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Estimation of Parameters for Recurrence Models of Earthquakes

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Abstract

Four renewal models have been applied to several seismic regions of Japan where large earthquakes occur repeatedly at fairly regular time intervals. The model parameters have been determined by the method of moments and the method of maximum likelihood. The four models represent the distributions of time intervals fairly well, though different models are best suited for different sets of data. The probability of the occurrence of the next large earthquake during a specified interval of time can be calculated easily for each model. Some sample results are presented.

1. Introduction

It has been recognized that in some seismic regions large earthquakes occur at fairly regular intervals as schematically shown in Fig. 1. Such a series of earthquakes is often represented by a renewal process, in which the time interval T between successive events has a certain distribution $w(T)$. It is possible that the time interval is related to the size or other parameters characterizing the preceding earthquake (e.g. SHIMAZAKI and NAKATA, 1980), but here we adopt the renewal model for the sake of simplicity. We describe the process by using the following notations.

T: variable representing the time interval between successive events,

T_i: observed time interval between the ith and the $(i+1)$th earthquakes,

t: time measured from the time of the last earthquake

$\mu(t)dt$: probability that the next earthquake will occur during the time interval between t and $t+dt$,

$\phi(t)$: probability that the next earthquake will occur at a time later than t,

$p(\tau|t)$: conditional probability that the next earthquake will occur during the time interval between t and $t+\tau$,

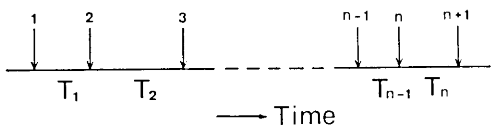

Fig. 1. Occurrence times of earthquakes (arrows) and time interval between successive earthquakes $T_i (i=1, 2, \cdots, n)$.

$w(T)$: density for the distribution of T.

The following relations are familiar in the reliability theory. Some of them have also been used in seismology (e.g. WATANABE, 1936; UTSU, 1972; HAGIWARA, 1974).

$$w(T)=\mu(T)\phi(T) , \tag{1}$$

$$\phi(t)=\exp\left\{-\int_0^t \mu(t)dt\right\}=\int_t^\infty w(T)dT , \tag{2}$$

$$p(\tau|t)=1-\exp\left\{-\int_t^{t+\tau} \mu(t)dt\right\}=1-\frac{\phi(t+\tau)}{\phi(t)} . \tag{3}$$

If $\mu(t)$ is independent of t, i.e., $\mu(t)=\alpha$, the process is called the Poisson precess. In this case T has the exponential distribution

$$w(T)=\alpha e^{-\alpha T} , \tag{4}$$

$$\phi(t)=e^{-\alpha t} , \tag{5}$$

$$p(\tau|t)=1-e^{-\alpha\tau} . \tag{6}$$

2. Four models for the recurrence of earthquakes

Recurrence of earthquakes in some seismic zones has been discussed by using the Weibull distribution for $w(T)$ (e.g. RIKITAKE, 1976), the gamma distribution for $w(T)$ (e.g. UDIAS and RICE, 1975), and the lognormal distribution for $w(T)$ (e.g. OGAWARA, 1955). UTSU (1972) assumed an exponential function for $\mu(t)$. In addition to these, the normal distribution is used for $w(T)$. The normal distribution is sometimes inconvenient, because it should be truncated at $T=0$, i.e., $w(T)=0$ for $T<0$. Here we compare four models; the Weibull model, the gamma model, the lognormal model, and the exponential probability model. The corresponding functions $w(T)$, $\phi(t)$, $\mu(t)$, and $p(\tau|t)$ are expressed by the following equations.

(1) Weibull model

$$w(T)=\alpha\beta T^{\beta-1}\exp(-\alpha T^\beta) , \quad \alpha>0,\ \beta>0 \tag{7}$$

$$\phi(t)=\exp(-\alpha t^\beta) , \tag{8}$$

$$\mu(t)=\alpha\beta t^{\beta-1}\ , \tag{9}$$

$$p(\tau|t)=1-\exp[-\alpha\{(t+\tau)^{\beta}-t^{\beta}\}]\ . \tag{10}$$

(2) Gamma model

$$w(T)=\frac{c}{\Gamma(r)}(cT)^{r-1}e^{-cT}\ , \quad c>0,\ r>0 \tag{11}$$

$$\phi(t)=\Gamma(r,\ ct)/\Gamma(r)\ , \tag{12}$$

$$\mu(t)=c^{r}t^{r-1}e^{-ct}/\Gamma(r,\ ct)\ , \tag{13}$$

$$p(\tau|t)=1-\frac{\Gamma(r,\ c(t+\tau))}{\Gamma(r,\ ct)}\ , \tag{14}$$

where $\Gamma(k,\ x)$ represents the incomplete gamma function of the second kind, i.e., $\Gamma(k,\ x)=\int_{x}^{\infty}e^{-u}u^{k-1}du$.

