Learning and Modal Logic: There and Back Again

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Among many interpretations of modal logic the one pertaining to knowledge and belief has been especially buoyant in recent years. The framework of epistemic logic offers a platform for a systematic study of knowledge and belief. Dynamic epistemic logic further extends that way of thinking to cover many kinds of transformations knowledge undergoes in communication, and under other informative events. Such iterated changes can be given a long-term horizon of learning, i.e., they can be seen as ways to acquire a desirable kind of epistemic state. Thus, the question arises: Can modal logic contribute to our understanding of learning processes in general?

The link between dynamic epistemic logic and computational learning theory was introduced in [10,11], where it was shown that exact learning in finite time (also known as finite identification, see [16,17]) can be modelled in dynamic epistemic logic, and that the elimination process of learning by erasing [15] can be seen as iterated upgrade of dynamic doxastic logic. This bridge opened a way to study truth-tracking properties of doxastic upgrade methods on positive, negative, and erroneous input [2,4]. Switching from relational to topological semantics for modal logic allowed characterising favourable conditions for learning in the limit in terms of general topology [3]. This line of research recently culminated in proposing a Dynamic Logic for Learning Theory, which extends Subset Space Logics [7] with dynamic observation modalities and a learning operator [1].

Finite identifiability and its connections with epistemic temporal logic have been further studied in [9]. Learning seen as conclusive epistemic update resulted in designing new types of learners, such as preset learners and fastest learners [14]. Some of those results were later adopted to study learning of action models in dynamic epistemic logic [5,6], and to investigate properties of finite identification from complete data [8]. For an overview of some above contributions one can also consult [12,13].

In my lecture I will overview the modal logic perspective on learnability, drawing from the line of work described above.

References

[1] A. Baltag, N. Gierasimczuk, A. Özgün, A. L. Vargas-Sandoval, and S. Smets. A dynamic logic for learning theory. *Journal of Logical and Algebraic Methods in Programming*, 109:100485, 2019.

[2] A. Baltag, N. Gierasimczuk, and S. Smets. Belief revision as a truth-tracking process. In K. Apt, editor, *TARK'11: Proceedings of the 13th Conference on Theoretical Aspects of Rationality and Knowledge, Groningen, The Netherlands, July 12-14, 2011*, pages 187–190. ACM, New York, NY, USA, 2011.

[3] A. Baltag, N. Gierasimczuk, and S. Smets. On the solvability of inductive problems: A study in epistemic topology. In R. Ramanujam, editor, *Proceedings Fifteenth Conference on Theoretical Aspects of Rationality and Knowledge, TARK 2015, Carnegie Mellon University, Pittsburgh, USA, June 4-6, 2015*, volume 215 of *EPTCS*, pages 81–98, 2015.

[4] A. Baltag, N. Gierasimczuk, and S. Smets. Truth-tracking by belief revision. *Studia Logica*, 107(5):917–947, 2019.

[5] T. Bolander and N. Gierasimczuk. Learning actions models: Qualitative approach. In W. van der Hoek, W. H. Holliday, and W. Wang, editors, *Logic, Rationality, and Interaction - 5th International Workshop, LORI 2015 Taipei, Taiwan, October 28-31, 2015, Proceedings*, volume 9394 of *Lecture Notes in Computer Science*, pages 40–52. Springer, 2015.

[6] T. Bolander and N. Gierasimczuk. Learning to act: qualitative learning of deterministic action models. *Journal of Logic and Computation*, 28(2):337–365, 2018.

[7] A. Dabrowski, L. S. Moss, and R. Parikh. Topological reasoning and the logic of knowledge. *Annals of Pure and Applied Logic*, 78(1):73 – 110, 1996. Papers in honor of the Symposium on Logical Foundations of Computer Science, Logic at St. Petersburg.

[8] D. de Jongh and A. L. Vargas-Sandoval. Finite identification with positive and with complete data. In A. Silva, S. Staton, P. Sutton, and C. Umbach, editors, *Language, Logic, and Computation*, pages 42–63, Springer, Berlin/Heidelberg, 2019. .

[9] C. Dégremont and N. Gierasimczuk. Finite identification from the viewpoint of epistemic update. *Information and Computation*, 209(3):383–396, 2011.

[10] N. Gierasimczuk. Bridging learning theory and dynamic epistemic logic. *Synthese*, 169(2):371–384, 2009.

[11] N. Gierasimczuk. Learning by erasing in dynamic epistemic logic. In A. H. Dediu, A. M. Ionescu, and C. Martin-Vide, editors, *LATA'09: Proceedings of 3rd International Conference on Language and Automata Theory and Applications, Tarragona, Spain, April 2-8, 2009*, volume 5457 of *Lecture Notes in Computer Science*, pages 362–373. Springer, The Netherlands, 2009.

[12] N. Gierasimczuk. *Knowing One's Limits. Logical Analysis of Inductive Inference*. PhD thesis, Universiteit van Amsterdam, The Netherlands, 2010.

[13] N. Gierasimczuk, D. de Jongh, and V. F. Hendricks. Logic and learning. In A. Baltag and S. Smets, editors, *Johan van Benthem on Logical and Informational Dynamics*. Springer, 2014.

[14] N. Gierasimczuk and D. de Jongh. On the complexity of conclusive update. *The Computer Journal*, 56(3):365–377, 2013.

[15] S. Lange, R. Wiehagen, and T. Zeugmann. Learning by erasing. In S. Arikawa and A. Sharma, editors, *ALT*, volume 1160 of *Lecture Notes in Computer Science*, pages 228–241. Springer, 1996.

[16] S. Lange and T. Zeugmann. Types of monotonic language learning and their characterization. In *COLT'92: Proceedings of the 5th Annual ACM Conference on Computational Learning Theory, Pittsburgh, PA, USA, July 27-29, 1992*, pages 377–390. ACM, New York, NY, USA, 1992.

[17] Y. Mukouchi. Characterization of finite identification. In K. Jantke, editor, *AII'92: Proceedings of the International Workshop on Analogical and Inductive Inference, Dagstuhl Castle, Germany, October 5-9, 1992*, volume 642 of *Lecture Notes in Computer Science*, pages 260–267. Springer, Berlin/Heidelberg, 1992.