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Vorwort

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Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

Modeling Profit Sharing in Combinatorial Exchanges by Network Flows

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Abstract

In this paper we study the possibilities of sharing profit in combinatorial procurement auctions and exchanges. Bundles of heterogeneous items are offered by the sellers, and the buyers can then place bundle bids on sets of these items. That way, both sellers and buyers can express synergies between items and avoid the well-known risk of exposure (see, e.g., [3]). The reassignment of items to participants is known as the Winner Determination Problem (WDP). We propose solving the WDP by using a Set Covering formulation, because profits are potentially higher than with the usual Set Partitioning formulation, and subsidies are unnecessary. The achieved benefit is then to be distributed amongst the participants of the auction, a process which is known as profit sharing.

The literature on profit sharing provides various desirable criteria. We focus on three main properties we would like to guarantee: Budget balance, meaning that no more money is distributed than profit was generated, individual rationality, which guarantees to each player that participation does not lead to a loss, and the core property, which provides every subcoalition with enough money to keep them from separating. We characterize all profit sharing schemes that satisfy these three conditions by a monetary flow network and state necessary conditions on the solution of the WDP for the existence of such a profit sharing. Finally, we establish a connection to the famous VCG payment scheme [2, 8, 19], and the Shapley Value [17].

1 Introduction and Prerequisites

In this paper we study the possibilities of sharing profit in combinatorial auctions and exchanges. Combinatorial auctions have attracted much attention

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during the last years, as they promise substantial advantages over classical auctions. While in the latter only a single item is offered on which bidders can place their bids, in combinatorial auctions there are multiple heterogeneous items for sale simultaneously from a single seller. Bidders can use bundle bids to express their valuations, which allows them to avoid the risk of exposure (see, e.g., [3]). Similarly, it can be useful to enable sellers to bundle their items, to let them express their wish to sell only the complete bundle. It is worth mentioning that this is not the same as trading a single "super-item", since the items of the offered bundle may be assigned to different bidders. Combinatorial exchanges take this concept even one step further, as they not only allow multiple buyers, but also multiple sellers, a feature they share with double auctions [18]. Furthermore, they allow participants to simultaneously buy and sell items. Hence, they are a true generalization of combinatorial auctions.

Motivated by our practical problem, a combinatorial freight exchange, we will consider procurement exchanges (also known as reverse exchanges). Here, the items sold are contracts, and buying means delivering the service to the seller. Accordingly, we seek a solution that minimizes procurement costs, whereas in forward exchanges the goal is to maximize the buyers' total valuations. We emphasize again that our model allows bundling both on the seller and the buyer side.

In both combinatorial auctions and exchanges, participants (also known as *agents* or *players*) place their offers and bids using a certain bidding language. Although designing such a language can be crucial for the auction's properties, it is also quite domain specific. For a comprehensive overview see [3], for example.

After all offers and bids have been placed, the auctioneer is facing the difficulty of the Winner Determination Problem (WDP). Compared to the single-item auctions this can be a hard task, as the WDP is equivalent to a Set Packing Problem (SPP) or a Set Covering Problem (SCP), which are both known to be NP-hard in general [1, 5, 7]. We propose to use the SCP formulation, which covers all offered items by at least one bundle, and then resolve the overlapping bundles in a post-processing step. This leads to an unambiguous distribution of items to participants, just as if using the SPP formulation, and can be seen as an automated sub-bundling [1]. One advantage of using SCP rather than SPP is that it increases the solution space and may thereby enhance the auction's revenue. Another nice property is that it does not lead to subsidies [2], which SPP solutions sometimes do, see Fig. 1. Because of these two reasons, we assume using the SCP formulation with a post-processing step throughout the remaining paper.

Having solved the WDP, the auction's profit has to be distributed among the participants. This process is known as *profit sharing*, which is the main topic of this paper. The literature on profit sharing provides various interesting properties and criteria, see for example the book by Moulin [11]. We aim to guarantee three properties that are essential for a practical usage of an exchange

[1]Note that removing the respective items from the conflicting bundles without penalty is generally no problem in a procurement scenario, as it means executing less service for the same money. One could say it is equivalent to *free disposal* in a forward auction, and we propose to call it *free omission*.

[2]In our case, a subsidy occurs if payments are necessary between participants, who did not trade items in the exchange. So, without subsidies all bidding bundles can be paid for by the related offering bundles.

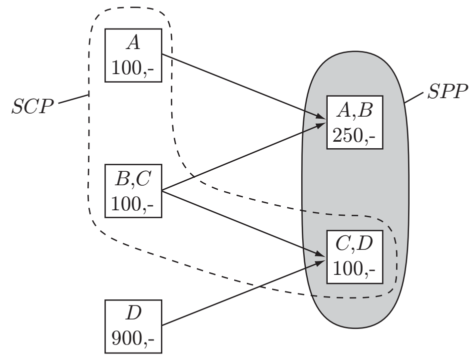

Figure 1: A simple procurement auction with three offering bundles and two bidding bundles. When using the SPP formulation for WDP, subsidies may appear even in optimal solutions. Here, the bidding bundle $(\{A, B\}, 250)$ demands more money for A and B than the offering participants are willing to pay. The optimal solution for the SCP, where items A, B, and C fall back to their owners, is also marked for comparison. Here, the two bundles $(\{B, C\}, 100)$ and $(\{C, D\}, 100)$ overlap, which has to be resolved before profit sharing. While the SPP solution generates 750,- profit, the SCP model yields 800,-. In the latter case, to meet the CORE property all profit has to be distributed to the two bundles $(\{C, D\}, 100)$ and $(\{D\}, 900)$, since they can form a subcoalition that generates 800,- profit.

platform: budget balance (BB), individual rationality (IR), and the "core property" (CORE). BB means that exactly that much money is distributed as profit was generated, so the exchange does not run at a deficit. IR guarantees to each player that participation does not lead to a personal loss. Besides these two almost inevitable conditions, we propose CORE, which provides every subcoalition with at least as much money as they could have realized among themselves. This keeps them from separating and thereby increases the stability of the profit sharing. It also adds some sense of fairness, since – conversely – not granting the subcoalition the profit they generate would be considered unfair.

