctx}, \Pi)$ and runs $\mathcal{A}_2(\mathsf{st}, (\mathsf{d}_{x_0}, \sigma_{\mathsf{d}_{x_0}}), (\mathsf{d}_{x_1}, \sigma_{\mathsf{d}_{x_1}}), \Sigma)$;
4. When $\mathcal{A}_2$ outputs $b'$ and terminates, $\mathcal{B}$ outputs $b'$ and terminates.

The above completes the description of $\mathcal{B}$. It is easy to see that $\mathcal{B}$ perfectly simulates $\mathsf{Game}_1$ for $\mathcal{A}$. Therefore, we have $\Pr[T_1] = \mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{PKE},\mathcal{B}}(\lambda)$.

□ (**Lemma 4.2**)

Theorem 4.1 now follows immediately from Lemmata 4.1 and 4.2.
□ (**Theorem 4.1**)

**Theorem 4.2.** *If the PKE scheme* PKE *is IND-CPA secure, the signature scheme* SIG *is EUF-CMA secure, the NIZK proof system* NIZK *is simulation sound and zero-knowledge, and the Circuit SNARG* CirS *is collision-resistant of the circuit digest and sound, then the above ABS scheme is unforgeable.*

*Proof.* Let us fix a PPT adversary $\mathcal{A}$ attacking the unforgeability of the ABS, the security parameter $\lambda$, and the attribute length $\ell$. The attack game used to define the unforgeability is in Definition 2.6. We define three games as follows:

$\mathsf{Game}_0$ : This game is the original attack game.

$\mathsf{Game}_1$ : This game is the game identical with $\mathsf{Game}_0$ except that we use modified algorithms, in which some steps are replaced as follows:

- In the $\mathsf{ABS.Setup}(1^\lambda, 1^\ell)$ algorithm, $\mathsf{crs}_{\mathsf{NIZK}} \leftarrow \mathsf{NIZK.Setup}(1^\lambda)$ is replaced by $(\widetilde{\mathsf{crs}}, \mathsf{td}) \leftarrow \mathsf{Sim}_0(1^\lambda)$.
- In the $\mathsf{ABS.Sign}(\mathsf{pp}, \mathsf{sk} = (\mathsf{d}_x, \sigma), x, C, m)$ algorithm, a NIZK proof $\Pi \leftarrow \mathsf{NIZK.Prove}(\mathsf{crs}_{\mathsf{NIZK}}, (\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_C, m, \mathsf{ctx}), (\mathsf{d}_x, \sigma, \pi, \mathsf{rand}))$ is replaced by $\widetilde{\Pi} \leftarrow \mathsf{Sim}_1(\widetilde{\mathsf{crs}}, \mathsf{td}, (\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_C, m, \mathsf{ctx}))$.

$\mathsf{Game}_2$ : This game is the game identical with $\mathsf{Game}_1$ except that the signing algorithm is modified in some step as follows:

- In the $\mathsf{ABS.Sign}(\mathsf{pp}, \mathsf{sk} = (\mathsf{d}_x, \sigma), x, C, m)$ algorithm, $\mathsf{ctx} \leftarrow \mathsf{PKE.Enc}(\mathsf{ek}, (m, \mathsf{d}_x, \sigma_{\mathsf{d}_x}, \pi))$ is replaced by $\widetilde{\mathsf{ctx}} \leftarrow \mathsf{PKE.Enc}(\mathsf{ek}, 0^t)$, where $t$ is the total length of $(m, \mathsf{d}_x, \sigma_{\mathsf{d}_x}, \pi)$.

For $i = 0, 1, 2$, let $T_i$ be the event that $\mathcal{A}$ wins the unforgeability experiment in the game $\mathsf{Game}_i$. We will show that the probability $|\Pr[T_0] - \Pr[T_1]| \leq \mathsf{negl}(\lambda)$, $|\Pr[T_0] - \Pr[T_1]| \leq \mathsf{negl}(\lambda)$, and $\Pr[T_1] \leq \mathsf{negl}(\lambda)$ in turn.

We first show to have $|\Pr[T_0] - \Pr[T_1]| \leq \mathsf{negl}(\lambda)$. Similar to Lemma 4.1, any difference between these two games $\mathsf{Game}_0$ and $\mathsf{Game}_1$ yields a PPT algorithm that distinguishes the real proof from the simulated proof.

**Lemma 4.3.** *There exists a PPT algorithm* $\mathcal{B}$ *such that*

$$|\Pr[T_0] - \Pr[T_1]| = \mathsf{Adv}^{\mathsf{zk}}_{\mathsf{NIZK},\mathcal{B}}(\lambda).$$

*Proof.* Assume that there exists a PPT adversary $\mathcal{A}$ which makes the probability $|\Pr[T_0] - \Pr[T_1]|$ non-negligible. Then, we can construct another PPT adversary $\mathcal{B}$ that breaks the zero-knowledge property of NIZK with non-negligible probability, which implies the lemma. The description of $\mathcal{B}$ is as follows.

