Discussion Paper Series A No.598

Trade in Value Added Revisited: A Comment on R. Johnson and G. Noguera, Accounting for Intermediates: Production Sharing and Trade in Value Added

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January, 2014

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Abstract

In light of growing intermediate goods trade, Johnson and Noguera (2012) developed theoretical and empirical research on the new concept of trade in value added in place of gross trade. However, they did not deal with the relationship between the new value added trade balance and the gross trade balance. Presented here is that in the case of two countries and many sectors the new value added trade balance always equals the gross trade balance. We verify this proposition by using an international input-output data compiled by Groningen University. In the case with three countries (China, the USA and the rest of the world; ROW) and many sectors, the China-USA trade balance or the USA-China imbalance measured in value added for 2010 is 23.5% smaller than that in gross terms, whereas the China-ROW trade balance in value added is 94% larger than that in gross terms.

JEL classification codes: F1, C67, D57, R15

Key words: trade in value added, gross trade, input-output tables, global supply chain

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1. Introduction

In light of growing intermediate goods trade, Johnson and Noguera (2012) presented theoretical and empirical research on the new concept of trade in value added, by using Trefler and Zhu (2010) and an international input-output table. The new concept of value added exports from an origin country to a destination country is defined as the origin county's value added induced by the destination country's final demand, excluding imports of intermediate goods for the world, whereas the traditional concept of value added exports is defined as the origin country's value added induced by its gross exports to the destination country, including exports of intermediate goods. This new concept is also presented by WTO and IDE JETRO (2011) and OECD and WTO (2012) although the latter provides such an exposition that the traditional concept is essentially compatible with the new concept without any mathematical proof. Anyway, Johnson and Noguera, OECD and WTO are addressing the importance of value added

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contents of foreign trade in place of the traditional gross trade concept. Currently, the phrase *global value chains* (GVC) or *trade in value added* (TiVA) in place of traditional terms for supply chains or gross trade relations across countries is becoming fashionable in academic literature, the publications of international organizations and the business world. Among others Johnson and Noguera's paper seems to provide the most detailed theoretical exposition of the new concept of trade in value added. However, they did not present any theoretical proposition of the relationship between the new value added trade balance and the gross trade balance for the general framework with many countries and many sectors, or its special case with two countries and many sectors, as well. Provided here are theoretical and empirical results of this relationship for the case with two countries and many sectors. We also present theoretical relationships between the new concept of value added in trade and the traditional one.

2. The new concept of trade in value added

2.1. A general framework

Following Isard (1951), WTO and IDE JETRO (2011), and Johnson and Noriega (2012), we reproduce an inter-country multi-sector model in a general framework.[1]

We assume there are r, s=1, 2,…, R countries (areas or regions) each of which produces and inputs $r(i)$, $s(j)$=1, 2,…, n products. We assume the classical Leontief open input-output model with fixed input coefficients and final demand for each country. In this model each sector produces a single commodity without joint production. We

[1] The model below is essentially equivalent to models presented by WTO and IDE JETRO (2011), and Johnson and Noriega (2012) except for our explicit exposition of a dual price system associated with an input-output system.

regard the last country *R* as the rest of the world (ROW). We consider an international input-output system not in physical terms but in *value terms*, as shown by Table 1.

Table 1

Data structure of an international input-output table.

	Intermediate demand/input								Final demand (destination)								
	Country 1	Country 2	⋯	Country *r*	⋯	Country *s*	⋯	ROW *R*	Country 1	Country 2	⋯	Country *r*	⋯	Country *s*	⋯	ROW *R*	Output
									$\boldsymbol{F}_1$	$\boldsymbol{F}_2$	⋯	$\boldsymbol{F}_r$	⋯	$\boldsymbol{F}_s$	⋯	$\boldsymbol{F}_R$	$\boldsymbol{X}$
Country 1	$\underline{\boldsymbol{X}}_{11}$	$\underline{\boldsymbol{X}}_{12}$	⋯	$\underline{\boldsymbol{X}}_{1r}$	⋯	$\underline{\boldsymbol{X}}_{1s}$	⋯	$\underline{\boldsymbol{X}}_{1R}$	$\boldsymbol{Y}_{11}$	$\boldsymbol{Y}_{12}$	⋯	$\boldsymbol{Y}_{1r}$	⋯	$\boldsymbol{Y}_{1s}$	⋯	$\boldsymbol{Y}_{1R}$	$\boldsymbol{X}_1$
Country 2	$\underline{\boldsymbol{X}}_{21}$	$\underline{\boldsymbol{X}}_{22}$	⋯	$\underline{\boldsymbol{X}}_{2r}$	⋯	$\underline{\boldsymbol{X}}_{2s}$	⋯	$\underline{\boldsymbol{X}}_{2R}$	$\boldsymbol{Y}_{21}$	$\boldsymbol{Y}_{22}$	⋯	$\boldsymbol{Y}_{2r}$	⋯	$\boldsymbol{Y}_{2s}$	⋯	$\boldsymbol{Y}_{2R}$	$\boldsymbol{X}_2$
⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯
Country *r*	$\underline{\boldsymbol{X}}_{r1}$	$\underline{\boldsymbol{X}}_{r2}$	⋯	$\underline{\boldsymbol{X}}_{rr}$	⋯	$\underline{\boldsymbol{X}}_{rs}$	⋯	$\underline{\boldsymbol{X}}_{rR}$	$\mathbf{Y}_{r1}$	$\mathbf{Y}_{r2}$	⋯	$\boldsymbol{Y}_{rr}$	⋯	$\boldsymbol{Y}_{rs}$	⋯	$\boldsymbol{Y}_{rR}$	$\boldsymbol{X}_r$
⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯
Country *s*	$\underline{\boldsymbol{X}}_{s1}$	$\underline{\boldsymbol{X}}_{s2}$	⋯	$\underline{\boldsymbol{X}}_{sr}$	⋯	$\underline{\boldsymbol{X}}_{ss}$	⋯	$\underline{\boldsymbol{X}}_{sR}$	$\boldsymbol{Y}_{s1}$	$\boldsymbol{Y}_{s2}$	⋯	$\boldsymbol{Y}_{sr}$	⋯	$\boldsymbol{Y}_{ss}$	⋯	$\boldsymbol{Y}_{sR}$	$\boldsymbol{X}_s$
⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯
ROW *R*	$\underline{\boldsymbol{X}}_{R1}$	$\underline{\boldsymbol{X}}_{R2}$	⋯	$\underline{\boldsymbol{X}}_{Rr}$	⋯	$\underline{\boldsymbol{X}}_{Rs}$	⋯	$\underline{\boldsymbol{X}}_{RR}$	$\mathbf{Y}_{R1}$	$\mathbf{Y}_{R2}$	⋯	$\boldsymbol{Y}_{Rr}$	⋯	$\boldsymbol{Y}_{Rs}$	⋯	$\boldsymbol{Y}_{RR}$	$\boldsymbol{X}_R$
Value added	$\boldsymbol{v}_1$	$\boldsymbol{v}_2$	⋯	$\boldsymbol{v}_r$	⋯	$\boldsymbol{v}_s$	⋯	$\boldsymbol{v}_R$									
Output	$\boldsymbol{X}_1$	$\boldsymbol{X}_2$	⋯	$\mathbf{X}_r$	⋯	$\boldsymbol{X}_s$	⋯	$\boldsymbol{X}_R$									

$\underline{\boldsymbol{X}}rs$ ($s \neq r$): country *r*'s gross export matrix of intermediate goods to country *s* or country *s*'s gross import matrix of intermediate goods from country *r*.