With the help of a monetary flow network, we characterize all profit sharing schemes that satisfy these conditions. We show as our main result that every profit sharing scheme that satisfies BB, IR and CORE can be represented by a monetary flow in the network, and vice versa. Furthermore, we establish a connection to the well-known Vickrey-Clarke-Groves (VCG) payment scheme [2, 8, 19] and the Shapley Value [17]. For some of our results, the solution of the WDP has to satisfy certain criteria. We characterize these conditions and show how they affect the subsequent profit sharing.

We could not find any approach in literature that models profit sharing with the help of a flow network. Many papers explore to which extent a profit sharing scheme can satisfy a set of certain properties simultaneously. One classical result is the Myerson-Satterthwaite theorem [12], which states that no mechanism can be IR, BB and (ex post) efficient, the latter meaning that the total gain from trading items is maximized [3]. Parkes et al. [15, 16] take BB and IR as hard constraints and try to achieve as much efficiency as possible, by lowering participants' incentives to misreport their valuations. The authors give a practical payment rule that minimizes distance to VCG-payments. Day et al. [4] aim at a similar target: Starting from VCG-payments, they implement a mechanism that iteratively generates constraints that force the profit sharing to satisfy BB and CORE. The constraints are derived from repeatedly solving an NP-hard integer program. Many other papers simply apply the Shapley Value to share profit or costs (see, e.g., [14]). In contrast to these approaches, we do not try to give concrete advice on how to share the profit, but rather outline the framework of possible profit sharing schemes that the monetary flow network allows. By gaining insights from this characterization, one may derive new profit sharing schemes as a future step.

The outline of this paper is as follows. In Section 2 we define our basic notation. In Section 3 we introduce the monetary flow network and show what requirements a solution of the WDP must meet to allow for a profit sharing that is BB, IR, and satisfies CORE. As our main result, we then prove the equivalence of profit sharing schemes with exactly these three properties and monetary flows in the network. In Section 4, we establish the connection between the flow network and the VCG discounts, and show that the Shapley Value can be represented in the flow network. We conclude in Section 5 and give some ideas for further research.

2 Basic Notation

In this section we introduce the notation for the remaining paper. Let A be the set of **agents** (or participants, or players) in the auction and I be the set of **items**. Agents would like to exchange items amongst each other. They can both offer bundles of items to the other players and also place bundle bids on items offered by other agents. For any offering or bidding bundle b, we denote by

	$v_b \in \mathbb{R}^{\geq 0}$	the value of b, i.e., the offering / bidding price,
and as	$I_b \subseteq I$	the set of items referred to by b.

As already mentioned, our context is a procurement auction. Here, for offering players the bundle price v_b means the maximum amount of money they are willing to pay for the items in b to be executed. For buyers, on the other hand, it stands for the minimum amount they want to receive if they execute all items in b. We call

$$O := \{b_1, \dots, b_m\}$$

the set of **offered** bundles. As all offered bundles may fall back to their owners, we introduce an artificial bid $\hat{b}_i$ for each offered bundle $b_i \in O$, with exactly the same properties. We combine all artificial bids and the "real" submitted bids $\{b_{m+1}, \dots, b_n\}$ into one set

$$B := \{\hat{b}_1, \dots, \hat{b}_m\} \cup \{b_{m+1}, \dots, b_n\},\ n > m,$$

and call it the set of bundle **bids**. Slightly abusing notation, for a set $\widetilde{B} \subseteq B$, let

$$I_{\widetilde{B}} := \bigcup_{b \in \widetilde{B}} I_b$$

be the set of items covered by the bundles in $\widetilde{B}$. A **solution** $\mathcal{L} \subseteq B$ has the property that $I_{\mathcal{L}} = I$. We call the bundles of a solution $\mathcal{L}$ **winning**. To simplify further notation, we define for a set $S \subseteq O \cup B$ the set

$$F_S := \{b \in O \mid I_b \subseteq I_{S \cap B}\}$$

of **satisfiable** offers. Note that for any solution $\mathcal{L} \subseteq B$ it holds that $F_{\mathcal{L}} = O$. For a set $S \subseteq (O \cup B)$, we define as

$$P(S) := \sum_{b \in S \cap F_S} v_b - \sum_{b \in S \cap B} v_b \tag{1}$$

the **profit** of S. The profit is composed of the value of all offers that can be satisfied by the bundles in S minus the prices of the bidding bundles. Note that $P(\emptyset) = 0$. We simply write $P_{\mathcal{L}}$ to refer to a solution's profit $P(O \cup \mathcal{L}) = \sum_{b \in O} v_b - \sum_{b \in \mathcal{L}} v_b$. We call a solution **minimal**, if $P(S) \leq P_{\mathcal{L}}$ for all subsets $S \subseteq O \cup \mathcal{L}$, and denote it by $\mathcal{L}^*$.

3 Profit Sharing and the Monetary Flow Network

In this section we will prove that every profit sharing scheme that is budget balanced (BB), incentive compatible (IR), and satisfies the core property (CORE) can be described by a monetary flow network, and vice versa. This is a useful result, since all three properties are crucial for practical application and therefore the combination defines a class of highly relevant profit sharing schemes. The flow network gives an intuitive model for sharing profit, and with the help of it we can get a better understanding of the degrees of freedom we have, if we demand BB, IR and CORE. The choices could be used, for example, to fulfill additional criteria focusing on fairness amongst participants, rather than practical operation of an exchange.

Before we proceed with the definition of the monetary flow network, let us formally define the profit sharing function.