1. $\mathcal{B}$ initially receives $\mathsf{crs}$, computes $(\mathsf{ek}, \mathsf{dk}) \leftarrow \mathsf{PKE}(1^\lambda)$, $(\mathsf{sk}, \mathsf{vk}) \leftarrow \mathsf{SIG.KG}(1^\lambda)$, and $\mathsf{crs}_{\mathsf{CirS}} \leftarrow \mathsf{CirS.Setup}(1^\lambda, 1^\ell)$, sets $L_{sig}, L_{corr}, L_{key} := \emptyset$, and $\mathsf{pp} := (\mathsf{ek}, \mathsf{vk}, \mathsf{crs}, \mathsf{crs}_{\mathsf{CirS}})$, and runs $\mathcal{A}(\mathsf{pp})$.
2. $\mathcal{B}$ responds to each of queries from $\mathcal{A}$ as follows:

– For each query to $\mathcal{O}_{sig}(x, C, m)$, $\mathcal{B}$ returns $\perp$ if $C(x) = 0$; otherwise, it proceeds as follows:
  (i) If $\exists(x, \cdot) \notin L_{key}$, then
    (a) compute $\mathsf{d}_x \leftarrow \mathsf{CirS.DStr}(\mathsf{crs}_{\mathsf{CirS}}, x)$ and $\sigma_{\mathsf{d}_x} \leftarrow \mathsf{SIG.Sign}(\mathsf{sk}, \mathsf{d}_x)$;
    (b) add $(x, (\mathsf{d}_x, \sigma_{\mathsf{d}_x}))$ to $L_{key}$;
    Otherwise, find $(x, \mathsf{sk}_x) \in L_{key}$ and parse $\mathsf{sk}_x = (\mathsf{d}_x, \sigma_{\mathsf{d}_x})$;
  (ii) Compute $\mathsf{d}_C \leftarrow \mathsf{CirS.DCir}(\mathsf{crs}_{\mathsf{CirS}}, C)$ and $(\cdot, \pi) \leftarrow \mathsf{CirS.Prove}(\mathsf{crs}_{\mathsf{CirS}}, x, C)$;
  (iii) Randomly choose $\mathsf{rand} \xleftarrow{\$} \{0,1\}^{\mathsf{poly}(\lambda)}$ and compute $\mathsf{ctx} \leftarrow \mathsf{PKE.Enc}(\mathsf{ek}, (m, \mathsf{d}_x, \sigma_{\mathsf{d}_x}, \pi); \mathsf{rand})$;
  (iv) Query $((\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_C, m, \mathsf{ctx}), (\mathsf{d}_x, \sigma_{\mathsf{d}_x}, \pi, \mathsf{rand}))$ to the challenge oracle, and receive $\Pi$.
  Then, $\mathcal{B}$ adds $(C, m)$ to $L_{sig}$ and returns $(\mathsf{ctx}, \Pi)$.
– For each query to $\mathcal{O}_{corr}(x)$, $\mathcal{B}$ computes as follows:
  - If $\exists(x, \cdot) \notin L_{key}$, then $\mathcal{B}$ computes $\mathsf{d}_x \leftarrow \mathsf{CirS.DStr}(\mathsf{crs}_{\mathsf{CirS}}, x)$ and $\sigma_{\mathsf{d}_x} \leftarrow \mathsf{SIG.Sign}(\mathsf{sk}, \mathsf{d}_x)$, sets $\mathsf{sk}_x := (\mathsf{d}_x, \sigma_{\mathsf{d}_x})$, and adds $(x, \mathsf{sk}_x)$ to $L_{key}$;
  - Otherwise, $\mathcal{B}$ finds $(x, \mathsf{sk}_x) \in L_{key}$.
  Finally, $\mathcal{B}$ adds $x$ to $L_{corr}$ and returns $\mathsf{sk}_x$.

3. When $\mathcal{A}$ outputs $(C^*, m^*, \Sigma^* = (\mathsf{ctx}^*, \Pi^*))$ and terminates, $\mathcal{B}$ computes $\mathsf{d}_{C^*} \leftarrow \mathsf{CirS.DCir}(\mathsf{crs}_{\mathsf{CirS}}, C^*)$ and outputs 1 if $C^*(x) = 0$ for all $x \in L_{corr}$, $(C^*, m^*) \notin L_{sig}$, and $\mathsf{NIZK.Ver}(\mathsf{crs}_{\mathsf{NIZK}}, (\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_{C^*}, m^*, \mathsf{ctx}^*), \Pi^*) = 1$; otherwise, it outputs 0.