$\underline{\boldsymbol{X}}rr$: country *r*'s input matrix of intermediate goods domestically produced.

We denote: $\boldsymbol{A}_{rs} = \left(a_{r(i)s(j)}\right)(n \times n)$: country *r*'s export coefficient matrix to country *s* or country *s*'s import coefficient matrix from country *r* if $r \neq s$, and country *r*'s input coefficient matrix of domestically produced intermediate goods if $r = s$; $\boldsymbol{Y}_r = \left[Y_{r(i)}\right](n \times 1)$: country *r*'s final demand vector in an international input-output table; $\widetilde{\boldsymbol{Y}}_r = \left[\widetilde{\boldsymbol{Y}}_{r(i)}\right](n \times 1)$: country *r*'s final demand vector, including exports of intermediate goods, in each country's input-output system; $\boldsymbol{Y}_{rs} = \left[Y_{r(i)s}\right](n \times 1)$: country *s*'s final demand vector for country *r* (*n*×1) or country *r*'s final goods export

vector to country s if $r \neq s$; $\boldsymbol{F}_s = [\boldsymbol{Y}_{rs}]\ \big((n \times R) \times 1\big)$: country s's final demand vector for all countries; $\boldsymbol{X}_r = \big[X_{r(i)}\big]\ (n \times 1)$: country r's output vector ; $\boldsymbol{X} = [\boldsymbol{X}_r]\ ((n \times R) \times 1)$: an overall output vector ; $\boldsymbol{I}$: an (n×R) dimensional identity matrix; $\boldsymbol{I}_n$: an n dimensional identity matrix. We assume that non-negative matrixes $\boldsymbol{A}$ and $\boldsymbol{A}_{rr}$ are productive. In Table 1, intermediate transactions among countries are denoted by $\underline{\boldsymbol{X}}_{rs} = \big(a_{r(i)s(j)}X_{s(j)}\big)\ (n \times n)$. Denoting $\boldsymbol{X}^*$ as the equilibrium output vector, an Isard type of non-competitive inter-country multi-sector input-output table in value terms can be written as:

$$\boldsymbol{X}^* = \boldsymbol{A}\boldsymbol{X}^* + \boldsymbol{Y}; \tag{1}$$

$$\boldsymbol{X}^* = \boldsymbol{B}\boldsymbol{Y}, \text{where } \boldsymbol{B} = (\boldsymbol{I} - \boldsymbol{A})^{-\boldsymbol{1}}\ ; \tag{2}$$

$$\boldsymbol{X}_r^* = \boldsymbol{B}^r\widetilde{\boldsymbol{Y}}_r, \text{where } \boldsymbol{B}^r = (\boldsymbol{I}_n - \boldsymbol{A}_{rr})^{-\boldsymbol{1}}\ . \tag{3}$$

Here,

$$\boldsymbol{A} = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1s} \dots & A_{1R} \\ \dots & \dots & \dots & \dots & \dots \\ A_{r1} & A_{r1} & \dots & A_{rs} \dots & A_{rR} \\ \dots & \dots & \dots & \dots & \dots \\ A_{R1} & A_{R2} & \dots & A_{Rs} \dots & A_{RR} \end{bmatrix},$$

$$\boldsymbol{B} = (\boldsymbol{I} - \boldsymbol{A})^{-\boldsymbol{1}} = \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1s} & B_{1R} \\ \dots & \dots & \dots & \dots & \dots \\ B_{r1} & B_{r1} & \dots & B_{rs} & B_{rR} \\ \dots & \dots & \dots & \dots & \dots \\ B_{R1} & B_{R2} & \dots & B_{Rs} & B_{RR} \end{bmatrix},$$

$$\boldsymbol{Y} = \begin{bmatrix} \boldsymbol{Y}_1 \\ \dots \\ \boldsymbol{Y}_r \\ \dots \\ \boldsymbol{Y}_R \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}_{11} \\ \dots \\ \boldsymbol{Y}_{r1} \\ \dots \\ \boldsymbol{Y}_{R1} \end{bmatrix} + \cdots + \begin{bmatrix} \boldsymbol{Y}_{1s} \\ \dots \\ \boldsymbol{Y}_{rs} \\ \dots \\ \boldsymbol{Y}_{Rs} \end{bmatrix} + \cdots + \begin{bmatrix} \boldsymbol{Y}_{1R} \\ \dots \\ \boldsymbol{Y}_{rR} \\ \dots \\ \boldsymbol{Y}_{RR} \end{bmatrix} = \boldsymbol{F}_1 + \cdots + \boldsymbol{F}_s + \cdots + \boldsymbol{F}_R; \boldsymbol{X} = \begin{bmatrix} \boldsymbol{X}_1 \\ \dots \\ \boldsymbol{X}_r \\ \dots \\ \boldsymbol{X}_R \end{bmatrix}.$$

Overall output $\boldsymbol{X}_{*s}^*$ induced by a fixed destination country *s's final demand $\boldsymbol{F}_{*s}$ is given by

$$\boldsymbol{X}^{*}_{*s} = \boldsymbol{A}\boldsymbol{X}^{*}_{*s} + \boldsymbol{F}_{*s};\ \boldsymbol{X}^{*}_{r*s} = A_{r*s}\boldsymbol{X}^{*}_{r*s} + \boldsymbol{Y}_{r*s}\ . \tag{4}$$

By definitions of $\boldsymbol{F}_s$ and $\boldsymbol{Y}_{rs}$ we have

$$\boldsymbol{X}^{*} = \Sigma_s \boldsymbol{X}^{*}_{*s};\ X^{*}_{r*s} = \Sigma_i X^{*}_{r(i)*s}\ . \tag{5}$$

Country *r*'s gross exports to country *s*, $\boldsymbol{E}_{rs}$ are given by $\boldsymbol{E}_{rs} = \boldsymbol{A}_{rs}\boldsymbol{X}^{*}_{s} + \boldsymbol{Y}_{rs}$ $(s \neq r)$. Hence, it follows from equation (3) that

$$\boldsymbol{X}^{*}_{r} = (\boldsymbol{I}_n - \boldsymbol{A}_{rr})^{-1}\tilde{\boldsymbol{Y}}_{r,} = (\boldsymbol{I}_n - \boldsymbol{A}_{rr})^{-1}\left(\Sigma_{s\neq r}\boldsymbol{E}_{rs} + \boldsymbol{Y}_{rr}\right)\ . \tag{6}$$

Let us define country *r*'s *i*-th value added ratio as $v_{r(i)} = V_{r(i)}/X_{r(i)}$ where $V_{r(i)}$ is country *r*'s *i*-th value added. Country *r*'s value added ratio vector and the overall value added vector are $\boldsymbol{v}_r = (v_{r(i)})\ (1 \times n)$ and $\boldsymbol{v} = (\boldsymbol{v}_r)\ (1 \times (n \times R))$ respectively. Then, by virtue of definitions of input coefficients and value added ratios, we have

$$\boldsymbol{u} = \boldsymbol{u}\boldsymbol{A} + \boldsymbol{v};\ \boldsymbol{u}_n = \boldsymbol{u}_n\Sigma_k\boldsymbol{A}_{kr} + \boldsymbol{v}_r\ . \tag{7}$$

Therefore, value added ratios are given by

$$\boldsymbol{v} = \boldsymbol{u}(\boldsymbol{I} - \boldsymbol{A});\ \boldsymbol{v}_r = \boldsymbol{u}_n(\boldsymbol{I}_n - \Sigma_k\boldsymbol{A}_{kr})\ . \tag{8}$$

where $\boldsymbol{u} = (1,1,\dots,1)\ (1 \times (n \times R))$ and $\boldsymbol{u}_n = (1,1,\dots,1)\ (1 \times n)$ are aggregation vectors of unities. That is to say, the price vector associated with an input-output system in value terms always equals an aggregation vector.

