VISUALIZING MUSIC ON THE METRICAL CIRCLE

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ABSTRACT

In this paper we propose a novel method, called Metrical Circle Map, for exploring the cyclic aspects of musical time. To this end, we give a short formalization introducing the notion of Metrical Markov Chains as transition probabilities of segments on the metrical circle. As an illustration we present a compact visualization of the zeroth- and first order metrical Markov transitions of 61 Irish folk songs.

1 INTRODUCTION

One important and distinctive feature of metrically-bound music is the double nature of its musical time, linear on one hand, cyclic on the other. However, in most of musicological and other music-related research the focus is laid on linear aspects of musical time. The cyclic nature of musical time is mainly investigated in the context of genuine meter and rhythm research, where rhythm and meters are occasionally visualized or operationalized using a circle representation ([3], [4], [5]). In this paper, we will extend this approach by introducing the so-called Metrical Circle Map and Metrical Markov Chains, opening up several interesting posibilities of visualizing and analyzing metrically bound music, including single monophonic or polyphonic pieces as well as entire corpora.

2 METRICAL CIRCLE MAP

The onsets of metrically-bound music are organized around underlying beats (or pulses), which are grouped into higher level units. This is even true for un-quantized music with tempo and meter variations, as for the concept of meter only a set of discrete time-points is needed along with a grouping prescription. We restrict ourselves here to the bar as the main metrical unit, and futhermore to the simplest form of a bar as a constantly-recurring time-span. For a rhythm conceived as a sequence of time-points t_i, and a bar time T - possibly inferred from the original sequence by some beat and meter induction algorithm (e.g. [2]), or given by annotation - the *Metrical Circle Map* M_T is defined as a mapping from the reals into the complex unit circle S^1:

$$M_T(t_i) = z_i = e^{2\pi i \frac{t_i}{T}} \quad (1)$$

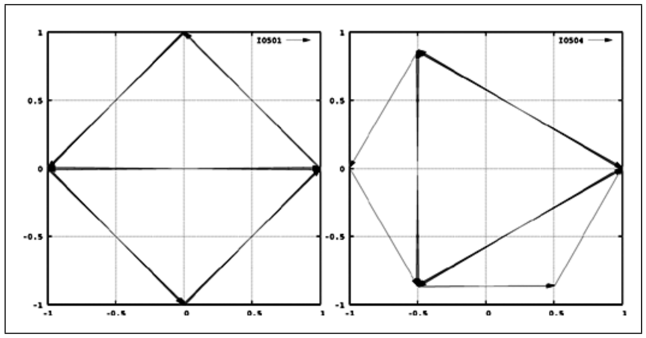

Figure 1. Metrical circle map of two Irish melodies. The left one (Essen Folksong Collection, I0501) shows 2/4 time, the right one (Essen Folksong Collection, I0504) 3/4 time. Thickness of lines corresponds to the frequencies of the transitions. Time is running counter-clockwise and the downbeat is located an the point (1,0) in the complex plane (three o'clock)

In this form we already normalized the map, so that time is running in the mathematical direction counter-clockwise and the beginning of bars always lie at three o'clock. In Figure 1 two simple examples of the Metrical Circle Map (MCM) for two Irish folk songs can be seen, one of them being in 2/4 and one in 3/4 time.

2.1 Metrical Markov Chains

The Metrical Circle Map (MCM) lends itself quite naturally to defining transition probabilities between segments on the metrical circle. To this end, we define N intervals on S^1 according to

$$I_k = \{z \in S^1 | z = e^{2\pi i \frac{\phi_k}{N}}, \phi_k \in [k - \frac{1}{2}, k + \frac{1}{2}]\} \quad (2)$$

with $0 \leq k < N$. With these intervals and the MCM we can map a sequence of time-points onto a sequence of intervals on the metrical circle. For these sequences of interval indices we define the usual Markov transitions probabilities. Particularly, we will be interested in zeroth- and first-order transitions

$$p(k) = p(z \in I_k), \quad p(k|j) = p(z_i \in I_k | z_{i-1} \in I_j) \quad (3)$$

For calculating transition probabilities, the choice of N is of course crucial, as it controls the time resolution. If, for example, one chooses $N = 2$, the first order probabilities indicate the probabilities of transitions between the two

Signature	2/4	3/4	4/4	6/4	6/8	9/8
Count	7	20	20	2	10	2

Table 1. Distribution of signatures in the collection of 61 Irish folk songs

halves of a bar. For more detailed and complete comparisions, particularly for whole corpora with a full range of meters, a choice of $N = 48$ seems appropriate, representing e.g. a 4/4 meter with a resolution of thirty-second note sextuplets.

