A SYMMETRY BASED APPROACH FOR MUSICAL TONALITY ANALYSIS

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ABSTRACT

We present a geometric approach for tonality analysis called symmetry model. To derive the symmetry model, Carol L. Krumhansl and E.J. Kessler's toroidal Multi Dimensional Scaling (MDS) solution is separated into a key spanning and a key related component. While the key spanning component represents relationships between different keys, the key related component is suitable for the analysis of inner relationships of diatonic keys, for example tension or resolution tendencies, or functional relationships. These features are directly related to the symmetric organisation of tones around the tonal center, which is particularly visualized by the key related component.

1 INTRODUCTION

Tonality is the basis of western tonal music. Tonality comprises several aspects like stability of tones within a given tonal context, aspects of consonance or dissonance, aesthetic properties of a given musical piece, or the prediction of tensions or resolution. Tonality also helps to explain the development and usage of chords and keys. To this day a plenty of theories have been developed to explain these different aspects of tonality. An intuitive and unified theory of tonality is required that not only supports the development of extended music information retrieval methods, e.g. chord and key recognition, transcription and similarity estimation, but also helps to understand the way the human auditory system processes tonal information.

2 RELATED WORK

The Analysis of musical tonality generally operates at three stages: 1. Frequency analysis, signal transformation, 2. Complexity, irrelevancy, redundancy reduction, 3. the analysis of the preprocessed audio signal by menans of a tonality model. The present publication is exclusively devoted to tonality models. For this aspects of frequency transformation and preprocessing (e.g. transient location, noise reduction, consonance filtering, pitch tracking, horizontal segmentation, dimensionality reduction) shall not be regarded in more detail. The Description of musical tonality with geometric tonality models has a long tradition. Early approaches are for example Heinichen's (1728) or Kellner's (1737) regional circles, the harmonic network proposed by Leonhard Euler (1739), and Weber's (1767) regional chart [6, p.43]. Known as cirle of fifth (Kellner's regional circle), Riemann's "Tonnetz" (Euler's harmonic network) or Schönberg's chart of key regions (Webers regional chart), these models are of great interest till this day. In the meantime, advanced geometric tonality models have been developed. Roger Shepard [12] proposes several helix models, which primarily describe aspects of octave equivalence or fifth and chroma proximity. Elaine Chew [2] proposes a so called Spiral Array. The model's core is the, harmonic network inspired, geometric arrangement of pitches on a spiral. The great breakthrough of Chew's model is an unified description of the relationship between tones, chords and keys within one model and the observation of functional relationships that build a tonal center. Fred Lerdahl's [6] "diatonic space" consists of "basic space", "chordal space" and "regional space". These spaces help to model different aspects of tonality. While "basic space" describes the relationships between different tonal hierarchies (octave, fifth, triadic, diatonic and chromatic), "chordal space" is specialized to model tonal relationships between chords (chord proximity, chord progressions, ...) and "regional space" helps to describe tonal relationships between keys (e.g. aspects of modulation). Dmitri Tymoczko [14] represents musical chords in a geometric space called "orbifold space". The mapping of the notes from one chord to those of another are represented with the help of line segments . The similarity of chords is represented by the length of these line segments. Aline Honingh [3] introduces the property of star convexity to describe and visualize the principle of shapeliness of tonal pitch structures. Another relevant geometric tonality model is Krumhansl and Kessler's [5] four dimensional MDS solution which is subject of the next chapter in which the symmetry model is derived.

3 THE SYMMETRY MODEL

Like Elaine Chew's Spiral Array [2], the symmetry model is a geometric tonality model. A particular feature is the organisation of tones in a way that tonal symmetries within western tonal music become apparent. The model differentiates between key related [1] and key spanning tonal phenomena [2] and additionally reveals the relationship between tones, major and minor chords and keys. There are different ways to derive the symmetry model [3], but to provide evidence for the close relationship between the symmetry model's output and the psychological reality, we derive the symmetry model from Carol L. Krumhansl and E.J.Kessler's pure cognition inspired MDS solution [5].