The above completes the description of $\mathcal{B}$. If $\mathsf{crs}$ is generated by the $\mathsf{NIZK.Setup}$ (resp., $\mathsf{Sim}_0$) algorithm and $\mathcal{B}$ accesses the $\mathsf{NIZK.Prove}$ (resp., $\mathsf{Sim}_1$) oracle, then $\mathcal{B}$ perfectly simulates $\mathsf{Game}_0$ (resp., $\mathsf{Game}_1$) for $\mathcal{A}$. Therefore, we have $|\Pr[T_0] - \Pr[T_1]| = \mathsf{Adv}^{\mathsf{zk}}_{\mathsf{NIZK},\mathcal{B}}(\lambda)$.

□ (**Lemma 4.3**)

Secondly, we will show to have $|\Pr[T_1] - \Pr[T_2]| \leq \mathsf{negl}(\lambda)$. Intuitively, any difference between these two games $\mathsf{Game}_1$ and $\mathsf{Game}_2$ yields a PPT algorithm that distinguishes two ciphertexts with different messages.

**Lemma 4.4.** *There exists a PPT algorithm $\mathcal{B}$ such that*

$$|\Pr[T_1] - \Pr[T_2]| = \mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{PKE},\mathcal{B}}(\lambda).$$

*Proof.* Assume that there exists a PPT adversary $\mathcal{A}$ which makes the probability $|\Pr[T_1] - \Pr[T_2]|$ non-negligible. Then, we can construct another PPT adversary $\mathcal{B}$ that breaks the IND-CPA security of the $\mathsf{PKE}$ with non-negligible probability, which implies the lemma. The description of $\mathcal{B}$ is as follows.

1. $\mathcal{B}$ initially receives $\mathsf{ek}$, computes $(\mathsf{sk}, \mathsf{vk}) \leftarrow \mathsf{SIG.KG}(1^\lambda)$, $(\widetilde{\mathsf{crs}}, \mathsf{td}) \leftarrow \mathsf{Sim}_0(1^\lambda)$, and $\mathsf{crs}_{\mathsf{CirS}} \leftarrow \mathsf{CirS.Setup}(1^\lambda, 1^\ell)$, sets $L_{sig}, L_{corr}, L_{key} := \emptyset$, and $\mathsf{pp} := (\mathsf{ek}, \mathsf{vk}, \widetilde{\mathsf{crs}}, \mathsf{crs}_{\mathsf{CirS}})$, and runs $\mathcal{A}(\mathsf{pp})$.

2. $\mathcal{B}$ responds to each of queries from $\mathcal{A}$ as follows:
   - For each query to $\mathcal{O}_{sig}(x, C, m)$, $\mathcal{B}$ returns $\perp$ if $C(x) = 0$; otherwise, it proceeds as follows:
     (i) If $\exists(x, \cdot) \notin L_{key}$, then
       (a) compute $\mathsf{d}_x \leftarrow \mathsf{CirS.DStr}(\mathsf{crs}_{\mathsf{CirS}}, x)$ and $\sigma_{\mathsf{d}_x} \leftarrow \mathsf{SIG.Sign}(\mathsf{sk}, \mathsf{d}_x)$;
       (b) add $(x, (\mathsf{d}_x, \sigma_{\mathsf{d}_x}))$ to $L_{key}$;
       Otherwise, find $(x, \mathsf{sk}_x) \in L_{key}$ and parse $\mathsf{sk}_x = (\mathsf{d}_x, \sigma_{\mathsf{d}_x})$;
     (ii) Compute $\mathsf{d}_C \leftarrow \mathsf{CirS.DCir}(\mathsf{crs}_{\mathsf{CirS}}, C)$ and $(\cdot, \pi) \leftarrow \mathsf{CirS.Prove}(\mathsf{crs}_{\mathsf{CirS}}, x, C)$;
     (iii) Query $((m, \mathsf{d}_x, \sigma_{\mathsf{d}_x}, \pi), 0^t)$ to the challenge oracle, where $t$ is the length of message $(m, \mathsf{d}_x, \sigma_{\mathsf{d}_x}, \pi)$, and receive $\mathsf{ctx}$;
     (iv) Compute $\Pi \leftarrow \mathsf{Sim}_1(\widetilde{\mathsf{crs}}, \mathsf{td}, (\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_C, m, \mathsf{ctx}))$;
     Then, $\mathcal{B}$ adds $(C, m)$ to $L_{sig}$ and returns $(\mathsf{ctx}, \Pi)$.
   - For each query to $\mathcal{O}_{corr}(x)$, $\mathcal{B}$ computes in the same way as described in the proof of Lemma 4.3.
3. When $\mathcal{A}$ outputs $(C^*, m^*, \Sigma^* = (\mathsf{ctx}^*, \Pi^*))$ and terminates, $\mathcal{B}$ computes $\mathsf{d}_{C^*} \leftarrow \mathsf{CirS.DCir}(\mathsf{crs}_{\mathsf{CirS}}, C^*)$ and outputs 1 if $C^*(x) = 0$ for all $x \in L_{corr}$, $(C^*, m^*) \notin L_{sig}$, and $\mathsf{NIZK.Ver}(\mathsf{crs}_{\mathsf{NIZK}}, (\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_{C^*}, m^*, \mathsf{ctx}^*), \Pi^*) = 1$; otherwise, it outputs 0.

