COMBIMA: Truthful, Budget Maintaining, Dynamic Combinatorial Market

JAAMAS Track

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1 EXTENDED ABSTRACT

One-sided auctions have long been studied in economics and the computer science multi-agent planning domain [10, 12, 21]. One-sided auctions aim to find a high-social welfare (SWF) (efficient) allocation of a commodity to a set of agents, while ensuring that agents' best strategy is to truthfully report their input. An important extension of one-sided auctions are one-sided combinatorial auctions [17, 19, 20] where multiple commodities are offered for sale. Agents bid on bundles of commodities, which allows agents to express complex preferences over subsets of commodities (see [9] for many examples within). An elegant and well-studied class of combinatorial one-sided auctions are the sequential posted price auctions in which agents are presented sequentially with a vector of prices and must choose their preferred bundle given the price vector (among the first studied are [1, 18]). One-sided combinatorial auctions have been applied to various problems, including airport time-slot allocation [17], distributed query optimization [20] and transportation service procurement [19].

Recent years have brought increased attention to the problems that arise in two-sided markets, where there are both buying and selling agents. As opposed to one-sided auctions where the auctioneer initially holds the commodity or the commodities and is not considered strategic, in two-sided markets commodities are initially held by the selling agents, who have costs for the commodities they hold and are expected to behave strategically. The market maker must match buying agents with selling agents and determine what price each matched buying agent pays the market and what price the market pays each selling agent.

The cornerstone method in auction theory for high-SWF (efficient) allocation and incentivizing agents' truth-telling strategy is the Vickrey-Clarke-Groves (VCG) mechanism [6, 13, 22]. In addition to motivating agents to report their true input, VCG is also individually rational (IR) in many settings. IR requires that no agent can lose by participating in the mechanism. In two-sided markets, a further important requirement is weak budget-balance (BB) meaning that the market does not end up with a loss. VCG is not BB except in special cases [14]. It is well known from [16] that maximizing SWF while maintaining IR and truthfulness perforce runs a deficit (is not BB) even in the bilateral trade setting, i.e., when there are just two agents trading with each other. A well-known circumvention of [16]'s impossibility, in the setting of single-commodity single-unit two-sided markets, relaxes efficiency in return for maintaining the other properties of truthfulness, IR and BB [15]. Other circumventions of [16]'s impossibility include relaxing determinism in addition to (SWF) efficiency, i.e., randomized solutions [2, 7] and randomized solutions in the combinatorial market setting [3, 8].

Current interest in two-sided markets is motivated by examples of successful practical applications of market mechanisms in supply chain markets, online advertising exchanges, and pollution-rights markets. Many of these examples represent dynamic and uncertain environments, and thus require dynamic markets where agents arrive over time. Moreover, the examples emphasize the need for solutions that involve multiple commodities and agents that can buy and sell a multiplicity of those commodities, i.e., two-sided combinatorial markets as opposed to unit demand/supply. To the best of our knowledge, the theoretical solutions of dynamic two-sided markets in the literature focus on a single commodity for sale and agents ask/offer one unit of the commodity [2, 4, 23][1].

We present, prove and evaluate a general solution that dynamically matches agents in a two-sided combinatorial market. Multiple commodities, each with multiple units, are bought and sold in different bundles by agents that arrive over time. Our dynamiC cOMBInatorial MArket solution (COMBIMA) provides the first dynamic two-sided combinatorial market that allows truthful and IR behavior for both buying and selling agents, keeps the market BB and approximates SWF efficiency.

The defining idea behind our COMBIMA solution is the transformation of the two-sided combinatorial market into a one-sided combinatorial auction. The transformation makes use of a novel principle: each selling agent is a buying agent of his own commodities. Thus all our dynamic market's selling agents become virtual buying agents who buy in a dynamic one-sided combinatorial auction along with our market's actual buying agents. COMBIMA is a

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[1]One exception in the literature is a recent work by [11] that has a single commodity with multiple demand however their market is mediated.