Definition 1 (Profit Sharing function)**.** *Let $\mathcal{L} \subseteq B$ be a solution. A profit sharing function $p : (O \cup \mathcal{L}) \to \mathbb{R}$ assigns to each offering or winning bundle b a profit $p(b)$. We sometimes refer to p as a profit sharing scheme interchangeably.*

The three essential properties budget balance, individual rationality, and the core property of a profit sharing function p are then defined w.r.t. a solution $\mathcal{L} \subseteq B$ as follows:

$$\text{BB} : \sum_{b \in O \cup \mathcal{L}} p(b) = P_{\mathcal{L}}$$

$$\text{IR} : p(b) \geq 0 \;\forall b \in O \cup \mathcal{L}$$

$$\text{CORE} : \sum_{b \in S} p(b) \geq P(S) \;\forall S \subseteq (O \cup \mathcal{L})$$

Please note that we adapted the classical definition of the core property (see, e.g., [13]) slightly to match our scenario: We will always carry out the profit sharing on a per bundle basis, since it should not matter in terms of profit share by which participant a certain bundle has been offered. As a nice side effect, this prevents pseudonymous bidding [3]. Nevertheless, all results can be adapted to an agent-based profit sharing. Secondly, we just consider the offering bundles and those bidding bundles belonging to the solution $\mathcal{L}$. One reason is that in combinatorial exchanges with multiple sellers, the CORE constraints can sometimes not be satisfied, if coalitions containing losing bundles are taken into account as well. Because of this, no stable profit sharing can be guaranteed in combinatorial exchanges anyways, and we focus on a stable profit sharing amongst the trading participants. The other reason is that we do not think that a mere participation in the exchange should be rewarded. This would also open the door for fake bidding to obtain some of the profit. Finally, please note again that, by the definition of the achievable profit $P(S)$ in Eq. (1), our definition of CORE builds on the SCP model.

3.1 Minimal Solutions and Profit Sharing

It is worth mentioning that the existence of a profit sharing scheme, which satisfies BB, IR, and CORE, depends on the WDP solution it is based on. For non-minimal solutions, there exists a set $S \subseteq \mathcal{L}$ with $P(S) > P_{\mathcal{L}}$. If we now try to distribute profit in a way that satisfies CORE, this set S has to be given more money than the whole solution generates, which in combination with IR directly rules out BB.

A solution is called optimal, if there is no other solution with higher profit. If the WDP is modeled as an SCP, any optimal solution is a minimal solution:

Lemma 1. *Any optimal (i.e., profit maximizing) solution $\mathcal{L}$ of the WDP modeled as an SCP is a minimal solution.*

Proof. Suppose there was a subset $S \subseteq (O \cup \mathcal{L})$ with $P(S) > P_{\mathcal{L}}$. Then taking just S plus the artificial bids $\hat{b}$ for all offering bundles not completely covered by S is a new solution $\bar{\mathcal{L}}$. The artificial bids generate zero profit, so $P_{\bar{\mathcal{L}}} = P(S) > P_{\mathcal{L}}$, which contradicts the optimality of $\mathcal{L}$.

There might be bundles in S though, which contain items of offering bundles that should now fall back to their owners. More precisely, there are overlaps between the bundles in S and the newly included artificial bids, which have to be resolved. This is only possible in an SCP model. □

Please note that Lemma 1 does not necessarily mean that we require an optimal solution. Also non-optimal solutions can be minimal, and there even might be special scenarios, in which every solution found by some solution algorithm is minimal.

3.2 The Monetary Flow Network

The distribution of the generated profit amongst the offers and the winning bids of the auction by means of a profit sharing function $p : (O \cup \mathcal{L}) \to \mathbb{R}^{\geq 0}$ can be easily modeled as a flow network. We introduce two types of nodes: Offering bundles correspond to sources that inject money into the network. On

[3] For an overview on pseudonymous bidding in combinatorial auctions see [3].

the other hand, we have sink nodes (corresponding to winning bundles) that are supplied (i.e., paid for) by the network for executing the acquired items. To transfer money between nodes, we have edges from sources to sinks. This naturally results in a bipartite flow network, typical for the so called Hitchcock Problem [9].

An important goal when sharing profit is that the profit should be distributed where it is generated. Accordingly, no payments should be made between players that do not exchange any items, as we do not want subsidies to be necessary. This "localization" property is established by restricting the set of edges to pairs of nodes that trade items:

Definition 2 (Monetary flow network)**.** *For a solution $\mathcal{L} \subseteq B$ and the set of offering bundles O, define the node-set $O \cup \mathcal{L}$ and the set of edges $E := \{(b_j, b_k) \in O \times \mathcal{L} \mid I_{b_j} \cap I_{b_k} \neq \emptyset\}$. We then call $G = (O \cup \mathcal{L}, E)$ a (monetary) flow network for the bundles in $O \cup \mathcal{L}$.*

Since we identify nodes with bundles, we usually use the letter b for a node of G. Note that the flow network is bipartite, and all $b \in O$ just have outgoing edges while all $b \in \mathcal{L}$ just have incoming edges. A **flow** is a mapping $f : E \to \mathbb{R}^{\geq 0}$. For an edge $(b_j, b_k) \in E$, we denote by $f(b_j, b_k)$ the flow from node b_j to node b_k. In reality, this flow represents a certain amount of money transfered from the owner of b_j to the owner of b_k. We shortly write

$$f(X, Y) := \sum_{(x,y) \in E \cap (X \times Y)} f(x, y)$$

for the total flow from the nodes in $X \subseteq O$ to the nodes in $Y \subseteq \mathcal{L}$.