The above completes the description of $\mathcal{B}$. If $\mathsf{ctx}$ is an encryption of a message $(m, \mathsf{d}_x, \sigma_{\mathsf{d}_x}, \pi)$ (resp., $0^t$), then $\mathcal{B}$ perfectly simulates $\mathsf{Game}_1$ (resp., $\mathsf{Game}_2$) for $\mathcal{A}$. Therefore, we have $|\Pr[T_1] - \Pr[T_2]| = 2 \cdot \mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{PKE},\mathcal{B}}(\lambda)$.

□ (**Lemma 4.4**)

Thirdly, we will show $\Pr[T_2] \leq \mathsf{negl}(\lambda)$, so we focus on the game $\mathsf{Game}_2$ only. We consider the winning condition for $\mathcal{A}$ in the game $\mathsf{Game}_2$. In the following, let $t$ be the fixed total length of a message $m$, an attribute digest $\mathsf{d}_x$, its signature $\sigma_{\mathsf{d}_x}$, and a Circuit SNARG proof $\pi$. In addition, let $(C^*, m^*, \Sigma^* = (\mathsf{ctx}^*, \Pi^*))$ be the $\mathcal{A}$'s output of the game.

In the following, we will use the notation $\star$ denoting that some value exists but is being ignored. We consider two events in the game as follows:

$\mathsf{E}_{sig}$ : is the event that there exists $(C, \star) \in L_{sig}$ such that $\mathsf{CirS.DCir}(\mathsf{crs}_{\mathsf{CirS}}, C^*) = \mathsf{CirS.DCir}(\mathsf{crs}_{\mathsf{CirS}}, C)$ and $C^* \neq C$.

$\mathsf{E}_{\bar{\rho}}$ : is the event that the statement $(\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_{C^*}, m^*, \mathsf{ctx}^*)$ is not in the language corresponding to the NIZK relation $\rho$, where $\mathsf{d}_{C^*} = \mathsf{CirS.DCir}(\mathsf{crs}_{\mathsf{CirS}}, C^*)$.

$\mathsf{E}_{key}$ : is the event that there exists $x \in L_{corr}$ such that $\mathsf{d}^* = \mathsf{CirS.DStr}(\mathsf{crs}_{\mathsf{CirS}}, x)$, where $(\star, \mathsf{d}^*, \star, \star) = \mathsf{PKE.Dec}(\mathsf{dk}, \mathsf{ctx}^*)$.

We clearly have

$$\Pr[T_2] \leq \Pr[T_2 \wedge \mathsf{E}_{sig}] + \Pr[T_2 \wedge \neg\mathsf{E}_{sig} \wedge \mathsf{E}_{\bar{\rho}}]$$

$$+ \Pr[T_2 \wedge \neg \mathsf{E}_{sig} \wedge \neg \mathsf{E}_{\bar{\rho}} \wedge \mathsf{E}_{key}] + \Pr[T_2 \wedge \neg \mathsf{E}_{sig} \wedge \neg \mathsf{E}_{\bar{\rho}} \wedge \neg \mathsf{E}_{key}].$$

In the following, we separate the winning condition in the game into four cases.

First, if the event $\mathsf{E}_{sig}$ occurs, it is easy to see that if $\mathcal{A}$ wins in the game $\mathsf{Game}_2$, then it breaks the collision resistance of the circuit digest of the Circuit SNARG scheme.

**Lemma 4.5.** *There exists a PPT algorithm $\mathcal{B}$ such that*

$$\Pr[T_2 \wedge \mathsf{E}_{sig}] \leq \mathsf{Adv}^{\mathsf{col\text{-}cir}}_{\mathsf{CirS},\mathcal{B}}(\lambda).$$

*Proof.* Assume that there exists a PPT adversary $\mathcal{A}$ which makes the probability $\Pr[T_2 \wedge \mathsf{E}_{sig}]$ non-negligible. Then, we can construct another PPT adversary $\mathcal{B}$ that breaks the collision resistance of the circuit digest of $\mathsf{CirS}$ with non-negligible probability, which implies the lemma. The description of $\mathcal{B}$ is as follows, where $\star$ denotes that some value exists but is being ignored.