3.1 Cognition inspired derivation

C.L. Krumhansl and E.J. Kessler [4] correlated the probe tone ratings [4] of 12 major and 12 minor keys and derived a similarity measure for every possible pair of keys. These data again had been fed into a Multi Dimensional Scaling analysis, resulting in a four dimensional spacial arrangement of keys (Figure 1). The distance between two objects within the MDS space represents the perceived similarity of two keys.

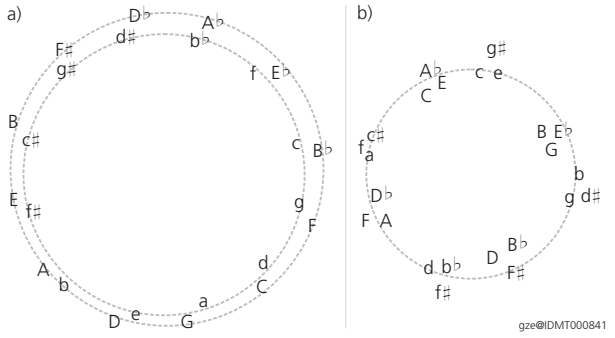

Figure 1. a) Dimensions 1 and 2 and b) dimensions 3 and 4 of Krumhansl and Kessler's four dimensional MDS solution [5, p.43].

Within the first two dimensions (Figure 1a) two circles of fifth result, one representing the major and another representing the minor keys. Within the second two dimensions (Figure 1b) 4 groups of 3 major and 4 groups of 3 minor keys are generated, primarily representing parallel and relative relationships between major and minor keys. The first step in deriving the symmetry model from the MDS solution is the development of the *spiral of thirds*: The angles of the keys within the first two dimensions are represented on the z-axis and the circle within the last two dimensions is represented on the xy-plane [5]. The two resulting spirals of fifth are conflated in such a way, that a single spiral composed of major and minor thirds evolves (Figure 2a). Cutting out one winding of the spiral of thirds results in a subspace containing 8 keys. The roots of these keys again form a diatonic set (Figure 2a). This set contains one pitch class twice, that is the pitch class forming the start (d) [6] and the end (D). Furthermore it can be denoted that also the geometric center of the spiral winding (that is between C and e) is occupied by the same pitch class forming the start and the end of the spiral winding. We call this "invisible" – for reference added – pitch class *symmetry tone* ($\tilde{d}$) because the whole diatonic set is symmetrically arranged around that tone [7] (Figure 2b,c).

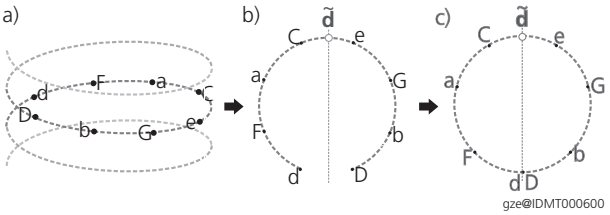

Figure 2. The key related circle of thirds evolves from dimensions 3 and 4 of the MDS solution. An interesting finding is the symmetric organisation of the diatonic set around the symmetry tone $\tilde{d}$.

Figure 2c shows a spiral winding where both ends are closed. This configuration is called *key related circle of thirds TR*. Within dimensions 1 and 2 the conflation of the two spiral of fifth results in a configuration of alternating major and minor thirds, called *key spanning circle of thirds* [8] *T* (Figure 3). This configuration represents the major-minor relationship between keys in a more interpretable way than the original solution does: It is difficult to interpret the generation of major-minor key pairs like C-Major and d-Minor within dimensions 1 and 2 of the original solution. Within the key related circle of thirds, the very important relative major/minor relationship is emphasized (e.g. C-Major/a-Minor).

[1] e.g. the resolution tendencies of the dominant seventh

[2] e.g. the parallel relationship between major and minor keys

[3] e.g. from music theory (Hugo Riemanns Harmonic Network [10]) or from Hendrik Purwins SOM based tonal representation [9]

[4] The probe tone ratings provide a measure how good each of the 12 chromatic pitch classes fits into a given key.