(3) Lognormal model

$$w(T)=\frac{1}{\sqrt{2\pi}\,\sigma T}\exp\left\{-\frac{(\ln T-m)^2}{2\sigma^2}\right\}\ , \quad m>0,\ \sigma>0 \tag{15}$$

$$\phi(t)=1-\Phi\left(\frac{\ln t-m}{\sigma}\right)\ , \tag{16}$$

$$\mu(t)=\frac{1}{\sqrt{2\pi}\ \sigma t}\exp\left\{-\frac{(\ln t-m)^2}{2\sigma^2}\right\}\Big/\left\{1-\Phi\left(\frac{\ln t-m}{\sigma}\right)\right\}\ , \tag{17}$$

$$p(\tau|t)=1-\left\{1-\Phi\left(\frac{\ln(t+\tau)-m}{\sigma}\right)\right\}\Big/\left\{1-\Phi\left(\frac{\ln t-m}{\sigma}\right)\right\}\ , \tag{18}$$

where $\Phi(x)$ represents the error integral defined by $\Phi(x)=(1/\sqrt{2\pi})\int_{-\infty}^{x}e^{-u^2/2}du$.

(4) Exponential probability model

$$w(T)=a\exp\left\{\frac{a}{b}(1-e^{bT})+bT\right\}\ , \quad a>0 \tag{19}$$

$$\phi(t)=\exp\left\{\frac{a}{b}(1-e^{bt})\right\}\ , \tag{20}$$

$$\mu(t)=ae^{bt}\ , \tag{21}$$

$$p(\tau|t)=1-\exp\left\{\frac{a}{b}e^{bt}(1-e^{b\tau})\right\}\ . \tag{22}$$

The Poisson process is a special case ($\beta=1$, $r=1$, or $b=0$) of models (1), (2), and (4). If $\beta<1$ or $r<1$ or $b<0$, the earthquakes have a tendency of clustering in time. If $\beta>1$ or $r>1$ or $b>0$, they have a tendency of intermittent occurrence.

From the occurrence times of $n+1$ earthquakes in the past, we obtain the time interval length $T_i (i=1, 2, \cdots, n)$, their mean $E[T_i]$, and the variance $V[T_i]$. To estimate the values for parameters of these models, (α, β), (c, r), (m, σ), and (a, b), the method of maximum likelihood or the method of moments are usually used. The formulas for these methods can be easily derived except the method of moments for the exponential probability model. They are shown below, together with the logarithm of the likelihood function $L=\prod_{i=1}^{n} w(T_i)$.

(1a) Weibull model—Moments

$$E[T_i]=\alpha^{-1/\beta}\Gamma\left(\frac{1}{\beta}+1\right), \tag{23}$$

$$V[T_i]=\alpha^{-2/\beta}\left\{\Gamma\left(\frac{2}{\beta}+1\right)-\Gamma^2\left(\frac{1}{\beta}+1\right)\right\}. \tag{24}$$