1. $\mathcal{B}$ receives $\mathsf{crs}_{\mathsf{CirS}}$, computes $(\mathsf{ek}, \mathsf{dk}) \leftarrow \mathsf{PKE}(1^\lambda)$, $(\mathsf{sk}, \mathsf{vk}) \leftarrow \mathsf{SIG.KG}(1^\lambda)$, and $(\widetilde{\mathsf{crs}}, \mathsf{td}) \leftarrow \mathsf{Sim}_0(1^\lambda)$, sets $L_{sig}, L_{corr}, L_{key} := \emptyset$, and $\mathsf{pp} := (\mathsf{ek}, \mathsf{vk}, \widetilde{\mathsf{crs}}, \mathsf{crs}_{\mathsf{CirS}})$, and runs $\mathcal{A}(\mathsf{pp})$.
2. $\mathcal{B}$ responds to each of queries from $\mathcal{A}$ as follows:
   - For each query to $\mathcal{O}_{sig}(x, C, m)$, $\mathcal{B}$ returns $\perp$ if $C(x) = 0$; otherwise, it proceeds as follows:
     (i) Compute $\mathsf{d}_C \leftarrow \mathsf{CirS.DCir}(\mathsf{crs}_{\mathsf{CirS}}, C)$;
     (ii) Compute $\widetilde{\mathsf{ctx}} \leftarrow \mathsf{PKE.Enc}(0^t)$, where $t$ is the fixed length in $\mathsf{Game}_2$;
     (iii) Compute $\Pi \leftarrow \mathsf{Sim}_1(\widetilde{\mathsf{crs}}, \mathsf{td}, (\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_C, m, \widetilde{\mathsf{ctx}}))$.
     Then, $\mathcal{B}$ adds $(C, m)$ to $L_{sig}$ and returns $(\mathsf{ctx}, \Pi)$.
   - For each query to $\mathcal{O}_{corr}(x)$, $\mathcal{B}$ computes in the same way as described in the proof of Lemma 4.3.
3. When $\mathcal{A}$ outputs $(C^*, m^*, \Sigma^* = (\mathsf{ctx}^*, \Pi^*))$ and terminates, $\mathcal{B}$ finds $C$ such that $(C, \star) \in L_{sig}$ and $\mathsf{CirS.DCir}(\mathsf{crs}_{\mathsf{CirS}}, C) = \mathsf{CirS.DCir}(\mathsf{crs}_{\mathsf{CirS}}, C^*)$. If $\mathcal{B}$ cannot find such $C$, then it outputs $\perp$; otherwise, it outputs $(C, C^*)$.

The above completes the description of $\mathcal{B}$. It is easy to see that $\mathcal{B}$ perfectly simulates the game $\mathsf{Game}_2$ for $\mathcal{A}$. Let $\mathsf{X}^* = (\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_{C^*}, m^*, \mathsf{ctx}^*)$. When the event $\mathsf{E}_{sig}$ occurs, $\mathcal{B}$ can always find $C$ such that $(C, \star) \in L_{sig}$, $C \neq C^*$, and $\mathsf{CirS.DCir}(\mathsf{crs}_{\mathsf{CirS}}, C) = \mathsf{CirS.DCir}(\mathsf{crs}_{\mathsf{CirS}}, C^*)$. Therefore, we have $\Pr[T_2 \wedge \mathsf{E}_{sig}] \leq \mathsf{Adv}^{\mathsf{col\text{-}cir}}_{\mathsf{CirS},\mathcal{B}}(\lambda)$.

□ (**Lemma 4.5**)

Second, if the event $\mathsf{E}_{sig}$ never occurs but the event $\mathsf{E}_{\bar{\rho}}$ occurs, it is easy to see that if $\mathcal{A}$ wins in the game $\mathsf{Game}_2$, then it breaks the simulation soundness of the NIZK scheme.

**Lemma 4.6.** *There exists a PPT algorithm $\mathcal{B}$ such that*

$$\Pr[T_2 \wedge \neg \mathsf{E}_{sig} \wedge \mathsf{E}_{\bar{\rho}}] \leq \mathsf{Adv}^{\mathsf{sim\text{-}sound}}_{\mathsf{NIZK},\mathcal{B}}(\lambda).$$

*Proof.* Assume that there exists a PPT adversary $\mathcal{A}$ which makes the probability $\Pr[T_2 \wedge \neg \mathsf{E}_{sig} \wedge \mathsf{E}_{\bar{\rho}}]$ non-negligible. Then, we can construct another PPT adversary $\mathcal{B}$ that breaks the simulation soundness of $\mathsf{NIZK}$ with non-negligible probability, which implies the lemma. The description of $\mathcal{B}$ is as follows.