2.2. The new concept of trade in value added

The new concept of value added trade is defined as follows.

Definition 1. The new concept of value added exports and trade balance: Johnson and Noguera (2012), and WTO and IDE JETRO (2011)

Country *r*'s value added exports to country *s* are defined as $\widehat{\boldsymbol{V}}_r\boldsymbol{X}^{*}_{rs}$ where $\widehat{\boldsymbol{V}}_r = diag\left\{v_{r_{(1)}},\dots,v_{r_{(n)}}\right\}$ $(n \times n)$. The total value added exports of origin country *r* to

destination country *s* amounts to $\boldsymbol{u}_n\widehat{\boldsymbol{V}}_r\boldsymbol{X}^*_{rs} = \boldsymbol{v}_r\boldsymbol{X}^*_{rs}$. Country *r*'s value added trade balance with country *s* is then

$$T^{va}_{rs} = \boldsymbol{u}_n\widehat{\boldsymbol{V}}_r\boldsymbol{X}^*_{rs} - \boldsymbol{u}_n\widehat{\boldsymbol{V}}_s\boldsymbol{X}^*_{sr} = \boldsymbol{v}_r\boldsymbol{X}^*_{rs} - \boldsymbol{v}_s\boldsymbol{X}^*_{sr}\,. \tag{9}$$

Country *r*'s gross trade balance with country *s* is

$$T^{g}_{rs} = \boldsymbol{u}_n(\boldsymbol{E}_{rs} - \boldsymbol{E}_{sr}) = \boldsymbol{u}_n(\boldsymbol{A}_{rs}\boldsymbol{X}^*_s + \boldsymbol{Y}_{rs}) - \boldsymbol{u}_n(\boldsymbol{A}_{sr}\boldsymbol{X}^*_r + \boldsymbol{Y}_{sr}); s \neq r\,. \tag{10}$$

Based on Definition 1, Johnson and Noguera (2012), and WTO and IDE JETRO (2011) tried to demonstrate empirical results of the relationship between value added trade balances and gross trade balances. However, rather surprisingly, they did not report any theoretical result and implication of this relationship mainly due to the complexity of equations.

2.3 The traditional value added trade concept

Origin country *r*'s output induced by its gross exports to destination country *s* is

$$\boldsymbol{X}^{\dagger}_{rs} = (\boldsymbol{I}_n - \boldsymbol{A}_{rr})^{-1}\boldsymbol{E}_{rs} = (\boldsymbol{I}_n - \boldsymbol{A}_{rr})^{-1}(\boldsymbol{A}_{rs}\boldsymbol{X}^*{}_s + \boldsymbol{Y}_{rs}); r \neq s \tag{11}$$

The traditional concept of value added trade is defined as follows.

Definition 2. The traditional value added trade balance. Miyazawa *et al*. (1975) and Koopman *et al*. (2010)

Country *r*'s value added exports to destination country *s* is defined as $\widehat{\boldsymbol{V}}_r\boldsymbol{X}^{\dagger}_{rs}$. Country *r*'s value added trade balance with country *s* is defined as:

$$\boldsymbol{T}^{\dagger}_{rs} = \boldsymbol{u}\widehat{\boldsymbol{V}}_r\boldsymbol{X}^{\dagger}_{rs} - \boldsymbol{u}\widehat{\boldsymbol{V}}_s\boldsymbol{X}^{\dagger}_{sr}\,. \tag{12}$$

This definition has widely been utilized in Japan and included in Japanese Statistics Bureau's official reports.[2] Koopman *et al*. (2010) developed this definition in the framework of an international input-output system.

[2] The latest version can be seen by Table 6 on the site: http://www.stat.go.jp/english/data/io/io05.htm.

The induced output $\boldsymbol{X}_{rs}^{\dagger}$ also induces imports $\boldsymbol{A}_{kr}\boldsymbol{X}_{rs}^{\dagger}$ $(k = 1,2, \dots, s; k \neq r)$. In view of equations (7) an (11), we have the following identity for the gross exports and their components:

$$\begin{aligned}\boldsymbol{u}_n\widehat{\boldsymbol{V}}_r\boldsymbol{X}_{rs}^{\dagger} + \boldsymbol{u}_n\Sigma_{k\neq r}\boldsymbol{A}_{kr}\boldsymbol{X}_{rs}^{\dagger} &= \boldsymbol{u}_n\Sigma_{k\neq r}\boldsymbol{A}_{kr}\,\boldsymbol{X}_{rs}^{\dagger} + \boldsymbol{u}_n(\boldsymbol{I}_n - \Sigma_{k\neq r}\boldsymbol{A}_{kr}\,)\boldsymbol{X}_{rs}^{\dagger} \\ &= \boldsymbol{u}_n(\boldsymbol{I}_n - \boldsymbol{A}_{rr})\boldsymbol{X}_{rs}^{\dagger} = \boldsymbol{u}\boldsymbol{E}_{rs}\ .\end{aligned}$$

Proposition 1. Identity between the total traditional value added exports *plus* total related imports and the total gross exports. Miyazawa *et al*. (1975) and Koopman *et al*. (2010)

$$\boldsymbol{u}_n\widehat{\boldsymbol{V}}_r\boldsymbol{X}_{rs}^{\dagger} + \boldsymbol{u}_n\Sigma_{k\neq r}\boldsymbol{A}_{kr}\boldsymbol{X}_{rs}^{\dagger} = \boldsymbol{u}\boldsymbol{E}_{rs}\ . \tag{13}$$

Equation (13) implies that gross exports can be decomposed into value added and imports induced by gross exports. This is a corollary of identity between GDPs on the production and expenditure sides. These imports of country *r*, which are exports of country $s \neq r$, also in turn induce value added in the country *s*. Due to this identity equation, the relationship between traditional value added exports and gross exports can clearly be understood in a well-defined manner. In the definition of the traditional value added exports on a country basis the exports of intermediates are exogenously given. When we measure the traditional value added trade on a country basis, we do not have to concern endogeneity issues of ROW. Given the information of a country's non-competitive type input-output table and gross exports by country and sector, we can easily compute the traditional value added exports of the country. However, this approach is insufficient to trace international transfers of value added and outputs in the world.