(1b) Weibull model—Maximum likelihood

$$\ln L=n\{\ln \alpha\beta+(\beta-1)E[\ln T_i]-\alpha E[T_i^\beta]\}, \tag{25}$$

$$E[T_i^\beta]=1/\alpha, \tag{26}$$

$$\alpha E[T_i^\beta \ln T_i]-E[\ln T_i]=1/\beta. \tag{27}$$

(2a) Gamma model—Moments

$$E[T_i]=r/c, \tag{28}$$

$$V[T_i]=r^2/c^4. \tag{29}$$

(2b) Gamma model—Maximum likelihood

$$\ln L=n\{r \ln c-\ln \Gamma(r)+(r-1)E[\ln T_i]-cE[T_i]\}, \tag{30}$$

$$E[T_i]=r/c,$$

$$E[\ln T_i]=\frac{\Gamma'(r)}{\Gamma(r)}-\ln c. \tag{31}$$

(3a) Lognormal model—Moments

$$E[T_i]=e^{m+\sigma^2/2} \tag{32}$$

$$V[T_i]=e^{2m+\sigma^2}(e^{\sigma^2}-1). \tag{33}$$

(3b) Lognormal model—Maximum likelihood

$$\ln L = -\frac{n}{2}\left\{ \ln 2\pi\sigma^2 + 2E[\ln T_i] + \frac{V[\ln T_i]}{\sigma^2} \right\}, \tag{34}$$

$$E[\ln T_i] = m\ , \tag{35}$$

$$V[\ln T_i] = \sigma^2\ . \tag{36}$$

(4) Exponential probability model—Maximum likelihood

$$\ln L = n\left\{ \ln a + \frac{a}{b}(1 - E[e^{bT_i}]) + bE[T_i] \right\}, \tag{37}$$

$$E[e^{bT_i}] = 1 + b/a\ , \tag{38}$$

$$E[T_i e^{bT_i}] - \frac{b}{a}E[T_i] = 1/a\ . \tag{39}$$

It is not difficult to find the values of $\alpha, \beta, c, r, m, \sigma, a$ and b by solving these equations for a given set of observed time intervals T_i. The method of moments for the gamma model is the easiest, but the computation of $p(\tau|t)$ for this model is somewhat time-consuming.

3. Examples from earthquake data in Japan

(1) Nankaido region

Great earthquakes ($M \gtrsim 8$) occurred repeatedly at the plate boundary off the Kii Peninsula and Shikoku Island (Fig. 2). Table 1 lists the dates of these earthquakes known in history. The source region of the 1498 earthquake is generally considered to lie in the Tonankai-Tokai region (east of the Nankaido region). This quake is included in the table because there is some evidence suggesting that this or another quake around 1498 broke at least part of the Nankaido region.

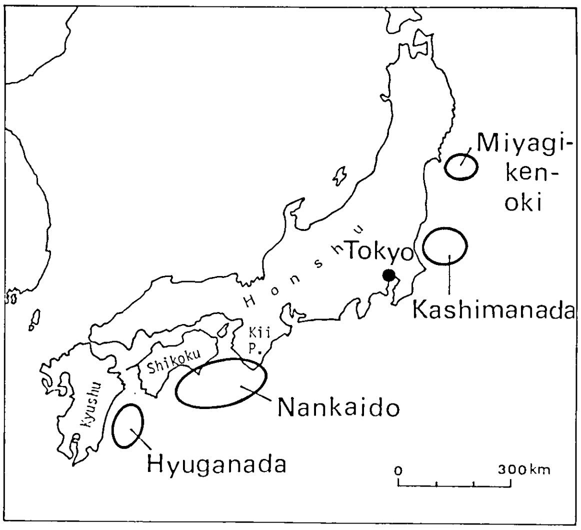

Fig. 2. Index map of the regions studied.

In Table 2 the parameter estimates and the corresponding $\ln L$ for the four models are given for three data sets, I (all events in Table 1), II (8

Table 2. Model parameters for the Nankaido region (data in Table 1)

Data set		I (all data)		II (excl. 1498)		III (1361-1946)	
		MOM	MLE	MOM	MLE	MOM	MLE
Weibull	α	1.87×10^{-7}	1.93×10^{-7}	4.81×10^{-8}	1.84×10^{-8}	2.98×10^{-14}	1.60×10^{-13}
	β	2.99	2.99	3.18	3.36	6.44	6.09
	$\ln L$	−43.56	−43.56	−38.70	−38.68	−22.49	−22.47
Gamma	c	0.0478	0.0498	0.0465	0.0408	0.259	0.254
	r	7.54	7.85	8.39	7.36	30.37	29.77
	$\ln L$	−43.24	−43.24	−39.01	−38.98	−22.27	−22.27
Lognormal	m	5.00	5.00	5.14	5.13	4.75	4.75
	σ	0.353	0.359	0.336	0.387	0.180	0.179
	$\ln L$	−43.12	−43.12	−39.32	−39.16	−22.23	−22.23
Exp. Prob.	a		9.89×10^{-4}		4.30×10^{-4}		8.34×10^{-5}
	b		0.0152		0.0180		0.0500
	$\ln L$		−44.23		−38.59		−22.67
Mean interval		157.8 yr		180.3 yr		117.1 yr	
S.D.		57.4 yr		62.3 yr		21.2 yr	

events excluding the 1498 quake), and III (6 events in 1361-1946). The parameter values are expressed using the time unit of years (β and γ are nondimensional). MOM and MLE indicate the estimate by the method of moments and the maximum likelihood method, respectively. There are no substantial differences between the values estimated by the two methods.