[5] Within dimensions 1,2 and 3,4 the keys are arranged on a circle which makes it possible to identify every key by a angle related to the circles origin.

[6] Capital letters like "D" represent major keys and chords, small letters like "d" represent minor keys and chords, letters with a tilde like "$\tilde{d}$" represent symmetry tones.

[7] The semitone distance between the symmetry tone and the tones to right of the symmetry tone are the same like that's of the tones to the left.

[8] Also known as Hauptmann's line-of-thirds[7].

3.2 Interpreting the symmetry model

According to Albert S. Bregman [1, p.124], the crucial issue in dealing with MDS solutions is to assign "meaningful properties" to the "mathematically derived dimensions". A first step towards that direction shall be done here: Actually the MDS solution is a low dimensional space composed of two circular dimensions. The first circular dimension (Figure 1a) can be interpreted as a *key spanning component*. Within the symmetry model it is represented by the key spanning circle of thirds *T* (Figure 3) which contains the two circles of fifth of the original MDS solution in a slightly contorted way. The second circular dimension (Figure 1b) can be interpreted as a *key related component*, represented by the key related circle of thirds *TR* (Figure 2c). The key related circle of thirds geometrically outlines *functional relationships*: Tones to the left of the symmetry axis form subdominant [9] chords (d-F-a, F-a-C), those to the right dominant chords (e-G-b, G-b-D) and those in the center tonic chords (a-C-e, C-e-G)(Figure 2b). Opposed to the symmetry tone, the *diminished chord* (b-D/d-f) appears, whereas a counter clockwise extension results in the *dominant seventh chord* (G-b-d/D-f) and a clockwise extension lets the *sixte ajoutée* (d-F-a-b) evolve. The resolution tendencies towards the tonic of that chords can be explained with the fact that these chords unambiguously define a spiral winding and therefore a tonal center.

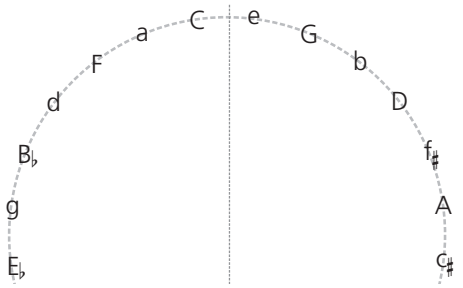

Figure 3. A part of the key spanning circle of thirds.

3.3 A first application

The angle of a tone within the key related circle of thirds α_{TR} can be taken as a measure for a tone's stability in a given tonal context. *TR* is composed of an underlying grid of 24 semi tones. To every of these semitones we assign a *base index m*:

$$-\frac{24}{2} \leq m < \frac{24}{2}, m \in \mathbb{Z} \tag{1}$$

[9] In 1726 Rameau introduced the terms tonic, dominant and subdominant to describe emotional aspects of modulations. Dominantic motions where linked to warmth, light, happiness, subdominant motions where characterized to be cold, dark, sad [11, p.68]. Rameau's descriptive framework became the basis of Hugo Riemann's "Funktionstheorie" [10].

The base index *m* is chosen such that it numerically represents real interval distances of tones and additionally outlines the symmetric organisation of the diatonic set around the symmetry tone (Figure 2): The base index $m = 0$ is assigned to the symmetry tone ($\tilde{d}$), values $m < 0$ are assigned to tones to the left of the symmetry tone, values $m > 0$ are assigned to tones right of the symmetry tone (Table 1).