By the way, it follows from equations (12) and (13) that

$$T_{rs}^{va\dagger} = \boldsymbol{u}\widehat{\boldsymbol{V}}_r\boldsymbol{X}_{rs}^{\dagger} - \boldsymbol{u}\widehat{\boldsymbol{V}}_s\boldsymbol{X}_{sr}^{\dagger} = \boldsymbol{u}(\boldsymbol{E}_{rs} - \boldsymbol{E}_{sr}) - \boldsymbol{u}_n(\Sigma_{k\neq r}\boldsymbol{A}_{kr}\boldsymbol{X}_{rs}^{\dagger} - \Sigma_{k\neq s}\boldsymbol{A}_{ks}\boldsymbol{X}_{sr}^{\dagger})\ . \tag{14}$$

Proposition 2. The relationship between the traditional value added trade balance and the gross trade balance

The bilateral, traditional value added trade balance is greater than the gross trade balance by the difference between the gross imports of the origin country, induced by its gross exports, and those of the destination country.

3. Two countries with multi-sectors

We return to the new value added trade. Following Johnson and Noguera (2012, §2.2.3), we consider a simple but important case with two countries $(r, s = 1,2)$ and multi-sectors $(r(i), s(j) = 1,2, \dots, n)$. Then we have

$$\boldsymbol{X}_1^* = \boldsymbol{X}_{11}^* + \boldsymbol{X}_{12}^*;\ \boldsymbol{X}_{12}^* = \boldsymbol{X}_1^* - \boldsymbol{X}_{11}^* \text{ and } \boldsymbol{X}_2^* = \boldsymbol{X}_{21}^* + \boldsymbol{X}_{22}^*;\ \boldsymbol{X}_{21}^* = \boldsymbol{X}_2^* - \boldsymbol{X}_{22}^* . \quad (5')$$

Equation (6) can be written as

$$\boldsymbol{X}_1^* = (\boldsymbol{I}_n - \boldsymbol{A_{11}})^{-1}(\boldsymbol{E}_{12} + \boldsymbol{Y}_{11}) \text{ and } \boldsymbol{X}_2^* = (\boldsymbol{I}_n - \boldsymbol{A}_{22})^{-1}(\boldsymbol{E}_{21} + \boldsymbol{Y}_{22}) . \quad (6')$$

Using equations (4) and (6'), we have

$$\begin{aligned} \boldsymbol{X}_{11}^* &= \boldsymbol{A}_{11}\boldsymbol{X}_{11}^* + \boldsymbol{A}_{12}\boldsymbol{X}_{21}^* + \boldsymbol{Y}_{11} = (\boldsymbol{I}_n - \boldsymbol{A}_{11})^{-1}(\boldsymbol{A}_{12}\boldsymbol{X}_{21}^* + \boldsymbol{Y}_{11}), \\ \boldsymbol{X}_{22}^* &= \boldsymbol{A}_{21}\boldsymbol{X}_{12}^* + \boldsymbol{A}_{22}\boldsymbol{X}_{22}^* + \boldsymbol{Y}_{22} = (\boldsymbol{I}_n - \boldsymbol{A}_{22})^{-1}(\boldsymbol{A}_{21}\boldsymbol{X}_{12}^* + \boldsymbol{Y}_{22}). \end{aligned} \quad (15)$$

Therefore, we have

$$\begin{aligned} \boldsymbol{X}_{12}^* &= \boldsymbol{X}_1^* - \boldsymbol{X}_{11}^* = (\boldsymbol{I}_n - \boldsymbol{A}_{11})^{-1}(\boldsymbol{E}_{12} - \boldsymbol{A}_{12}\boldsymbol{X}_{21}^*), \\ \boldsymbol{X}_{21}^* &= \boldsymbol{X}_2^* - \boldsymbol{X}_{22}^* = (\boldsymbol{I}_n - \boldsymbol{A}_{22})^{-1}(\boldsymbol{E}_{21} - \boldsymbol{A}_{21}\boldsymbol{X}_{12}^*). \end{aligned} \quad (16)$$

When we add imports $\boldsymbol{A}_{21}\boldsymbol{X}_{12}^*$ induced by output transfer $\boldsymbol{X}_{12}^*$ to value added exports, in view of equations (7) and (8) we have

$$\begin{aligned} \boldsymbol{v}_1\boldsymbol{X}_{12}^* + \boldsymbol{u}_n\boldsymbol{A}_{21}\boldsymbol{X}_{12}^* &= \boldsymbol{u}_n(\boldsymbol{I}_n - \boldsymbol{A}_{11} - \boldsymbol{A}_{21})\boldsymbol{X}_{12}^* + \boldsymbol{u}_n\boldsymbol{A}_{21}\boldsymbol{X}_{12}^* \\ &= \boldsymbol{u}_n(\boldsymbol{I}_n - \boldsymbol{A}_{11})\boldsymbol{X}_{12}^* = \boldsymbol{u}_n(\boldsymbol{E}_{12} - \boldsymbol{A}_{12}\boldsymbol{X}_{21}^*) . \end{aligned} \quad (17)$$

Similarly, by virtue of $\boldsymbol{v}_2 = \boldsymbol{u}_n(\boldsymbol{I}_n - \boldsymbol{A}_{12} - \boldsymbol{A}_{22})$, we have

$$\boldsymbol{v}_2\boldsymbol{X}_{21}^* + \boldsymbol{u}_n\boldsymbol{A}_{12}\boldsymbol{X}_{21}^* = \boldsymbol{u}_n(\boldsymbol{E}_{21} - \boldsymbol{A}_{21}\boldsymbol{X}_{12}^*) . \quad (18)$$

Hence, we arrive at the following important result:

$$
\begin{aligned}
T_{12}^{va} = \; & \boldsymbol{v}_1 \boldsymbol{X}_{12}^{*} - \boldsymbol{v}_2 \boldsymbol{X}_{21}^{*} \\
& = \boldsymbol{u}_n (\boldsymbol{E}_{12} - \boldsymbol{A}_{12} \boldsymbol{X}_{21}^{*}) - \; \boldsymbol{u}_n \boldsymbol{A}_{21} \boldsymbol{X}_{12}^{*} - \boldsymbol{u}_n (\boldsymbol{E}_{21} - \boldsymbol{A}_{21} \boldsymbol{X}_{12}^{*}) + \boldsymbol{u}_n \boldsymbol{A}_{12} \boldsymbol{X}_{21}^{*} \\
& = \boldsymbol{u}_n (\boldsymbol{E}_{12} - \; \boldsymbol{E}_{21}) = T_{12}^{g} \; . \qquad (19)
\end{aligned}
$$

Proposition 3. Identity between the total new value added trade balance and the gross trade balance

In the case with two countries and multi-sectors the total new value added trade balance is equivalent to the total gross trade balance.