The cumulative distribution of the time intervals for data set I is shown in Fig. 3. Curves of $1-\phi(t)\left(=\int_0^t w(T)dT\right)$ for the four models computed on the basis of the parameters determined by the maximum likelihood method are drawn in the same figure. It is difficult to conclude which curve (model) fits best. The value of $\ln L$ is an indication of how well the model fits. The log-normal model gives the largest (best) $\ln L$ for data set I. On the other hand, the exponential probability model provides the largest $\ln L$ for data set II. The order of $\ln L$ is completely reversed between data sets I and II, but the differences among models are small in

Table 1. Great earthquakes in the Nanaido region

Nov.	29,	684
Aug.	26,	887
Feb.	22,	1099
Aug.	3,	1361
(Sep.	20,	1498)
Feb.	3,	1605
Oct.	28,	1707
Dec.	24,	1854
Dec.	21,	1946

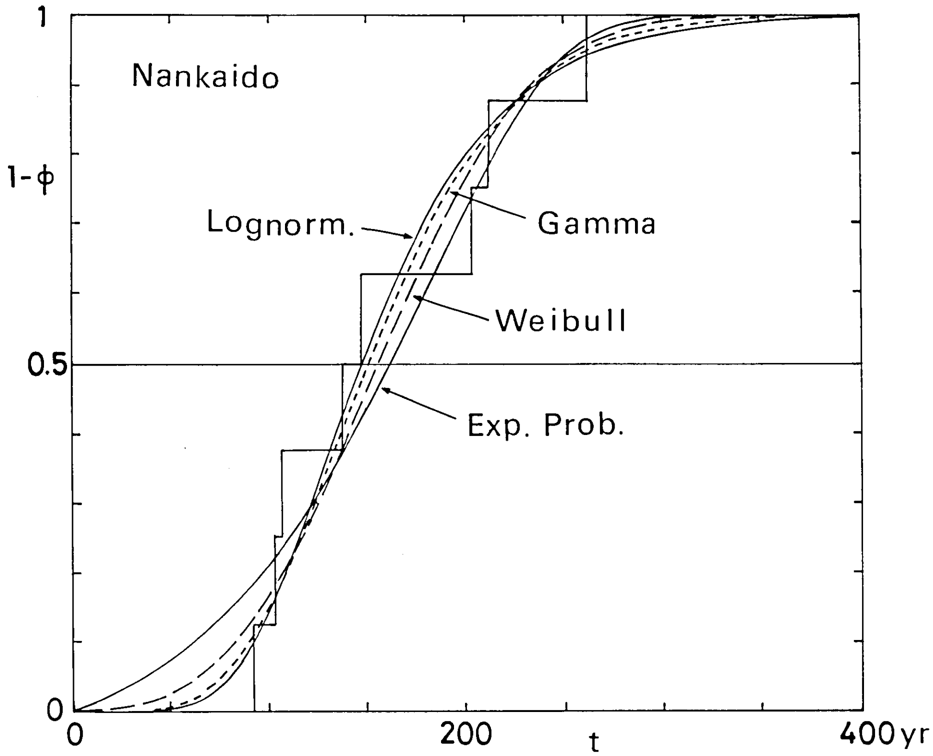

Fig. 3. Comulative distribution of T_i and the curves of $1-\phi(t)$ for four models. Great earthquakes in Nankaido region, data set I.