To reflect the freedom of choice in terms of profit sharing, we parametrize the sources' and sinks' capacities in G by a vector $\kappa_G := (\kappa_b)_{b \in O \cup \mathcal{L}}$: For sources, we set $\kappa_b \in [0, v_b]$, while for sinks we have $\kappa_b \in [-P_{\mathcal{L}}, -v_b]$. So, for an offering bundle, the owner will pay at most his offering price, and for a bidding bundle the owner will get at least the price he asked for. The remaining profit then simply is

$$p(b) = \begin{cases} v_b - \kappa_b & \text{for offering bundles } b \in O \\ -\kappa_b - v_b & \text{for bidding bundles } b \in B \end{cases} \tag{2}$$

and we can directly deduce the range of $p(b)$ by the ranges for κ_b:

$$p(b) \in [0, v_b] \; \forall b \in O \quad \text{and} \quad p(b) \in [0, P_{\mathcal{L}}] \; \forall b \in \mathcal{L} \; . \tag{3}$$

Just like for the capacities, we shortly write p_G for $(p(b))_{b \in O \times \mathcal{L}}$. Note that we can of course determine the nodes' capacity setup for any given profit distribution p_G by solving equation (2) for κ_b. Finally, as in all flow networks, the usual flow conservation constraints should hold:

$$f(\{b\}, \mathcal{L}) - f(O, \{b\}) = \kappa_b \quad \forall b \in O \cup \mathcal{L} \tag{4}$$

A flow f in a monetary flow network $G = (O \cup \mathcal{L}, E)$ that obeys constraints (4) with respect to capacities κ_G is called a **valid κ_G-flow**. A profit sharing function $p_G : (O \cup \mathcal{L}) \to \mathbb{R}^{\geq 0}$ that can be generated by some valid κ_G-flow in G is called a **flow-admitting** profit sharing. We define as

$$\Phi_G := \{ \; p_G \mid p_G \text{ is a flow-admitting profit sharing in } G\}$$

the set of all flow-admitting profit sharing schemes for a solution $\mathcal{L}$.

Because we assume that every item is assigned to exactly one bundle (see introduction), we can state the following lemma:

Lemma 2. *For all subsets $S \subseteq \mathcal{L}$ of a solution $\mathcal{L} \subseteq B$, in a monetary flow network all edges from nodes in F_S lead to nodes in S:*

$$f(F_S, \mathcal{L} \setminus S) = 0 \ . \tag{5}$$

Proof. Generally, for every bundle $b \in O$ it holds that there is only one edge going out for each item $i \in I_b$. By the definition of F_S, all bundles $b \in F_S$ can be completely satisfied by S, i.e., there cannot be any $i \in I_b$ that is taken by a bid in $\mathcal{L} \setminus S$. Therefore there cannot be any edges and the flow is zero. □

3.3 Properties of the Monetary Flow Network

In this subsection we will show that the monetary flow network just introduced has some connections to game theoretic properties of profit sharing schemes.

Theorem 1. *Let $\mathcal{L} \subseteq B$ be a solution. If $p : (O \cup \mathcal{L}) \to \mathbb{R}$ is a flow-admitting profit sharing scheme on the corresponding flow network, then p satisfies* BB*,* IR*, and* CORE*.*

Proof. First of all, any profit sharing implied by the flow network is of course budget balanced, because of the flow conservation constraints (4) and the bounds on κ_G. As stated in (3) we do not allow negative values for p and therefore we also have individual rationality.

To show that p satisfies CORE, we use some easy set transformations that can be best understood with the help of Figure 3. For all $S \subseteq O \cup \mathcal{L}$ it holds that

$$\begin{aligned}
\sum_{b \in S} p(b) = \sum_{b \in S \cap \mathcal{L}} p(b) + \sum_{b \in S \cap O} p(b) &\overset{(2)}{\underset{(4)}{=}} \sum_{b \in S \cap \mathcal{L}} (f(O, \{b\}) - v_b) + \sum_{b \in S \cap O} (v_b - f(\{b\}, \mathcal{L})) \\
&= f(O, S \cap \mathcal{L}) - f(S \cap O, \mathcal{L}) + \Big(\sum_{b \in S \cap O} v_b - \sum_{b \in S \cap \mathcal{L}} v_b \Big) \\
&= f(O, S \cap \mathcal{L}) - f(S \cap O, \mathcal{L}) + \Big(\sum_{b \in (S \cap O) \setminus F_S} v_b + P(S) \Big) \\
&= \Big(f(O \setminus S, S \cap \mathcal{L}) + f(S \cap O, S \cap \mathcal{L}) \Big) \\
&\quad - \Big(f(S \cap O, \mathcal{L} \setminus S) + f(S \cap O, S \cap \mathcal{L}) \Big) + \sum_{b \in (S \cap O) \setminus F_S} v_b + P(S) \\
&= f(O \setminus S, S \cap \mathcal{L}) - f(S \cap O, \mathcal{L} \setminus S) + \sum_{b \in (S \cap O) \setminus F_S} v_b + P(S) \\
&= \underbrace{f(O \setminus S, S \cap \mathcal{L})}_{\geq 0} - \underbrace{f((S \cap O) \setminus F_S, \mathcal{L} \setminus S)}_{\leq \sum_{b \in (S \cap O) \setminus F_S} v_b, \text{ since } \kappa_b \leq v_b} + \underbrace{f(F_S \cap S, \mathcal{L} \setminus S)}_{=0 \text{ by Lemma 2}} \Big) \\
&\quad + \sum_{b \in (S \cap O) \setminus F_S} v_b + P(S) \\
&\geq P(S)
\end{aligned}$$

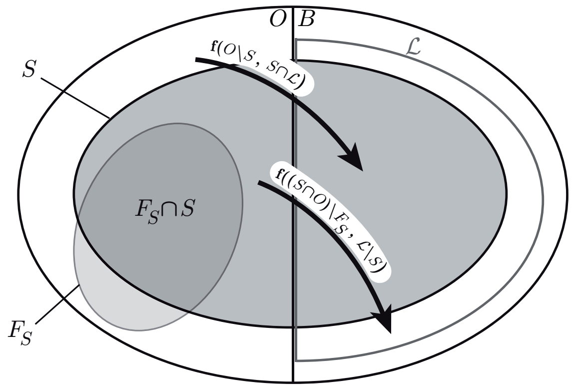

Figure 2: The sets used in Theorem 1. The arrows illustrate two of the terms used in the last transformation of the proof.