	d	F	a	C	e	G	b	D
m	-12	-9	-5	-2	+2	+5	+9	+12

Table 1. The assignment of the base index *m* to the diatonic set of C-Major

So the angle α_{TR} of a tone within the key related circle of thirds can be calculated as follows:

$$\alpha_{TR}(m) = 2\pi * \frac{m}{24} \tag{2}$$

With the help of these equations the stability *s* of an given real tone can be calculated as follows:

$$s = 1.0 - |\frac{\alpha_{TR}(m)}{\pi}| \tag{3}$$

Figure 4 shows the result of this prediction in comparison to Krumhansl and Kessler's probe tone ratings [5, Table 2.1]. It is important to denote, that the symmetry model to this point considers the major and minor mode simultaneously [10] . To allow a comparison of the probe tone profiles with the symmetry model's prediction, the profiles have to be summed. Furthermore the calculated stability curve is slightly shifted to the right. This can be explained due to the fact that the symmetry model totally eliminates the root relationship.

4 RESULTS

The *spiral of thirds* has been shown as a three dimensional representation of Krumhansl and Kessler's four dimensional MDS space. With the spiral's help, the *key related* and *key spanning circle of thirds* have been identified as important subspaces of the MDS solution. It has been outlined that the generation of a *tonal center* can be explained with the *symmetric organisation of tonal objects* around a so called *symmetry tone*. *Functional relationships* between tones and the tonal stability of a tone within a given tonal context can be explained with the tone's relation to the tonal center, which is not – like traditionally done – represented by the key's root but by it's symmetry tone. The *dominant seventh chord* has been shown as evolving from the 2D representation of one winding of the spiral of thirds. The chord's key defining

[10] To predict the exact tension and resolution values of distinct modes, the symmetry model has to be extended by laws that describe the polarisation of music towards major or minor.

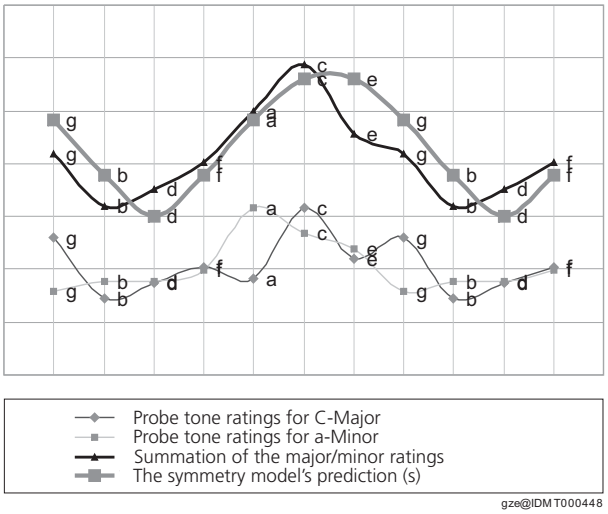

Figure 4. Krumhansl and Kessler's probe tone ratings in comparison with the symmetry model's prediction.

function and it's resolution tendencies towards the tonic haven been explained with the fact, that the diminished seventh unambiguously defines a "spiral winding" which again defines a tonal center.

5 CONCLUSION

Krumhansl and Kessler's pure cognition inspired MDS solution does not only represent relationships between keys. It also provides strong evidence, that the generation of major as well minor chords, the factors that generate a tonal center, aspects of musical consonance and dissonance, as well as functional relationships have it's origin in the neuronal self organisation of tonal events. The symmetry model provides an analytic description of these relationships and separates the MDS solution in semantic relevant subspaces. Therefore the symmetry model is predestinated for musical feature analysis, which can comprise of chord and key finding, tension and resolution analysis or tonal fingerprint estimation. The close relationship to Krumhansl and Kessler's pure cognition inspired MDS solution shows, that the model describes important aspects of the psychological reality. Next steps will be to investigate, if there are more semantic properties encoded within the symmetry model than regarded to date. Algorithms have to be developed, that extract the structural relationships – represented by the symmetry model – from real audio. Another necessary step is to combine the pure "pitch chroma" based model with approaches that incorporate aspects of "pitch height", e.g. the principle of root relationship. This can be for example by incorporating psychoacoustical approaches like Ernst Terhardt's virtual pitch theory [13] or Richard Parncutt's theory of harmony [8].

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