Table 3. Probabilities $p(\tau|t)$ for the Nankaido region (Roman: lognormal model, italics: exp. prob. model)

Data	τ \ t	0	40	80	120	160	200 yr
I	20 yr	0.000 *0.023*	0.006 *0.041*	0.099 *0.075*	0.222 *0.133*	0.294 *0.230*	0.329 *0.380*
	40	0.000 *0.053*	0.043 *0.095*	0.248 *0.167*	0.426 *0.285*	0.516 *0.459*	0.558 *0.676*
	60	0.006 *0.092*	0.138 *0.162*	0.414 *0.277*	0.595 *0.449*	0.675 *0.641*	0.712 *0.864*
	80	0.043 *0.143*	0.280 *0.246*	0.568 *0.404*	0.722 *0.613*	0.786 *0.824*	0.814 *0.959*
	100	0.138 *0.207*	0.440 *0.350*	0.695 *0.541*	0.814 *0.760*	0.861 *0.927*	0.880 *0.992*
III	20	0.000 *0.003*	0.000 *0.021*	0.198 *0.145*	0.662 *0.687*	0.804 *1.000*	0.784 *1.000*
	40	0.000 *0.011*	0.021 *0.076*	0.581 *0.442*	0.919 *0.987*	0.962 *1.000*	0.946 *1.000*
	60	0.000 *0.031*	0.215 *0.210*	0.859 *0.825*	0.984 *1.000*	0.992 *1.000*	0.984 *1.000*
	80	0.021 *0.086*	0.590 *0.485*	0.966 *0.993*	0.997 *1.000*	0.998 *1.000*	0.995 *1.000*
	100	0.215 *0.218*	0.862 *0.839*	0.993 *1.000*	0.999 *1.000*	0.999 *1.000*	0.998 *1.000*

Table 5. Model parameters for the Miyagi-ken-oki region (data in Table 4)

Data set		I (all data) MOM	I (all data) MLE	II (excl. 1770) MOM	II (excl. 1770) MLE
Weibull	α	1.05×10^{-7}	4.52×10^{-7}	6.47×10^{-8}	1.56×10^{-7}
	β	4.36	3.97	4.37	4.14
	ln L	−36.97	−36.86	−34.00	−33.97
Gamma	c	0.411	0.431	0.371	0.382
	r	14.9	15.6	14.9	15.4
	ln L	−36.08	−36.07	−33.48	−33.48
Lognormal	m	3.56	3.56	3.66	3.66
	σ	0.255	0.252	0.255	0.254
	ln L	−35.96	−35.96	−33.38	−33.38
Exp. Prob.	a		0.00239		0.00151
	b		0.0892		0.0905
	ln L		−38.11		−34.73
Mean interval		36.18 yr		40.19 yr	
S.D.		9.38 yr		10.41 yr	

both cases.

The conditional probabilities $p(\tau|t)$ are calculated for various combinations of t and τ. Table 3 lists parts of the results for data sets I and III. The Roman and italic numerals indicate the probabilities based on the lognormal and the exponential probability models, respectively. The probabilities for the Weibull and gamma models fall between the two listed values in most combinations of t and τ.

Table 4. Large earthquakes off Miyagi Prefecture (Miyagi-ken-oki)

Date	M
Sep. 9, 1616	
June 9, 1646	
Oct. 2, 1678	
Apr. 30, 1736	
(May 27, 1770)	
Feb. 17, 1793	
July 20, 1835	
Oct. 21, 1861	
Feb. 20, 1897	7.4
Nov. 3, 1936	7.5
June 12, 1978	7.4

(2) Miyagi-ken-oki region

Large earthquakes with magnitudes around 7.5 seem to occur off the coast of Miyagi Prefecture (Miyagi-ken-oki) at intervals of 30 to 50 years. Table 4 (after SENO, 1979) lists these earthquakes including somewhat uncertain events in respect to the source location. It is certain that the last three earthquakes (1897, 1936, and 1978) occurred at almost the same spot with almost equal magnitudes. The earthquake of 1736 has been regarded as being of inland origin, but there