□

Next we will proof the contrary, namely that every profit sharing that obeys BB, IR, and CORE is a flow-admitting profit sharing scheme:

Theorem 2. *Let $\mathcal{L} \subseteq B$ be a solution. If p is a profit sharing w.r.t. $\mathcal{L}$, which satisfies* BB*,* IR*, and* CORE*, then it is a flow-admitting profit sharing scheme $p \in \Phi_G$ on the corresponding monetary flow network $G = (O \cup \mathcal{L}, E)$.*

Proof. We set up an extended flow network consisting of a digraph $H = (V, A)$ and arc capacities $u : A \to \mathbb{R}^{\geq 0}$. The set of nodes consists of all nodes of the monetary flow network $G = (O \cup \mathcal{L}, E)$ and two additional nodes: The source node s is connected to all nodes $v \in O$, and all nodes $v \in \mathcal{L}$ are connected to the sink node t. In addition to that, we maintain all edges from E. More formally this means that

$$\begin{aligned} V &:= \{s, t\} \cup O \cup \mathcal{L} \\ \text{and} \quad A &:= E \cup (\{s\} \times O) \cup (\mathcal{L} \times \{t\}) \, . \end{aligned}$$

The arc capacities are given as follows:

$$\begin{aligned} u((b_1, b_2)) &:= \infty && \forall (b_1, b_2) \in E \\ u((s, b)) &:= v_b - p(b) && \forall (s, b) \in \{s\} \times O \\ u((b, t)) &:= v_b + p(b) && \forall (b, t) \in \mathcal{L} \times \{t\} \end{aligned}$$

Note that the arc capacities are chosen in a way that each bundle is able to receive exactly the profit specified by p, cf. (2). As p is BB, we have

$$\begin{aligned} & \sum_{b \in O \cup \mathcal{L}} p(b) = P_{\mathcal{L}} = \sum_{b \in O} v_b - \sum_{b \in \mathcal{L}} v_b \\ \Leftrightarrow \quad & \sum_{b \in O} v_b - \sum_{b \in O} p(b) = \sum_{b \in \mathcal{L}} v_b + \sum_{b \in \mathcal{L}} p(b) \end{aligned}$$

and therefore

$$H^{\max} := \sum_{b \in O} u((s, b)) = \sum_{b \in \mathcal{L}} u((b, t)) \, . \tag{6}$$

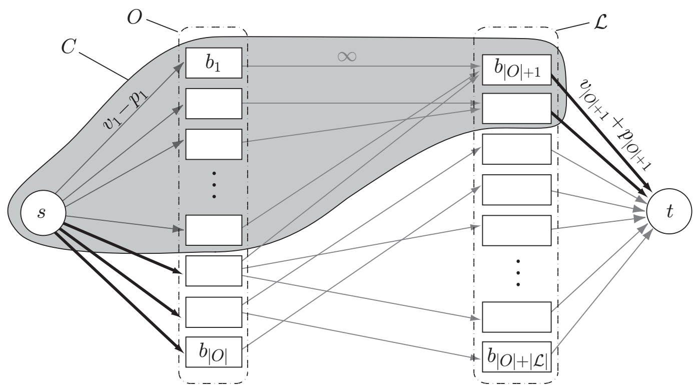

Figure 3: Visualization of a minimum cut as used in the proof of Theorem 2.

We now seek a maximum flow $f^H : A \to \mathbb{R}^{\geq 0}$ in the network H. Call

$$\text{val}(f^H) := \sum_{b \in O} f^H(s, b)$$

the value of the flow. If $\text{val}(f^H) = H^{\max}$, it is easy to see that we have also found a valid flow on the original monetary flow network $G = (O \cup \mathcal{L}, E)$, simply by using the flow values on the edges $O \times \mathcal{L}$.

So assume $\text{val}(f^H) < H^{\max}$. By the MaxFlowMinCut-Theorem [6], we can then find an s-t-cut $C \subseteq V$ whose capacity equals the value of the flow, i.e., a subset of nodes where the leaving edges have a combined capacity of $\text{val}(f^H)$. We assumed that $\text{val}(f^H) < H^{\max}$, so by (6) this cut can neither be $\{s\}$ nor $V \setminus \{t\}$. Furthermore, no edges from $O \times \mathcal{L}$ can leave the cut, since they have infinite capacity. Thus, the edges leaving the cut consist of one or more edges $(s, b), b \in O$, and one or more edges $(b, t), b \in \mathcal{L}$.

We know by (6) that the capacity of C is smaller than the capacity of the cut $C' = \{s\}$, and therefore

$$\sum_{b \in O \setminus C} u((s, b)) + \sum_{b \in C \cap \mathcal{L}} u((b, t)) < \sum_{b \in O} u((s, b)) \, .$$

The corresponding edges are displayed bold in Figure 3. This directly implies

$$\begin{aligned} & \sum_{b \in C \cap \mathcal{L}} u((b, t)) < \sum_{b \in C \cap O} u((s, b)) \\ \Leftrightarrow \quad & \sum_{b \in C \cap \mathcal{L}} v_b + p(b) < \sum_{b \in C \cap O} v_b - p(b) \\ \Leftrightarrow \quad & \sum_{b \in C \setminus \{s\}} p(b) < \sum_{b \in C \cap O} v_b - \sum_{b \in C \cap \mathcal{L}} v_b = P(C \setminus \{s\}) \, , \end{aligned}$$

which contradicts the CORE property of p. To realize the last equality, please note again that there cannot be any arcs in the cut from $C \cap O$ to $\mathcal{L} \setminus C$, since they have infinite capacity. Therefore $C \cap F_C = C \cap O$ and the equality holds (cf. (1)). □

Theorem 1 and Theorem 2 show the equivalence of flows in the monetary flow network and profit sharing schemes that are BB, IR, and CORE. This directly gives us two possibilities to find a profit sharing that satisfies the three properties, or to test a given profit sharing for them: As this type of flow network can be represented by a Linear Program (LP), it can be solved (with any linear objective function, e.g., maximizing a certain bundle's profit) by a polynomial-time algorithm (like the Ellipsoid method or an interior point method). Secondly, one can use specialized algorithms for the Minimum Cost Flow Problem. For an overview see, e.g., the corresponding chapter in [9].