This proposition may be very important because a country's trade balance with its partners can always be summarized by that with one aggregate partner, or all partners including the rest of the world. It is also important in that trade balances in value added by sector differ from those in gross concept, depending upon sectoral value added ratios and international input-output relations within the macro identity. This macro identity may verify the significance of value added trade in place of gross trade since this identity clearly demonstrates that trade in value added, the total of which is linked with the total gross trade balance, focuses on the distributions and linkages of value added among sectors and countries in place of that of gross output. Anyhow, our proposition may make the new concept of trade in value added legible from the viewpoint of foreign trade. In fact, Proposition 3 suggests that, in the case of country 1's trade with many countries, we have the following zero-sum relation:

$$
\begin{aligned}
& T_{12}^{va} + \; T_{13}^{va} + \cdots = \; T_{12}^{g} + T_{13}^{g} + \cdots ; \\
& (T_{12}^{va} - T_{12}^{g}) + \; (T_{13}^{va} - T_{13}^{g}) + \cdots = 0 \, . \qquad (20)
\end{aligned}
$$

We suppose country 1 (e.g., China) exports to, and imports from, country 2 (e.g., the USA) and country 3 (ROW). If country 1's value added trade balance with country 2 is smaller than the gross trade balance ($T_{12}^{va} < T_{12}^{g}$), this difference $(T_{12}^{va} - T_{12}^{g} < 0)$ should be offset by the difference between country 1's value added and gross trade balances with country 3 ($T_{13}^{va} - T_{13}^{g} > 0$).

In the case with two countries and many sectors, the relationship between the traditional value added trade balance and the new value added trade balance is also straightforward. It follows from equations (14) and (19) that

$$T_{12}^{va} - T_{12}^{va\dagger} = \boldsymbol{u}_n\left(\boldsymbol{A}_{21}\boldsymbol{X}_{12}^{\dagger} - \boldsymbol{A}_{12}\boldsymbol{X}_{21}^{\dagger}\right). \tag{21}$$

The new value added trade balance is greater than the traditional one by the difference between the gross imports of country 1, induced by gross exports and those of country 2.

It may be noteworthy to briefly describe the generalized case with many countries and multi-sectors for further research. We focus on country 1's new value added trade balance with country 2 in a generalized framework.

$$\boldsymbol{X}_{12}^{*} = \boldsymbol{X}_{1}^{*} - \boldsymbol{X}_{11}^{*} - \boldsymbol{X}_{13}^{*} - \cdots \text{ and } \boldsymbol{X}_{21}^{*} = \boldsymbol{X}_{2}^{*} - \boldsymbol{X}_{22}^{*} - \boldsymbol{X}_{23}^{*} - \cdots \tag{22}$$

$$\begin{aligned}
&\boldsymbol{v}_1\boldsymbol{X}_{12}^{*} + \boldsymbol{u}_n\boldsymbol{A}_{21}\boldsymbol{X}_{12}^{*} + \boldsymbol{u}_n\boldsymbol{A}_{31}\boldsymbol{X}_{12}^{*} + \cdots \\
&\quad = \boldsymbol{u}_n(\boldsymbol{I}_n - \boldsymbol{A}_{11})\boldsymbol{X}_{12}^{*} + \boldsymbol{u}_n\boldsymbol{A}_{21}\boldsymbol{X}_{12}^{*} + \boldsymbol{u}_n\boldsymbol{A}_{31}\boldsymbol{X}_{12}^{*} + \cdots \\
&\quad = \boldsymbol{u}_n\{\boldsymbol{E}_{12} + \boldsymbol{E}_{13} + \cdots - \boldsymbol{A}_{12}(\boldsymbol{X}_{21}^{*} + \boldsymbol{X}_{23}^{*} + \cdots) \\
&\qquad - \boldsymbol{A}_{13}(\boldsymbol{X}_{31}^{*} + \boldsymbol{X}_{33}^{*} + \cdots) - \boldsymbol{Y}_{13} - \cdots\}
\end{aligned} \tag{23}$$

$$\begin{aligned}
&\boldsymbol{v}_2\boldsymbol{X}_{21}^{*} + \boldsymbol{u}_n\boldsymbol{A}_{12}\boldsymbol{X}_{21}^{*} + \boldsymbol{u}_n\boldsymbol{A}_{32}\boldsymbol{X}_{21}^{*} + \cdots \\
&\quad = \boldsymbol{u}_n(\boldsymbol{I}_n - \boldsymbol{A}_{22})\boldsymbol{X}_{21}^{*} + \boldsymbol{u}_n\boldsymbol{A}_{12}\boldsymbol{X}_{21}^{*} + \boldsymbol{u}_n\boldsymbol{A}_{32}\boldsymbol{X}_{21}^{*} + \cdots \\
&\quad = \boldsymbol{u}_n\{\boldsymbol{E}_{21} + \boldsymbol{E}_{23} + \cdots - \boldsymbol{A}_{21}(\boldsymbol{X}_{12}^{*} + \boldsymbol{X}_{13}^{*} + \cdots) \\
&\qquad - \boldsymbol{A}_{23}(\boldsymbol{X}_{32}^{*} + \boldsymbol{X}_{33}^{*} + \cdots) - \boldsymbol{Y}_{23} - \cdots\}.
\end{aligned} \tag{24}$$

Therefore,

$$\begin{aligned}
T_{12}^{va} &= \boldsymbol{v}_1\boldsymbol{X}_{12}^{*} - \boldsymbol{v}_2\boldsymbol{X}_{21}^{*} \\
&= \boldsymbol{u}_n[\boldsymbol{E}_{12} - \boldsymbol{E}_{21} - \{\boldsymbol{A}_{12}(\boldsymbol{X}_{23}^{*} + \cdots) - \boldsymbol{A}_{21}(\boldsymbol{X}_{13}^{*} + \cdots)\} \\
&\quad + \boldsymbol{E}_{13} - \boldsymbol{E}_{23} + \cdots - \{\boldsymbol{A}_{13}(\boldsymbol{X}_{31}^{*} + \boldsymbol{X}_{33}^{*} + \cdots) - \boldsymbol{A}_{23}(\boldsymbol{X}_{32}^{*} + \boldsymbol{X}_{33}^{*} + \cdots)\} \\
&\quad - \{(\boldsymbol{Y}_{13} + \cdots) + (\boldsymbol{Y}_{23} + \cdots)] - \boldsymbol{u}_n\{(\boldsymbol{A}_{31}\boldsymbol{X}_{12}^{*} + \cdots) - (\boldsymbol{A}_{32}\boldsymbol{X}_{21}^{*} + \cdots)\}.
\end{aligned} \tag{25}$$

In the generalized case with more than three countries, country 1's value added trade balance with partner country 2 depends on gross trade balance between these two countries, gross trade balances with other countries and complicated international input-output relations as well. This needs further research.

Needless to say, we can immediately compute $\boldsymbol{X}_{rs}^{*}$ and T_{rs}^{va} through the generalized international Leontief inverse as follows:

$$\begin{aligned}
&\boldsymbol{X}_{sr}^{*} = (\boldsymbol{B}_{r1} \ldots \boldsymbol{B}_{rs} \ldots \boldsymbol{B}_{rR})\boldsymbol{F}_s = \boldsymbol{B}_{r1}\boldsymbol{Y}_{1s} + \cdots + \boldsymbol{B}_{rs}\boldsymbol{Y}_{rs} + \cdots + \boldsymbol{B}_{rR}\boldsymbol{Y}_{Rs} \\
&T_{rs}^{va} = \boldsymbol{u}_n\hat{\boldsymbol{V}}_r\boldsymbol{X}_{rs}^{*} - \boldsymbol{u}_n\hat{\boldsymbol{V}}_s\boldsymbol{X}_{sr}^{*} = \boldsymbol{v}_r\boldsymbol{X}_{rs}^{*} - \boldsymbol{v}_s\boldsymbol{X}_{sr}^{*}
\end{aligned} \tag{26}$$

Equation (26) provides us with the impact of the partner countries' imports from the origin country or the origin country's exports to partner countries on the value added trade. This equation also suggests the importance of the term of ROW, $\boldsymbol{B}_{rR}\boldsymbol{Y}_{Rs}$ on the value added trade.