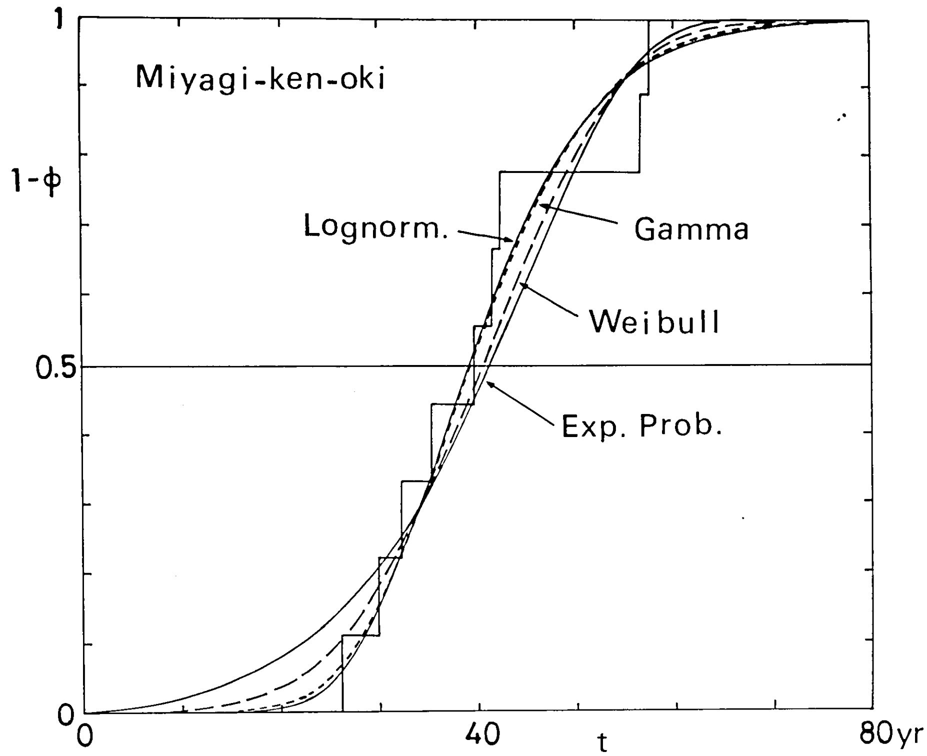

Fig. 4. Cumulative distribution of T_i and the curves of $1-\phi(t)$ for four models. Large earthquake off Miyagi Prefecture, data set II.

is a possibility that it belongs to the offshore series. The earthquake of 1646 may have been an inland one. The epicenter of the 1770 quake is quite uncertain. It may have been an inland shock in Iwate Prefecture.

Table 5 shows the parameter estimates for the two data sets I (including the 1770 quake) and II (excluding the 1770 quake). Fig. 4 is the same kind of graph as Fig. 3 for data set II. The lognormal model gives the largest in L.

(3) Ibaraki-ken-oki (Kashimanada) region and Miyazaki-ken-oki (Hyuga-nada) region

These two regions at subducting plate boundaries (Fig. 2) have a similar nature in seismicity pattern. No great earthquakes are known in his-

Table 6. Large earthquakes off Ibaraki Prefecture (Kashimanada)

Date	M
Jan. 9, 1896	7.3
⌈June 2, 1923	7.3 & 7.1
⌊Aug. 15, 1924	7.1
May 23, 1938	7.0
Apr. 14, 1943	6.7
Jan. 16, 1961	6.8
July 23, 1982	7.0

Table 7. Large earthquakes off Miyazaki Prefecture (Hyuganada)

Date	M	M_s
Nov. 25, 1899	7.1 & 6.9	7.2 & –
Nov. 2, 1931	7.1	7.6
Nov. 19, 1941	7.2	7.8
Apr. 1, 1968	7.5	7.6

tory, but earthquakes of magnitudes up to about 7.5 are relatively frequent and they occur sometimes in swarms. Tables 6 and 7 list earthquakes with an M of about 7 or more. In

Table 8. Model parameters for the Kashimanada and Hyuganada regions (data in Tables 6-7)

Data set		Kashimanada I (incl. 1943)		Kashimanada II (excl. 1943)		Hyuganada	
		MOM	MLE	MOM	MLE	MOM	MLE
Weibull	α	6.43×10^{-4}	5.62×10^{-4}	1.87×10^{-8}	1.07×10^{-8}	1.92×10^{-4}	1.03×10^{-4}
	β	2.47	2.52	5.64	5.82	2.64	2.83
	$\ln L$	−17.19	−17.18	−11.54	−11.54	−10.90	−10.89
Gamma	c	0.310	0.213	1.10	0.996	0.264	0.203
	r	5.36	3.69	23.8	21.5	6.01	4.63
	$\ln L$	−17.81	−17.60	−11.73	−11.73	−11.17	−11.11
Lognormal	m	2.77	2.17	3.05	3.05	3.05	3.01
	σ	0.414	0.597	0.203	0.219	0.392	0.506
	$\ln L$	−18.98	−18.06	−11.83	−11.80	−11.50	−11.26
Exp. Prob.	a		8.46×10^{-3}		5.11×10^{-4}		3.79×10^{-3}
	b		0.139		0.263		0.134
	$\ln L$		−16.81		−11.58		−10.60
Mean interval		17.31 yr		21.63 yr		22.78 yr	
S.D.		7.48 yr		4.43 yr		9.29 yr	