As a final note for this chapter, we would like to emphasize a remarkable implication of Theorem 2: We are able to represent an arbitrary profit sharing that satisfies BB, IR, and CORE as a flow in the monetary flow network for the corresponding solution $\mathcal{L}$. Since the set of edges in this network is restricted to pairs of bundles that exchange items in $\mathcal{L}$, this means that we do not need to transfer money between unrelated bundles, so no subsidies are necessary.

4 Connection to Existing Approaches

4.1 Connection to VCG Discounts

In this subsection, we use the flow network to share profit amongst the bundles of the exchange. Consider a solution $\mathcal{L} \subseteq B$ and the corresponding monetary flow network $G = (O \cup \mathcal{L}, E)$. For each bundle $b \in O \cup \mathcal{L}$, we define as $p^*(b)$ the maximum possible profit that any valid network flow admits:

$$p^*(b) := \max_{p_G \in \Phi_G} p(b), \quad b \in O \cup \mathcal{L}$$

For any bundle $b \in O \cup B$, p^* can be imagined as the maximum profit that can be shifted to b through the edges of the network (granting all other bundles at least their required minimum payments). We now show how p^* is connected to VCG-discounts that are based on an auction held just amongst the winning bundles.

Theorem 3. *Let $\mathcal{L}^* \subseteq B$ be a minimal solution and $G = (O \cup \mathcal{L}^*, E)$ the corresponding monetary flow network. Let $p^*(b)$ be the maximal profit achievable for some $b \in O \cup \mathcal{L}^*$ by any flow-admitting profit sharing $p^* \in \Phi_G$. Then*

$$p^*(b) = P_{\mathcal{L}^*} - \Delta_b \ ,$$

where

$$\Delta_b := \max_{S \subseteq O \cup \mathcal{L}^* \setminus \{b\}} P(S)$$

is the maximum profit of a partial exchange of $\mathcal{L}^*$, b is not involved in.

Before we can start the proof, we present some auxiliary definitions and lemmata. The actual proof is then rather short and comprehensive.

Lemma 3 (Supermodularity of $P(\cdot)$). *For any solution $\mathcal{L} \subseteq B$ and any two subsets $S, T \subset (O \cup \mathcal{L})$, it holds that*

$$P(S \cup T) + P(S \cap T) \geq P(S) + P(T) \ .$$

Proof. First, note that

$$F_{S\cap T} = F_S \cap F_T \tag{7}$$

since, on the one hand, $F_{S\cap T} \subseteq F_S$ and $F_{S\cap T} \subseteq F_T$, and on the other hand it holds that $x \in F_S \wedge x \in F_T \Rightarrow x \in F_{S\cap T}$. Using this, and realizing by simple counting arguments that

$$F_{S\cup T} \supseteq F_S \cup F_T \tag{8}$$

and
$$\sum_{b\in(S\cup T)\cap(F_S\cup F_T)} v_b + \sum_{b\in(S\cap T)\cap(F_S\cap F_T)} v_b \geq \sum_{b\in S\cap F_S} v_b + \sum_{b\in T\cap F_T} v_b \ , \tag{9}$$

we get:

$$\begin{aligned}
&P(S\cup T) + P(S\cap T)\\
&= \sum_{b\in(S\cup T)\cap F_{S\cup T}} v_b - \sum_{b\in\mathcal{L}\cap(S\cup T)} v_b + \sum_{b\in(S\cap T)\cap F_{S\cap T}} v_b - \sum_{b\in\mathcal{L}\cap(S\cap T)} v_b\\
&\underset{(7)}{=} \sum_{b\in(S\cup T)\cap F_{S\cup T}} v_b + \sum_{b\in(S\cap T)\cap(F_S\cap F_T)} v_b - \sum_{b\in\mathcal{L}\cap(S\cup T)} v_b - \sum_{b\in\mathcal{L}\cap(S\cap T)} v_b\\
&\underset{(8)}{\geq} \sum_{b\in(S\cup T)\cap(F_S\cup F_T)} v_b + \sum_{b\in(S\cap T)\cap(F_S\cap F_T)} v_b - \sum_{b\in\mathcal{L}\cap(S\cup T)} v_b - \sum_{b\in\mathcal{L}\cap(S\cap T)} v_b\\
&= \sum_{b\in(S\cup T)\cap(F_S\cup F_T)} v_b + \sum_{b\in(S\cap T)\cap(F_S\cap F_T)} v_b - \sum_{b\in\mathcal{L}\cap S} v_b - \sum_{b\in\mathcal{L}\cap T} v_b\\
&\underset{(9)}{\geq} \sum_{b\in S\cap F_S} v_b + \sum_{b\in T\cap F_T} v_b - \sum_{b\in\mathcal{L}\cap S} v_b - \sum_{b\in\mathcal{L}\cap T} v_b\\
&= P(S) + P(T)
\end{aligned}$$

□

Definition 3. *Let $\mathcal{L} \subseteq B$ be a solution. A set $S \subseteq (O\cup\mathcal{L})$ is called* tight*, if*

$$\forall T \subseteq S : P(T) \leq P(S) \ .$$

Note that the concept of tightness may be also used to define a minimal solution: A solution $\mathcal{L}$ is minimal, iff $\mathcal{L}\cup O$ is tight.