1. $\mathcal{B}$ initially receives $\widetilde{\mathsf{crs}}$, computes $(\mathsf{ek}, \mathsf{dk}) \leftarrow \mathsf{PKE}(1^\lambda)$, $(\mathsf{sk}, \mathsf{vk}) \leftarrow \mathsf{SIG.KG}(1^\lambda)$, and $\mathsf{crs}_{\mathsf{CirS}} \leftarrow \mathsf{CirS.Setup}(1^\lambda, 1^\ell)$, sets $L_{sig}, L_{corr}, L_{key} := \emptyset$, and $\mathsf{pp} := (\mathsf{ek}, \mathsf{vk}, \widetilde{\mathsf{crs}}, \mathsf{crs}_{\mathsf{CirS}})$, and runs $\mathcal{A}(\mathsf{pp})$.
2. $\mathcal{B}$ responds to each of queries from $\mathcal{A}$ as follows:
   - For each query to $\mathcal{O}_{sig}(x, C, m)$, $\mathcal{B}$ returns $\perp$ if $C(x) = 0$; otherwise, it proceeds as follows:
     (i) Compute $\mathsf{d}_C \leftarrow \mathsf{CirS.DCir}(\mathsf{crs}_{\mathsf{CirS}}, C)$;
     (ii) Compute $\widetilde{\mathsf{ctx}} \leftarrow \mathsf{PKE.Enc}(0^t)$, where $t$ is the fixed length in $\mathsf{Game}_2$;
     (iii) Query $(\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_C, m, \widetilde{\mathsf{ctx}})$ to the simulation oracle, and receive $\Pi$.
     Then, $\mathcal{B}$ adds $(C, m)$ to $L_{sig}$ and returns $(\mathsf{ctx}, \Pi)$.
   - For each query to $\mathcal{O}_{corr}(x)$, $\mathcal{B}$ computes in the same way as described in the proof of Lemma 4.3.
3. When $\mathcal{A}$ outputs $(C^*, m^*, \Sigma^* = (\mathsf{ctx}^*, \Pi^*))$ and terminates, $\mathcal{B}$ computes $\mathsf{d}_{C^*} \leftarrow \mathsf{CirS.DCir}(\mathsf{crs}_{\mathsf{CirS}}, C^*)$ and outputs $((\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_{C^*}, m^*, \mathsf{ctx}^*), \Pi^*)$.

The above completes the description of $\mathcal{B}$. It is easy to see that $\mathcal{B}$ perfectly simulates the game $\mathsf{Game}_2$ for $\mathcal{A}$. Let $\mathsf{X}^* = (\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_{C^*}, m^*, \mathsf{ctx}^*)$. When the event $\mathsf{E}_{\bar{\rho}}$ occurs, we have $\mathsf{X}^* \notin L_\rho$, where $L_\rho := \{x \mid \exists w \text{ s.t. } (x, w) \in \rho\}$. On the other hand, when $\mathcal{A}$ wins, we have $\mathsf{NIZK.Ver}(\widetilde{\mathsf{crs}}, \mathsf{X}^*, \Pi^*) = 1$. In addition, $\mathsf{X}^*$ is never queried to the simulation oracle from the following reason: if $(\star, m^*) \notin L_{sig}$, then $\mathsf{X}^*$ is never queried; otherwise, there is no $C$ such that $(C, m^*) \in L_{sig}$ and $\mathsf{CirS.DCir}(\mathsf{crs}_{\mathsf{CirS}}, C) = \mathsf{CirS.DCir}(\mathsf{crs}_{\mathsf{CirS}}, C^*)$ since the event $\mathsf{E}_{sig}$ never occurs and we have $(C^*, m^*) \notin L_{sig}$ due to the winning condition for $\mathcal{A}$. Therefore, we have $\Pr[T_2 \wedge \neg \mathsf{E}_{sig} \wedge \mathsf{E}_{\bar{\rho}}] \leq \mathsf{Adv}^{\mathsf{sim\text{-}sound}}_{\mathsf{NIZK},\mathcal{B}}(\lambda)$.

□ (**Lemma 4.6**)

Third, if the events $\mathsf{E}_{sig}$ and $\mathsf{E}_{\bar{\rho}}$ never occur but the event $\mathsf{E}_{key}$ occurs, $\mathcal{A}$ must generate a circuit SNARG proof that passes the verification of the Circuit SNARG. However, we have $C^*(x) = 0$ for all $x \in L_{corr}$ when $\mathcal{A}$ wins the game. Therefore, to win the game, $\mathcal{A}$ has to break the soundness of the $\mathsf{CirS}$ scheme.