Table 9. Strong earthquakes in Tokyo area (intensity 5+ or more)

June 26, 1615
┌June 16, 1647
│July 30, 1649
└Sep. 1, 1649
(Nov. 25, 1697)
Dec. 31, 1703
Aug. 23, 1782
Dec. 7, 1812
Nov. 11, 1855
Aug. 20, 1894
Sep. 1, 1923

calculating the parameter values, three quakes in 1923 and 1924 in Table 6 are regarded as a single event originating at the time of the largest quake (M 7.3). In Table 7, the two quakes on Nov. 25, 1899 are treated as a single event. As seen from Table 7, surface wave magnitude M_s (ABE, 1981; ABE and NOGUCHI, 1983) are larger than M (JMA magnitude or equivalent) for large Hyuganada earthquakes. Some earthquake catalogs list a great

Table 10. Model parameters for strong earthquakes in Tokyo area (data in Table 9)

Data set		I (incl. 1697)		II (excl. 1697)	
		MOM	MLE	MOM	MLE
Weibull	α	3.53×10^{-4}	4.47×10^{-4}	1.01×10^{-5}	1.62×10^{-5}
	β	2.11	2.05	2.95	2.83
	$\ln L$	−34.88	−34.87	−29.29	−29.28
Gamma	c	0.104	0.0765	0.167	0.199
	r	4.02	2.95	7.35	8.75
	$\ln L$	−35.49	−35.23	−28.60	−28.55
Lognormal	m	3.54	3.47	3.72	3.73
	σ	0.471	0.695	0.357	0.328
	$\ln L$	−37.89	−36.22	−28.27	−28.22
Exp. Prob.	a		0.00888		0.00499
	b		0.0349		0.0447
	$\ln L$		−35.04		−30.32
Mean interval		38.52 yr		44.02 yr	
S.D.		19.21 yr		16.24 yr	

Table 11. Probabilities $p(\tau|t)$ for the next strong earthquake in Tokyo area on the basis of data set I and the Weibull model

τ \ t	0	20	40	60	80	100 yr
5 yr	0.012	0.112	0.207	0.294	0.372	0.442
10	0.049	0.234	0.390	0.516	0.617	0.698
15	0.108	0.357	0.543	0.678	0.773	0.842
20	0.186	0.476	0.668	0.792	0.870	0.919
25	0.278	0.585	0.766	0.869	0.928	0.960
30	0.377	0.680	0.839	0.920	0.961	0.981
35	0.477	0.761	0.893	0.953	0.979	0.991
40	0.573	0.826	0.931	0.973	0.990	0.996
45	0.662	0.877	0.957	0.985	0.995	0.998
50	0.740	0.916	0.974	0.992	0.998	0.999

earthquake in Hyuganada on Nov. 10, 1909, but this was undoubtedly a large intermediate-depth earthquake ($M=7.6$, $m_B=7.5$) beneath central Kyushu (UTSU, 1979). The earthquakes in 1943 and 1961 in Kashimanada were smaller than other quakes in Table 6, but they were the largest shocks in remarkable swarms (the 1961 swarm was especially large). The energy released by these swarms was comparable to an $M \gtrsim 7$ quake.