3 EXAMPLES: ZEROTH- AND FIRST-ORDER MARKOV CHAINS FOR IRISH FOLK SONGS

In order to illustrate our method, we transformed 61 (monophonic) Irish folk songs taken from the Essen Folksong Database with the MCM, and accumulated zeroth- and first-order transition probabilities over all songs, using an $N = 48$ segmentation of the circle. These probabilities are jointly depicted in Figure 2, where the thickness of a line connecting two metric positions is proportional to the first-order transition probability (transitions with $p < 0.001$ left out). The size of the smaller circles at the metric positions are proportional to the logarithm of occupation probability. One clearly identifies the square from duple, the triangle from triple, and the hexagon from compound duple meters. Not surprisingly, the downbeat is the most frequent position (21%), with the most arrows ending at this point, reached from 6 different positions, noticably from points right before the downbeat, the upbeats, which play an important role in establishing and reinforcing the meter. The next most frequent position, the half-bar position from duple and ternary compound meter, is occupied in 13% of the cases, with 3 arrows ending there. Furthermore, a nearly complete absence of syncopations can be read off from the graph, notably, the vertical axis from the transition $12 \rightarrow 36$ is missing, as well as syncopational transitions from positions 6, 18, 30, 42 in duple meter, or from 8, 20, 40 in compound duple and triple meter. In contrast to this, examination of a set of pop songs showed a considerably more chaotic picture (not shown here for the lack of space), with frequent syncopations, a higher diversity of possible transitions and less emphasis on the down- and half-bar-beats.

4 DISCUSSION & OUTLOOK

We propose the method of the Metrical Circle Map. Single or entire corpora of melodies, as well as polyphonic music can be visualized on the metrical circle using metrical transition probabilities, giving new analytical insight in the cyclic aspects of musical time and in the metrical peculiarities of different genres, which will be explored further in the future.

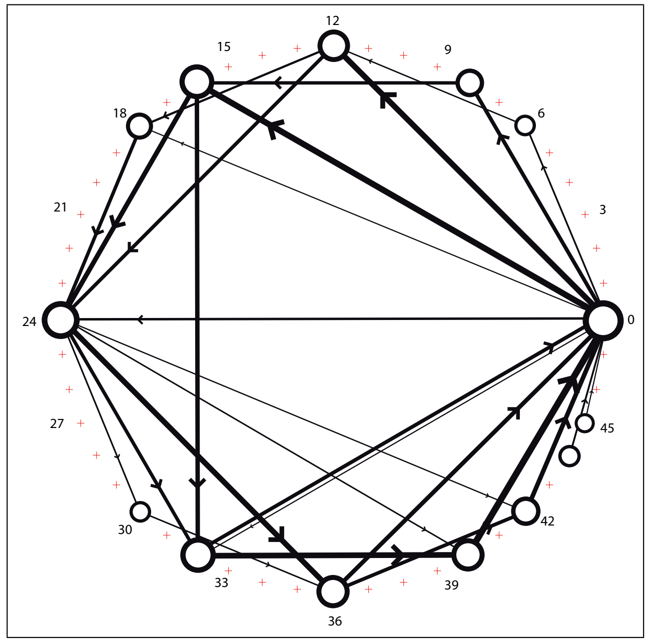

Figure 2. Zeroth- and first-order metric transitions probabilities of 61 Irish folk songs from the Essen collection. The thickness of lines is proportional to the frequency of first-order transitions; transitions with $p < 0.01$ were left out. The size of the circles is proportional to the logarithm of occupation probabilities.

5 ACKNOWLEDGEMENTS

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6 REFERENCES

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