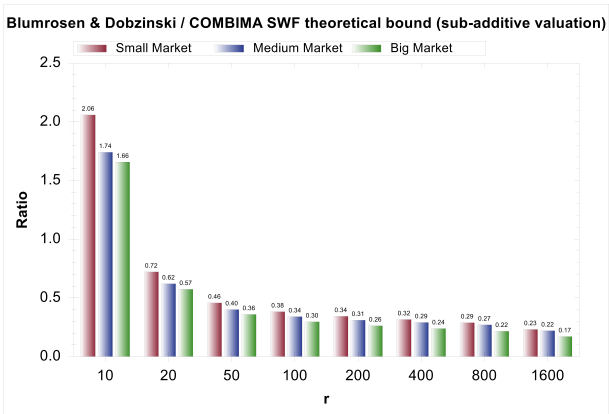

Figure 1: COMBIMA's theoretical SWF approximation ratio under different bounds of agents' demand/supply level with respect to the overall supply of commodities in three market size ranges (small with $10^3.5$, medium with 10^5 and large with 10^7 sellers respectively) vs. Blumrosen and Dobzinski 2014's theoretical SWF approximation ratio.

primal-dual sequential posted-price mechanism that builds upon a combinatorial auction studied in the literature [5]. Our proposed solution is based on solving a linear relaxation of the combinatorial problem in a dynamic fashion using the means of a Primal Dual approch. We prove that

Theorem 1.1. *COMBIMA is a truthful, IR and WBB combinatorial dynamic market that approximates the SWF of the trading buying agents and the remaining commodities with in* $O(r[(1+s_{\max}(v_{\max}-c_{\min}))^{\frac{1}{r-1}}-1]+1)$.

Where r is a known bound on the demand/supply of each commodity in each bundle. $s_{\max}$ is the largest bundle demanded (supplied) in the market however not pre-known. $v_{\max}$ is a priori known upper bound valuation and $c_{\min}$ is a priori known lower bound cost.

To evaluate the quality of COMBIMA's SWF efficiency approximation ratio we compare its theoretical bound with all the other known two sided market's SWF efficiency approximation ratio theoretical bounds; the dynamic two sided market by [2][2] and the non-dynamic two sided market by [3, 7, 8]. The most notable achievement of COMBIMA's theoretical bound was when compared with [3, 7]. In large markets with 10^7 sellers where the demand/supply of each agent is bounded by 1/1600 of the total number of units of commodities, COMBIMA's SWF efficiency approximation ratio guaranty approaches more than a 11 times improvement over that of [3]'s and is slightly better (a 1.4 times) than [7], though [3, 7]'s ratio guaranty is tailored for randomized non-dynamic markets. See figure 1.

To validate the performance of our suggested solution we experimentally tested the SWF efficiency of COMBIMA under a variety of agent bid distributions, agent demand, and market sizes, i.e., participating agent and commodity counts. We compared these results to all known SWF efficiency maximizing two-sided market benchmarks.

- An optimal non-dynamic and non-truthful allocation algorithm (simplex). Here COMBIMA's approximation approaches over 0.57 of that of the optimal SWF, as agents' demand/supply level decreases for large markets (10^7 sellers), and 0.38 and 0.4 times that of an optimal SWF approximation, as agents' demand/supply level decreases for small ($10^3.5$ sellers) and medium (10^5 sellers) markets respectively.
- [4] and [23]'s dynamic single-commodity unit-demand markets. Here COMBIMA's approximation is about twice the SWF of [4]'s for large markets (10^7 sellers) where the demand/supply is bounded by 1/800. It starts to improve on [4]'s SWF even when demand supply is only bounded by 1/100 in small markets. COMBIMA?s approximation is 1.4 the SWF of [23]'s in large markets where the demand/supply of each agent is bounded by 1/1600 of the total number of commodities' units. It starts to improve on [23]'?s SWF even when demand supply is bounded by 1/800 in small markets. The above SWF improvements are surprising given that COMBIMA is tailored for a completely general combinatorial setting, unlike [4, 23], and consequently was not expected to perform better than [4, 23].
- [7]'s randomized non-dynamic single-commodity unit-demand market. In this comparison COMBIMA's approximation approaches more than a 1.4 times improvement over that of [7]'s SWF approximation for large markets where demand/supply of each agent is bounded by 1/1600 of the total number of commodities' units even though COMBIMA is deterministic and dynamic, unlike [7], and as such it was not expected to perform better than [7].
- [3]'s randomized non-dynamic combinatorial market. In this comparison COMBIMA's approximation is more than 11 times improvement over that of [3]'s SWF approximation for large markets where demand/supply of each agent is bounded by 1/1600 of the total number of commodities' units, even though COMBIMA is deterministic and dynamic unlike [3] and as such it was not expected to perform better than [3].

Our paper's contributions are threefold. First, we provide the first dynamic two-sided combinatorial market that is truthful, IR and BB for all agents that approximates SWF efficiency. Second, our experimental tests show that our solution is general and practical as it performs better than existing two-sided market solutions whether dynamic or non-dynamic, single commodity or combinatorial. Third, our two-sided combinatorial market transformation to a one-sided combinatorial auction is of independent interest for future work on simplifying other forms of multi-sided exchanges to the well studied form of one-sided auctions.

[2] [4] and [23] did not provide an approximation guaranty.

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