Definition 4. *Let $\mathcal{L} \subseteq B$ be a solution. The directed graph $\widetilde{G} = (\widetilde{V}, \widetilde{E})$, where*

$$\begin{aligned}
\widetilde{V} &:= \{v_S \mid S \subseteq O\cup\mathcal{L}, P(S) > 0 \text{ and } S \text{ is tight}\}\\
\text{and}\quad \widetilde{E} &:= \{(v_S, v_T) \mid v_S, v_T \in \widetilde{V},\ S \supset T\}\ ,
\end{aligned}$$

is called the partial exchange graph*. It contains a node for each subset S of offering and winning bundles that enables a profitable partial exchange of items, and edges from a node to all of its subsets. A* leaf node *is defined as a node with outdegree zero and represents a minimal partial exchange of $O\cup\mathcal{L}$.*

We now present an alternative method to generate a flow-admitting profit sharing (besides the two options mentioned at the end of Section 3).

Algorithm 1. *Let $\mathcal{L}^*$ be a minimal solution and $G = (O\cup\mathcal{L}, E)$ the corresponding flow network.*

1. Construct $\widetilde{G} = (\widetilde{V}, \widetilde{E})$

2. Set $p(b) = 0$ *for all* $b \in O \cup \mathcal{L}$

3. Repeat until $\widetilde{G}$ *has no more nodes:*

(a) Pick a leaf node $v_S \in \widetilde{V}$

(b) Distribute $\max(0, P(S) - \sum_{b \in S} p(b))$ *profit arbitrarily amongst the nodes of* S*, adding it to their* $p(\cdot)$*-values*

(c) Remove v_S *from* $\widetilde{V}$

Lemma 4. *Algorithm 1 generates a profit distribution* $p \in \Phi_G$*.*

Proof. We show that we can apply Theorem 2 to any profit distribution generated by the algorithm. Obviously, CORE and IR are satisfied during the whole algorithm. Untight sets are not a problem, since they receive enough profit by their respective tight subsets. It remains to show that we do not distribute too much profit, i.e., it holds that

$$\sum_{b \in O \cup \mathcal{L}} p(b) \leq P_{\mathcal{L}} \ . \tag{10}$$

Let $\{T_1, \ldots, T_n\}$ be the sequence of partial exchanges as they are processed by the algorithm. Recall that $T_j \subseteq O \cup \mathcal{L}$, T_j is tight, and $P(T_j) > 0$ for all $j = 1, \ldots, n$. Let

$$\mathcal{T}_k := \bigcup_{j=1}^{k} T_j \ , \quad \mathcal{T}_k \subseteq O \cup \mathcal{L}$$

be the set of all bids belonging to partial exchanges already processed, including T_k. Note that $T_n = O \cup \mathcal{L}$ and therefore also $\mathcal{T}_n = O \cup \mathcal{L}$. Define as p_b^j the total amount of profit distributed to bundle $b \in (O \cup B)$ *directly after* processing v_{T_j} in the algorithm. We now prove a stronger statement than (10), namely that

$$\sum_{b \in \mathcal{T}_j} p_b^j \leq P(\mathcal{T}_j) \quad \forall j \in \{1, \ldots, k\} \ , \tag{11}$$

which contains (10) for $j = n$.

For $j = 1$ the statement is obviously true: In the first distribution phase of the algorithm, exactly $P(T_1)$ profit is distributed amongst the nodes of T_1 and no other profit has been distributed yet. Now assume (11) holds for $j = k$ and the next node to be processed by the algorithm is $v_{T_{k+1}}$. The additional amount of profit that will be distributed is obviously bounded by 0 and $P(T_{k+1})$. Besides, if $T_{k+1} \cap \mathcal{T}_k$ (see fig. 4) is a partial exchange with strictly positive profit $P(T_{k+1} \cap \mathcal{T}_k)$, the corresponding node $v_{T_{k+1} \cap \mathcal{T}_k}$ will already have been processed by the algorithm (as it is a proper subset of T_{k+1}), so all profit of $v_{T_{k+1} \cap \mathcal{T}_k}$ must have been distributed to nodes of T_{k+1}. As a direct consequence, the profit newly distributed can be at most $P(T_{k+1}) - P(T_{k+1} \cap \mathcal{T}_k)$. Recall that by Lemma 3 $P(\mathcal{L})$ is supermodular, so it holds that

$$P(T_{k+1} \cup \mathcal{T}_k) + P(T_{k+1} \cap \mathcal{T}_k) \geq P(T_{k+1}) + P(\mathcal{T}_k) \ . \tag{12}$$

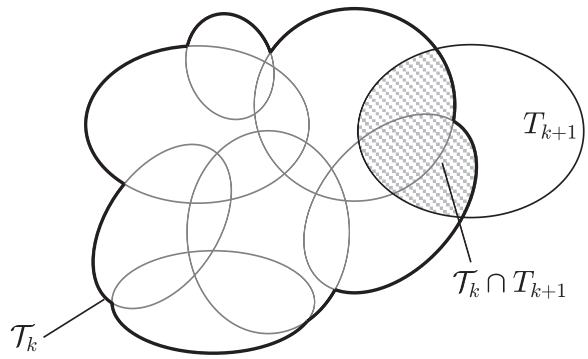

Figure 4: The sets used in the proof of Lemma 4, when processing node $v_{T_{k+1}}$.

Putting everything together, we get the desired statement for $j = k+1$:

$$\begin{aligned}\sum_{b \in \mathcal{T}_{k+1} \cup F_{\mathcal{T}_{k+1}}} p_b^{k+1} &\leq \sum_{b \in \mathcal{T}_k \cup F_{\mathcal{T}_k}} p_b^k + (P(T_{k+1}) - P(T_{k+1} \cap \mathcal{T}_k)) \\ &\leq P(\mathcal{T}_k) + (P(\mathcal{T}_k \cup T_{k+1}) - P(\mathcal{T}_k)) \\ &= P(\mathcal{T}_k \cup T_{k+1}) = P(\mathcal{T}_{k+1})\end{aligned}$$

□

This concludes the proof. The algorithm allows us to directly state that $\Phi_G \neq \emptyset$ for an arbitrary minimal solution $\mathcal{L}^* \subseteq B$ and the corresponding flow network $G = (O \cup \mathcal{L}^*, E)$: If we apply the algorithm, we always end up with a flow-admitting profit sharing. Note that the algorithm is related to the algorithm `SubmodularCostShare` in [13], p.397.