4. Empirical results

4.1. A numerical example

To make our proposition legible, we would like to begin with a numerical example. Table 2 displays an international input-output table with two countries each of which produces a differentiated aggregate product.

Table 2

A numerical example with two countries and two sectors. (in billion US$)

	Intermediate demand		Final demand			
	country 1	country 2	country 1	country 2	final demand	Output
Country 1	2	6	8	4	12	20
Country 2	8	18	7	7	14	40
Value added	10	16				
Output	20	40				

It follows from Table 2 that

$$\boldsymbol{A} = \begin{bmatrix} 0.1 & 0.15 \\ 0.4 & 0.45 \end{bmatrix},\ (\boldsymbol{I} - \boldsymbol{A})^{-1} = \begin{bmatrix} 1.264 & 0.345 \\ 0.920 & 2.069 \end{bmatrix},\ \boldsymbol{F}_1 = \begin{pmatrix} 8 \\ 7 \end{pmatrix},\ \boldsymbol{F}_2 = \begin{pmatrix} 4 \\ 7 \end{pmatrix}.$$

$$\boldsymbol{v} = (0.5 \quad 0.4).$$

Using equation (4) or (20), we immediately reach:

$$\begin{pmatrix} X_{12}^* \\ X_{22}^* \end{pmatrix} = (\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{F}_2 = \begin{pmatrix} 7.471 \\ 18.161 \end{pmatrix};\ \begin{pmatrix} X_{11}^* \\ X_{21}^* \end{pmatrix} = (\boldsymbol{I} - \boldsymbol{A})^{-1}F_1 = \begin{pmatrix} 12.529 \\ 21.839 \end{pmatrix}.$$

$$v_1X_{12}^* = 0.5 \times 7.471 = 3.736;\ v_2X_{21}^* = 0.4 \times 21.839 = 8.736.$$

$$T_{12}^{va} = v_1X_{12}^* - v_2X_{21}^* = -5;\ \ T_{12}^{g} = E_{12} - E_{21} = (6 + 4) - (8 + 7) = -5.$$

Therefore, we can verify $T_{12}^{va} = T_{12}^{g} = -5$.

Using equation (13), we also have

$$v_1 X_{12}^{*} = E_{12} - A_{12}X_{21}^{*} - A_{21}X_{12}^{*} = 10 - 0.15 \times 21.839 - 0.4 \times 7.471 = 3.736;$$

$$v_2 X_{21}^{*} = E_{21} - A_{21}X_{12}^{*} - A_{12}X_{21}^{*} = 15 - 0.4 \times 7.471 - 0.15 \times 21.839 = 8.736.$$

Our proposition $T_{12}^{va} = T_{12}^{g}$ also holds.

4.2. Empirical results

We employ WIOD (World Input-Output Database) compiled by Groningen University. WIOD consists of 40 countries and ROW with 35 sectors (see Timmer ed., 2012). First we consider country 1 (China) and country 2 (other 39 countries and ROW) for 2010. We redefine the aggregate country 2 as ROW. In WIOD, there are several vectors including net tax on products and international transport margins, which are not distributed to intermediate quadrant, or value added one. We aggregate these undistributed vectors into a single dummy vector. We define country *r*'s *i*-th dummy ratio as $d_{r(i)} = D_{r(i)}/X_{r(i)}$ where $D_{r(i)}$ is country *r*'s *i*-th dummy value. Country *r*'s dummy ratio vector and the overall dummy vector are $\boldsymbol{d}_r = (d_{r(i)})\ (1 \times n)$ and $\boldsymbol{d} = (\boldsymbol{d}_r)\ (1 \times (n \times R))$ respectively. Then, by virtue of definitions of input coefficients and value added ratios, equation (7) can now be rewritten as

$$\boldsymbol{u} = \boldsymbol{u}\boldsymbol{A} + \boldsymbol{d} + \boldsymbol{v};\ \boldsymbol{u}_n = \boldsymbol{u}_n \Sigma_k \boldsymbol{A}_{kr} + \boldsymbol{d}_r + \boldsymbol{v}_r\ . \qquad (7')$$

Accordingly, in the two-country world, equation (8) is rewritten as

$$T_{12}^{va} + \boldsymbol{d}_1 \boldsymbol{X}_{12}^{*} - \boldsymbol{d}_2 \boldsymbol{X}_{21}^{*} = T_{12}^{g}\ . \qquad (8')$$

We call the left-hand side of this equation as value added trade balance adjusted for dummy sector.

Table 3 shows our empirical result for two countries in 2010. With the paradigm shift from gross to value added trade, some sectoral gross imbalances (agriculture, pulp and paper, coke and oil products, chemicals, electricity, financial intermediation, real estate) changes into positive value added balances. The mining sector value added imbalance is largely improved as a half of its gross balance. On the other hand, value added balances of China's major exports of textiles and electrical equipment (including iPhones) becomes 0.4 times and 0.2 times their gross balances, respectively. In all, this table clearly demonstrates that China's total value added balance with the rest of the world, adjusted for the dummy sector, exactly equals its total gross balance of 310.1 billion US$ for 2010.

Table 3

China's trade with the rest of the world (ROW) in 2010. (in billion US$)

	Gross exports to ROW	Gross exports to China	Value added exports to ROW	Value added exports to China	Gross trade balance	Value added trade balance
	$\boldsymbol{E}_{12}$	$\boldsymbol{E}_{21}$	$\widehat{\boldsymbol{V}}_1\boldsymbol{X}_{12}^*$	$\widehat{\boldsymbol{V}}_2\boldsymbol{X}_{21}^*$	$\boldsymbol{T}_{12}^{g}$	$\boldsymbol{T}_{12}^{va}$
Agriculture	15.2	53.8	97.1	39.3	-38.6	57.8
Mining	8.3	249.5	71.8	182.4	-241.1	-110.6
Food	40.0	31.9	33.7	12.8	8.1	20.8
Textiles	199.8	15.4	86.6	5.3	184.4	81.4
Leather	42.3	6.6	15.6	1.8	35.8	13.9
Wood	9.1	5.0	11.8	3.6	4.1	8.2
Pulp and paper	7.0	14.5	18.6	13.2	-7.6	5.4
Coke and oil products	11.1	31.0	16.4	12.7	-19.9	3.6
Chemicals	89.7	141.1	71.2	51.5	-51.3	19.7
Rubber and plastics	54.3	21.5	34.1	13.5	32.8	20.6
Other non-metallic mineral	21.5	8.0	17.5	6.6	13.5	10.9
Basic metal products	99.3	98.8	100.0	63.7	0.5	36.3
Machinery, NEC	118.0	126.2	56.1	50.3	-8.2	5.8
Electrical equipment	639.0	383.8	158.2	117.5	255.1	40.8
Transport Equipment	79.3	74.0	32.3	24.4	5.3	7.9
Manufacturing NEC	61.2	17.1	25.8	7.1	44.1	18.7
Electricity, Gas & Water	1.4	2.2	41.8	20.1	-0.7	21.7
Construction	7.3	5.3	2.7	8.1	2.1	-5.4
Trade	86.9	24.0	140.3	93.0	62.9	47.3
Transport & communications	82.5	42.6	100.3	60.4	39.9	39.9
Financial intermediation	1.6	3.3	62.1	36.7	-1.6	25.4
Real estate	0.0	0.2	22.2	17.7	-0.2	4.5
Renting of M&Eq	55.3	50.7	61.8	103.0	4.6	-41.3
Other services	13.2	27.0	27.2	31.9	-13.8	-4.7
Total excluding dummy	1743.5	1433.4	1305.2	976.6	310.1	328.6
Dummy sector			22.6	41.1		-18.5
Total	1743.5	1433.4	1327.9	1017.8	310.1	310.1

Source: Author's calculation using WIOD for 2010.