**Lemma 4.7.** *There exist PPT algorithms $\mathcal{B}$ such that*

$$\Pr[T_2 \wedge \neg \mathsf{E}_{sig} \wedge \neg \mathsf{E}_{\bar{\rho}} \wedge \mathsf{E}_{key}] \leq \mathsf{Adv}^{\mathsf{sound}}_{\mathsf{CirS},\mathcal{B}}(\lambda).$$

*Proof.* Assume that there exists a PPT adversary $\mathcal{A}$ which makes the probability $\Pr[T_2 \wedge \neg \mathsf{E}_{sig} \wedge \neg \mathsf{E}_{\bar{\rho}} \wedge \mathsf{E}_{key}]$ non-negligible. Then, we can construct another PPT adversary $\mathcal{B}$ that breaks the soundness of $\mathsf{CirS}$ with non-negligible probability, which implies the lemma. The description of $\mathcal{B}$ is as follows, where $\star$ denotes that some value exists but is being ignored.

1. $\mathcal{B}$ receives $\mathsf{crs}_{\mathsf{CirS}}$, computes $(\mathsf{ek}, \mathsf{dk}) \leftarrow \mathsf{PKE}(1^\lambda)$, $(\mathsf{sk}, \mathsf{vk}) \leftarrow \mathsf{SIG.KG}(1^\lambda)$, and $(\widetilde{\mathsf{crs}}, \mathsf{td}) \leftarrow \mathsf{Sim}_0(1^\lambda)$, and sets $L_{sig}, L_{corr}, L_{key} = \emptyset$, and $\mathsf{pp} := (\mathsf{ek}, \mathsf{vk}, \widetilde{\mathsf{crs}}, \mathsf{crs}_{\mathsf{CirS}})$, and runs $\mathcal{A}(\mathsf{pp})$.
2. $\mathcal{B}$ responds to each of queries from $\mathcal{A}$ in the same way as described in the proof of Lemma 4.5.
3. When $\mathcal{A}$ outputs $(C^*, m^*, \Sigma^* = (\mathsf{ctx}^*, \Pi^*))$ and terminates, $\mathcal{B}$ computes $(m^*, \mathsf{d}^*, \sigma_{\mathsf{d}^*}, \pi^*) \leftarrow \mathsf{PKE.Dec}(\mathsf{dk}, \mathsf{ctx}^*)$ and finds $x^*$ such that $(x^*, (\mathsf{d}^*, \star)) \in L_{key}$. If $\mathcal{B}$ cannot find such $x^*$, then it outputs $\bot$; otherwise, it outputs $(x^*, C^*, 1, \pi^*)$.

The above completes the description of $\mathcal{B}$. It is easy to see that $\mathcal{B}$ perfectly simulates the game $\mathsf{Game}_2$ for $\mathcal{A}$. Let $\mathsf{X}^* = (\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_{C^*}, m^*, \mathsf{ctx}^*)$. When the event $\mathsf{E}_{key}$ occurs, $\mathcal{B}$ can always find $x^*$ such that $(x^*, (\mathsf{d}^*, \star)) \in L_{key}$. On the other hand, when $\mathcal{A}$ wins, we have $C^*(x) = 0$ for all $(x, \star) \in L_{key}$ since $\{x \mid x \in L_{corr}\} = \{x \mid (x, \star) \in L_{key}\}$ from the above description of $\mathcal{B}$. We also have $\mathsf{CirS.Ver}(\mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}^*, \mathsf{d}_{C^*}, 1, \pi^*) = 1$ since we have $\mathsf{X} \in L_\rho$ and both $\mathsf{d}^*$ and $\mathsf{d}_{C^*}$ are calculated deterministically. Therefore, we have $\Pr[T_2 \wedge \neg \mathsf{E}_{sig} \neg \mathsf{E}_{\bar{\rho}} \wedge \mathsf{E}_{key}] \leq \mathsf{Adv}^{\mathsf{sound}}_{\mathsf{CirS},\mathcal{B}}(\lambda)$.

□ (**Lemma 4.7**)

Finally, if all the events $\mathsf{E}_{sig}$, $\mathsf{E}_{\bar{\rho}}$, and $\mathsf{E}_{key}$ never occur, $\mathcal{A}$ must generate a valid signature for $\mathsf{d}_{x^*}$. Thus, if $\mathcal{A}$ wins, then it breaks the EUF-CMA security of the signature scheme.