We can now finally prove Theorem 3 with the help of Algorithm 1:

Proof. Let S^* be the subset of $(O \cup \mathcal{L}^*) \setminus \{b\}$ with $P(S^*) = \Delta_b$. We show that the algorithm can distribute all profit to bundles in S^*, or to b.

Assume the algorithm processes a node belonging to a tight set $T \subseteq (O \cup \mathcal{L}^*) \setminus \{b\}$. If there exists a substructure $X \subseteq (T \setminus S^*)$ with $P(X) > 0$. Then

$$P(S^* \cup X) \underset{Lemma\ 3}{\geq} P(S^*) + P(T) > P(S^*),$$

which contradicts the maximality of S^*. Next, note that there cannot be an edge

$$e \in O \cap (S^* \setminus T) \times \mathcal{L}^* \cap (T \setminus S^*),$$

since this means the bundle b, which the edge departs from, cannot be satisfied by S^*, contradicting its tightness (removing b from S^* increases the profit). Similarly, there cannot be an edge $e \in O \cap (T \setminus S^*) \times \mathcal{L}^* \cap (S^* \setminus T)$, since that would contradict the tightness of T. Hence, all profit of T has to be realized within S^*. The algorithm will therefore distribute all profit to bundles in S^*. On the other hand, the profit distributed amongst nodes of S^* can only add up to $P(S^*)$ because of eq. (11), so exactly $P(S^*)$ will be distributed. For the remaining sets containing bundle b, we can always account the whole profit of that node to b, which adds up to a total of $P_{\mathcal{L}^*} - P(S^*)$. As Algorithm 1 always generates a flow-admitting profit sharing scheme, and as there is no possibility to distribute more than $P_{\mathcal{L}^*} - P(S^*)$ to b, the statement follows. □

Theorem 3 shows an interesting connection. First, recall that the VCG-discount is calculated as the amount saved by the presence of the particular player, or here: the particular bundle. Applied to just the winning bundles, this gets us exactly $p^*(b)$: It is defined as the solution's original profit minus the profit with b taken out. So it is equivalent to a bundle's VCG-discount in an auction held just amongst the winning bundles.

4.2 Connection to the Shapley Value

A profit sharing scheme that is often applied, since it has a couple of nice properties, is the well-known Shapley Value [17]. As we did before, we will apply it to just the offering and the winning bundles and adapt the definition accordingly:

Definition 5 (Shapley Value)**.** *Let $\mathcal{L}^* \subseteq B$ be a minimal solution. The Shapley Value for bundle $b \in O \cup \mathcal{L}^*$ is defined as*

$$p^{\mathrm{sh}}(b) = \sum_{S \subseteq O \cup \mathcal{L}^*} \frac{(n - |S|)!(|S| - 1)!}{n!} (P(S) - P(S \setminus \{b\})) .$$

It sort of generalizes the concept of VCG-payments, as it does not only calculate the marginal profit for a bundle when added last to the remaining bundles, but averages over all possible permutations of bundles and all inserting positions. Hence, it does not over-emphasize the threat of a single bundle leaving the coalition, but switches to a more global view, where every bundle's significance is put into perspective of all dependencies.

One can easily show that the Shapley Value is a flow-admitting profit sharing scheme. It is known that for games with a supermodular characteristic function (like $P(\cdot)$, see Lemma 3), the Shapley Value lies in the *core*, which means it satisfies conditions BB and CORE (see, e.g., [13]). Besides, it is also IR, as $p^{\mathrm{sh}}(b) \geq 0$ for all bundles $b \in O \cup \mathcal{L}^*$. Therefore the next corollary directly follows from Theorem 2.

Corollary 1. *Let $\mathcal{L}^* \subseteq B$ be a minimal solution and $G = (O \cup \mathcal{L}^*, E)$ the corresponding monetary flow network. Then p^{sh} is a flow-admitting profit sharing scheme, i.e., $p^{\mathrm{sh}} \in \Phi_G$.*

So, for all minimal solutions the Shapley Value can be represented by a flow in the corresponding flow network $G = (O \cup \mathcal{L}^*, E)$.

5 Conclusion and Future Research

In this paper we examined profit sharing schemes in the context of combinatorial exchanges. We introduced a monetary flow network and were able to show three interesting connections to existing theory. First, every profit sharing scheme that is budget balanced, incentive compatible, and satisfies the core property is equivalent to a flow in a corresponding monetary flow network. This defines the class of profit sharing schemes satisfying the three mentioned properties in a very comprehensive way. Secondly, we discovered a connection to the classical VCG discounts. Shifting all profit to a certain bundle within the flow network exactly tells us the VCG discount for that bundle, calculated just within the set of traded bundles. Lastly, we showed that the Shapley Value, applied to the set of traded bundles, can be represented in the flow network.

The question that remains to be answered is, of course, how to use the degrees of freedom that the flow network points out. Easy examples show that simply scaling down p^* to force both a relation to VCG payments and keep budget balance may lose the core property, i.e., will result in a profit sharing scheme that is no longer flow-admitting. The same holds for other simple approaches, like an even distribution of the profit to all participants. It remains to show whether there are meaningful and interesting flow-admitting profit sharing schemes besides the Shapley Value. Focusing on fairness amongst participants seems a promising next step.

Finally, it seems worth to further characterize how demanding the concept of a minimal solution is: In which cases is a non-optimal SCP solution still minimal? We are not aware of any literature on such a connection between the employed WDP model, the solution quality, and the properties of the subsequent profit sharing.

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