Computation results performed with 35 sectors are aggregated for several trade related sectors, transport related sectors and other service related sectors.

Table 4

China's trade balance with the USA and ROW in 2010. (in billion US$)

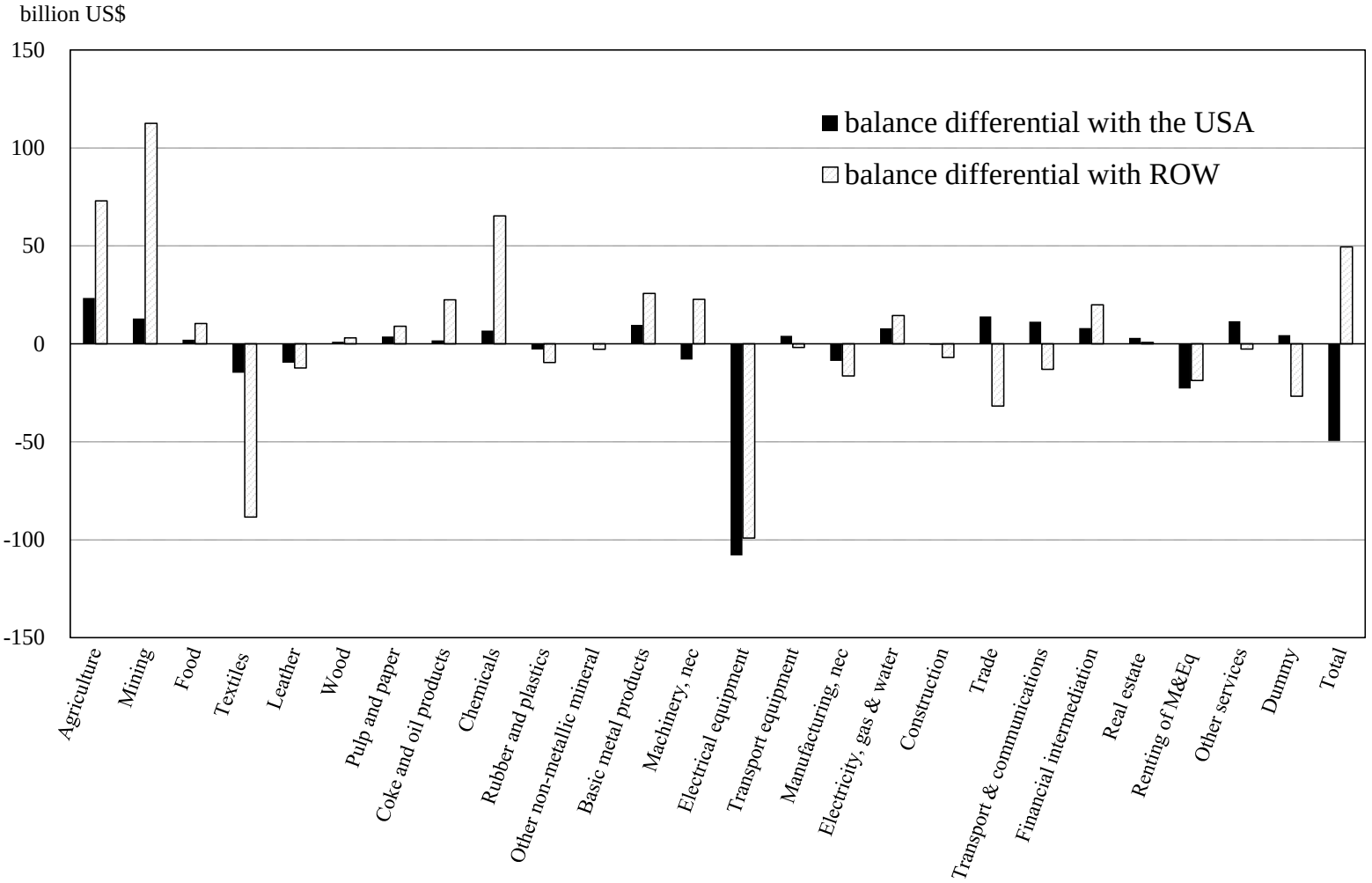
	Gross exports				Value added exports				Gross trade balance			Value added trade balance		
	China to the USA	The USA to China	China to ROW	ROW to China	China to the USA	The USA to China	China to ROW	ROW to China	China with the USA	China with ROW	China with the USA and ROW	China with the USA	China with ROW	China with the USA and ROW
	E_{12}	E_{21}	E_{13}	E_{31}	$\hat{V}_1 X^*_{12}$	$\hat{V}_2 X^*_{21}$	$\hat{V}_1 X^*_{13}$	$\hat{V}_3 X^*_{31}$	T^g_{12}	T^g_{13}	$T^g_{1,2\&3}$	T^{va}_{12}	T^{va}_{13}	$T^{va}_{1,2\&3}$
Agriculture	1.5	10.4	13.8	43.4	19.5	5.0	77.6	34.2	-8.9	-29.6	-38.6	14.5	43.4	57.9
Mining	0.6	0.6	7.8	248.8	15.5	2.6	56.2	184.7	-0.1	-241.1	-241.1	12.9	-128.5	-115.6
Food	5.7	2.4	34.3	29.5	6.6	1.2	27.1	11.9	3.3	4.8	8.1	5.4	15.2	20.6
Textiles	32.5	1.1	167.2	14.3	17.2	0.5	69.4	4.8	31.4	153.0	184.4	16.7	64.6	81.3
Leather	14.3	0.1	28.0	6.5	4.7	0.0	11.0	1.8	14.3	21.5	35.8	4.6	9.2	13.9
Wood	2.4	0.8	6.7	4.3	3.0	0.4	8.7	3.3	1.6	2.4	4.1	2.6	5.5	8.1
Pulp and paper	2.2	3.2	4.8	11.3	4.8	2.2	13.7	11.2	-1.1	-6.5	-7.6	2.6	2.5	5.2
Coke and oil products	0.8	1.0	10.3	30.0	3.3	1.8	13.1	10.2	-0.2	-19.7	-19.9	1.5	2.8	4.3
Chemicals	14.4	11.9	75.4	129.2	15.8	6.5	55.4	43.9	2.5	-53.8	-51.3	9.3	11.5	20.7
Rubber and plastics	11.4	1.6	42.9	19.9	8.2	1.3	25.9	12.3	9.8	23.1	32.8	6.9	13.6	20.4
Other non-metallic mineral	3.9	0.8	17.6	7.3	3.8	0.6	13.7	6.2	3.2	10.3	13.5	3.3	7.5	10.8
Basic metal products	16.0	8.7	83.3	90.1	23.2	6.2	76.7	57.7	7.3	-6.8	0.5	17.0	19.0	35.9
Machinery, NEC	27.2	12.5	90.8	113.7	13.2	6.5	42.9	43.1	14.7	-22.9	-8.2	6.7	-0.2	6.4
Electrical equipment	161.6	29.8	477.4	354.1	43.1	19.3	114.9	90.6	131.8	123.3	255.1	23.8	24.3	48.1
Transport Equipment	9.3	11.1	70.1	63.0	5.4	3.1	26.9	21.7	-1.8	7.1	5.3	2.2	5.3	7.5
Manufacturing NEC	16.6	1.7	44.6	15.4	7.1	1.1	18.6	5.8	14.8	29.3	44.1	6.1	12.9	18.9
Electricity, Gas & Water	0.2	0.0	1.3	2.1	9.5	1.4	32.3	18.7	0.2	-0.9	-0.7	8.1	13.6	21.8
Construction	0.0	0.0	7.3	5.3	0.2	0.7	2.4	7.4	0.0	2.1	2.1	-0.4	-4.9	-5.3
Trade	1.9	0.3	85.0	23.7	24.7	9.2	115.4	85.8	1.6	61.3	62.9	15.6	29.6	45.2
Transport & communications	6.7	9.5	75.8	33.2	17.5	8.9	82.7	53.1	-2.8	42.6	39.9	8.6	29.6	38.2
Financial intermediation	0.4	1.5	1.3	1.7	14.5	7.7	47.6	28.1	-1.2	-0.5	-1.6	6.8	19.4	26.3
Real estate	0.0	0.0	0.0	0.2	5.1	2.0	17.1	16.6	0.0	-0.2	-0.2	3.1	0.5	3.6
Renting of M&Eq	37.1	13.1	18.2	37.6	23.2	22.0	38.6	76.7	24.0	-19.4	4.6	1.2	-38.1	-36.8
Other services	0.2	15.4	13.0	11.5	5.1	8.9	22.1	23.3	-15.3	1.4	-13.8	-3.7	-1.2	-5.0
Total excluding dummy	366.7	137.5	1,376.8	1,295.9	294.2	119.0	1,010.0	853.0	229.2	80.9	310.1	175.3	157.1	332.3
Dummy sector					5.5	1.1	17.1	43.8				4.4	-26.7	-22.2
Total	366.7	137.5	1,376.8	1,295.9	299.8	120.1	1,027.1	896.7	229.2	80.9	310.1	179.7	130.4	310.1