**Lemma 4.8.** *There exists a PPT algorithm $\mathcal{B}$ such that*

$$\Pr[T_2 \wedge \neg \mathsf{E}_{sig} \wedge \neg \mathsf{E}_{\bar{\rho}} \wedge \neg \mathsf{E}_{key}] \leq \mathsf{Adv}^{\mathsf{euf\text{-}cma}}_{\mathsf{SIG},\mathcal{B}}(\lambda).$$

*Proof.* Assume that there exists a PPT adversary $\mathcal{A}$ which makes the probability $\Pr[T_2 \wedge \neg \mathsf{E}_{sig} \wedge \neg \mathsf{E}_{\bar{\rho}} \wedge \neg \mathsf{E}_{key}]$ non-negligible. Then, we can construct another PPT adversary $\mathcal{B}$ that breaks the EUF-CMA security of $\mathsf{SIG}$ with non-negligible probability, which implies the lemma. The description of $\mathcal{B}$ is as follows.

1. $\mathcal{B}$ initially receives $\mathsf{vk}$, computes $(\mathsf{ek}, \mathsf{dk}) \leftarrow \mathsf{PKE}(1^\lambda)$, $(\widetilde{\mathsf{crs}}, \mathsf{td}) \leftarrow \mathsf{Sim}_0(1^\lambda)$, and $\mathsf{crs}_{\mathsf{CirS}} \leftarrow \mathsf{CirS.Setup}(1^\lambda, 1^\ell)$, and sets $L_{sig}, L_{corr}, L_{key} := \emptyset$, and $\mathsf{pp} := (\mathsf{ek}, \mathsf{vk}, \widetilde{\mathsf{crs}}, \mathsf{crs}_{\mathsf{CirS}})$, and runs $\mathcal{A}(\mathsf{pp})$.
2. $\mathcal{B}$ responds to each of queries from $\mathcal{A}$ as follows, where $\star$ denotes that some value exists but is being ignored:
   - For each query to $\mathcal{O}_{sig}(x, C, m)$, $\mathcal{B}$ computes in the same way as described in the proof of Lemma 4.5.
   - For each query to $\mathcal{O}_{corr}(x)$, $\mathcal{B}$ computes as follows:
     - If $\exists (x, \star) \notin L_{key}$, then $\mathcal{B}$ proceeds as follows:
       (i) Compute $\mathsf{d}_x \leftarrow \mathsf{CirS.DStr}(\mathsf{crs}_{\mathsf{CirS}}, x)$;
       (ii) Query $\mathsf{d}_x$ to the signing oracle and receive $\sigma_{\mathsf{d}_x}$;
       (iii) Set $\mathsf{sk}_x := (\mathsf{d}_x, \sigma_{\mathsf{d}_x})$, and add $(x, \mathsf{sk}_x)$ to $L_{key}$.

- Otherwise, $\mathcal{B}$ finds $(x, \mathsf{sk}_x) \in L_{key}$.

Then, $\mathcal{B}$ adds $x$ to $L_{corr}$ and returns $(\mathsf{d}_x, \sigma_{\mathsf{d}_x})$.

3. When $\mathcal{A}$ outputs $(C^*, m^*, \Sigma^* = (\mathsf{ctx}^*, \Pi^*))$ and terminates, $\mathcal{B}$ computes $(m^*, \mathsf{d}^*, \sigma_{\mathsf{d}^*}, \pi^*) \leftarrow \mathsf{PKE.Dec}(\mathsf{dk}, \mathsf{ctx}^*)$ and outputs $(\mathsf{d}^*, \sigma_{\mathsf{d}^*})$.

The above completes the description of $\mathcal{B}$. It is easy to see that $\mathcal{B}$ perfectly simulates the game $\mathsf{Game}_2$ for $\mathcal{A}$. Let $\mathsf{X}^* = (\mathsf{ek}, \mathsf{vk}, \mathsf{crs}_{\mathsf{CirS}}, \mathsf{d}_{C^*}, m^*, \mathsf{ctx}^*)$. Since the event $\mathsf{E}_{key}$ never occurs, $\mathcal{B}$ never queries $\mathsf{d}^*$ to the signing oracle in the EUF-CMA security experiment. On the other hand, when $\mathcal{A}$ wins, $\mathsf{SIG.Ver}(\mathsf{vk}, \mathsf{d}^*, \sigma_{\mathsf{d}^*}) = 1$ since we have $\mathsf{X} \in L_\rho$. Therefore, we have $\Pr[T_2 \wedge \neg \mathsf{E}_{sig} \wedge \neg \mathsf{E}_{\bar{\rho}} \wedge \neg \mathsf{E}_{key}] \leq \mathsf{Adv}^{\mathsf{euf\text{-}cma}}_{\mathsf{SIG},\mathcal{B}}(\lambda)$.

□ (**Lemma 4.8**)

Theorem 4.2 now follows immediately from Lemmata 4.3 to 4.8.

□ (**Theorem 4.2**)

*Acknowledgement.* This work was partially supported by JSPS KAKENHI Grant Number JP23KJ0548.

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