The estimated parameter values are listed in Table 8. Two data sets, one including the 1943 quake, and the other excluding it, are

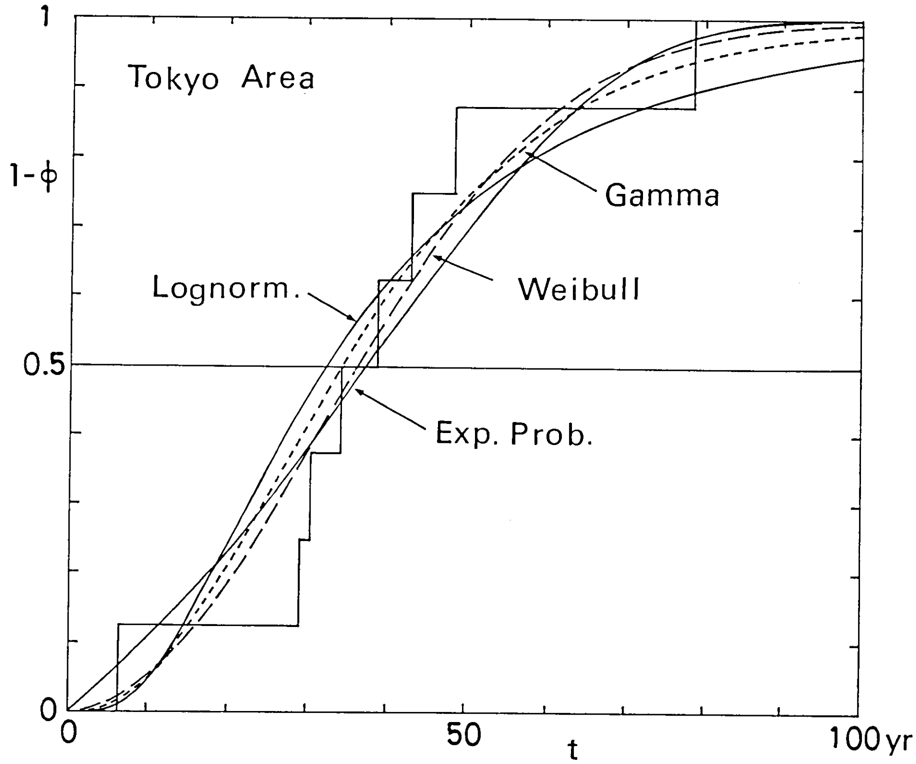

Fig. 5. Cumulative distribution of T_i and the curves of $1-\phi(t)$ for four models. Destructive earthquakes in Tokyo area, data set I.

examined for the Kashimanada region. The lognormal or the exponential probability models provide the largest $\ln L$ for these data sets.

(4) Tokyo and vicinity

The city of Tokyo and its surrounding area have been hit by many strong earthquakes during the past 400 years. More than 50 shocks caused damage in the Tokyo area. Of these the 11 shocks listed in Table 9 were the most destructive (intensity 5+ to 6 in JMA scale at some localities). Although these earthquakes did not originate from the same source, the parameters for the recurrence models are calculated. The three shocks in 1647 and 1649 are considered as a single sequence and represented by the largest one on July 30, 1649. The 1697 shock may, in a broad sense, be regarded as a foreshock of the great earthquake in 1703. Therefore, the parameters are calculated and listed in Table 10 for two data sets, one including the 1697 event, the other not. The Weibull model and the lognormal model yield the largest $\ln L$ for the respective data sets. Fig. 5 is the same kind of graph as Figs. 3–4 for the first data set. Sixty years have passed since the last destructive earthquake in the Tokyo area. The conditional probabilities $p(\tau|t)$ calculated by using the Weibull model (MLE) are given in Table 11. The column of $t=60$ years indicates the present situation.

4. Conclusion

This paper describes the estimation of parameters for recurrence models of earthquakes employing several sets of data on large earthquakes in Japan. We have compared four renewal models using different distributions of time intervals. It is found that different models are best suited for different sets of data. The lognormal model gives the best results in some cases, but the worst in others. The exponential probability model shows similar characteristics. The Weibull and gamma models give intermediate results. Anyway the differences among the four models are usually small, and it is hard to say which is the best model. All models seem to be acceptable.

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地震のリカレンス・モデルのパラメータの推定

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大地震が繰り返し発生している地域について，その時系列を表す簡単なモデルとして，時間間隔の分布が異る四種類の更新過程を取り上げ，それぞれについてパラメータの値を最尤法及びモーメント法により求めた．日本付近のいくつかの地域について調べた結果によれば四種類のモデルは多少違う結果を与えるが，大きな差はなく，どれか一つが特に良く適合するとはいえない．どのモデルによっても，前回の大地震から t だけ経過した時点から τ という長さの期間に次の大地震が起こる確率は容易に計算できる．その計算結果の一部を掲げた．