Source: Author's calculation using WIOD for 2010.

Computation results performed with 35 sectors are aggregated for several trade related sectors, transport related sectors and other service related sectors.

Next, we consider China's trade balances with two countries, that is to say, the USA and ROW. Here, the ROW denotes an aggregate of 38 countries and ROW in the original data. Table 4 displays our empirical result for 2010. The China-USA trade balance or the USA-China imbalance measured in value added for 2010 is 23.5% smaller than that in gross terms, excluding the dummy sector. This is due to the facts that China's major exports to the USA have consisted of the machinery products assembled (iPhone etc.) and textile products fabricated with rather low value added-output ratios, and that China's imports from the USA have been very small in any terms. In contrast, the China-ROW trade balance in value added is 94% or about twice larger than that in gross terms, excluding the dummy sector. The China-ROW trade balance in value added, adjusted for the dummy sector, is also 61% larger than that in gross terms. Thus, a negative differential between the China-USA trade balances in value added and gross terms is offset by a positive differential between the China-ROW trade balances in value added and gross terms. Comparing Tables 3 and 4, each sectoral sum of the China-USA/ROW trade balances in value added and gross terms in Table 4 is different from that given in Table 3, due to the Leontief inverse matrix's dependence on country aggregation levels. However, it is noteworthy to find that the total sum of the China-USA/ROW trade balances in value added adjusted for the dummy sector in Table 4 (179.7 billion US$+130.4 billion US$=310.1 billion US$) is exactly equal to that in Table 3 (310.1 billion US$). Fig. 1 shows how sectoral differentials of the China-USA balances in value added and gross terms are canceled out by those of the China-ROW balances. With regard to agriculture, mining and chemicals, sectoral differentials of the China-ROW balances are much larger than those of the China-USA balances. Textiles' differential of the China-ROW balances is much smaller than that of the China-USA balances. Electrical equipment's differential of the China-USA balances is slightly smaller than that of the China-ROW balances. These canceling-out processes result in a zero-sum relation of total differentials of the China-USA and China-ROW balances.

Fig. 1. Differentials between China's value added and gross trade balances with the USA and ROWin 2010.



5. Concluding remarks

Growing intermediate goods trade in the world needs further developments of theoretical and empirical investigations in international trade. Responding to this task, we tried to further develop Jonhson and Noguera (2012)'s theoretical and empirical studies on their new concept of trade in value added. We proved theoretically and empirically that in the two country and many sectors world a country's new trade balance with its partner in value added equals that in gross terms. This proposition led us to the fact that in the three country and many sectors world the differential of a country's (e.g., China) balances with a partner (e.g., the USA) in value added and gross terms must be offset by that of the country's balances with another partner (ROW). We also discussed the relationship between the traditional value added trade and the new to nourish an appropriate coordination of the two concepts. We expect this paper will contribute to deeper analyses of trade in value added.

Acknowledgement

I am grateful to colleagues of Institute of Developing Economies (IDE), JETRO and Professor Bart Los of Groningen University for their encouragement and helpful comments on earlier versions of this paper. This study was supported by the JSPS Grants-in-Aid for Scientific Research (B), number 24330085 and the Sumitomo Mitsui Banking Overseas Cooperation Foundation Grant. The usual disclaimer applies.

References

Isard, W., 1951. Interregional and regional input-output analysis: a model of a space-economy. The Review of Economics and Statistics 33 (4), 318-328.

Johnson R., Noguera, G., 2012. Accounting for intermediates: production sharing and trade in value added. Journal of International Economics 86 (2), 224-236.

Miyazawa, K. (ed.), 1975. Sangyou-renkan bunseki nyumon, (*The introduction of input-output analysis*, in Japanese), the first edition. Nikkei, Tokyo. (The seventh edition was published in 2002.)

Koopman, R., Powers, W., Wang, Z., Wei, S., 2010. Give credit to where credit is due: tracing value added in global production chains. NBER Working Paper No. 16426 (first published in September 2010); revised version, 2011. HKIMR Working Paper No. 31/2011 (October).

OECD and WTO, 2012. Trade in value-added: concepts, methodologies and challenges (Joint OECD- WTO Note)
(Available at OECD: http://www.oecd.org/sti/ind/49894138.pdf).

Timmer, M. (ed.), 2012. The world input-output database (WIOD): contents, sources and methods. WIOD Working Paper No. 10 (August, version 0.9).

Trefler, D., Zhao, S., 2010. The structure of factor content predictions. Journal of International Economics 82 (2), 195-207.

WTO and IDE JETRO, 2011. Trade patterns and global value chains in East Asia: from trade in goods to trade in tasks, edited by Escaith, H., Inomata, S. World Trade Organization, Geneva.