Syllogistic Reasoning under the Weak Completion Semantics

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Abstract. In a recent meta-analysis, Khemlani & Johnson-Laird (2012) showed that the conclusions drawn by human reasoners in psychological experiments about syllogistic reasoning are not the conclusions predicted by classical first-order logic. Moreover, current cognitive theories deviate significantly from the empirical data. In this paper we show how human syllogistic reasoning can be modelled under the weak completion semantics, a new cognitive theory.

1 Introduction

The way of how humans ought to reason correctly about syllogisms has already been investigated by Aristotle. A *syllogism* consists of two quantified statements using some of the four quantifiers *all* (A), *no* (E), *some* (I), and *some are not* (O)[1] about sets of entities which we denote in the following by a, b, c. An example is:

$$\begin{array}{c} \textit{Some a are b} \\ \textit{No b are c} \end{array} \qquad (\mathsf{IE}1)$$

and the task is to draw a conclusion. Implicitly, most experiments expect to draw a logical consequence from these so-called premises, e.g., '*some a are not c*' in classical first-order logic (FOL). The four quantifiers and their formalization in FOL are given in Table 1. The entities can appear in four different orders called *figures* as shown in Table 2. Hence, a problem can be completely specified by the quantifiers of the first and second premise and the figure. E.g., the example discussed above is denoted by IE1.

Altogether, there are 64 syllogisms and, if formalized in FOL, we can compute their logical consequence in classical logic. However, a meta-analysis [15] based on six experiments has shown that humans do not only systematically deviate from the predictions of FOL but from any other of at least 12 cognitive theories. In the case of IE1, besides the above mentioned logical consequence, a significant number of humans answered '*no a are c*' which does not follow from IE1 in FOL.

⋆ The authors are mentioned in alphabetical order.
[1] We are using the classical abbreviations.

Mood	Natural Language	FOL	Short
affirmative universal (A)	*all a are b*	$\forall X(a(X) \rightarrow b(X))$	Aab
affirmative existential (I)	*some a are b*	$\exists X(a(X) \wedge b(X))$	Iab
negative universal (E)	*no a are b*	$\forall X(a(X) \rightarrow \neg b(X))$	Eab
negative existential (O)	*some a are not b*	$\exists X(a(X) \wedge \neg b(X))$	Oab

Table 1. The four syllogistic moods together with their logical formalization.

	Figure 1	Figure 2	Figure 3	Figure 4
First Premise	a-b	b-a	a-b	b-a
Second Premise	b-c	c-b	c-b	b-c

Table 2. The four figures used in syllogistic reasoning.

In recent years, a new cognitive theory based on the weak completion semantics (WCS) has been developed. It has its roots in the ideas first expressed by Stenning and van Lambalgen [23], but is mathematically sound [11], and has been successfully applied – among others – to the suppression task [6], the selection task [7], the belief bias effect [18,19,2], to reasoning about conditionals [3,5] and to spatial reasoning [4]. Hence, it was natural to ask whether WCS is competitive in syllogistic reasoning and how it performs with respect to the cognitive theories considered in [15].

In the following, three important cognitive approaches are presented and compared to predictions made by WCS. Firstly, we show how to adequately represent the syllogisms in logic programs and, secondly, we demonstrate how to reason with respect to them. Afterwards we compare the results under the WCS with the results of FOL, the syntactic rule based theory PSYCOP [22], the Verbal Model Theory [21] and the Mental Model Theory [13].[2] These two model-based theories performed the best in the meta-analysis.

The predictions of the theories FOL, PSYCOP, Verbal, and Mental Models for the syllogisms OA3, EA3, and AA4 and those of the participants, taken from [15], are depicted in Table 3, where the participants were 156 high school to university students. The significant percentage of participants means that the number of participants who chose for the particular conclusion, was too high for

[2] `http://mentalmodels.princeton.edu/models/mreasoner/`

	Participants	FOL	PSYCOP	Verbal Models	Mental Models
OA4	Oca	Oca	Oca, Ica, Iac	Ocs, NVC	Oca, Oac, NVC
EA3	Eac	Eac, Eca Oac, Oca	Eac, Eca Oac, Oca	NVC, Eca	Eac, Eca
AA4	Aac, NVC	Iac, Ica	Iac, Ica	NVC, Aca	Aca, Aac, Iac, Ica

Table 3. The conclusions drawn by a significant percentage of participants are highlighted in gray and compared to the predictions of the theories FOL, PSYCOP, Verbal, and Mental Models for the syllogisms OA4, EA3, and AA4. NVC stands for *no valid conclusion.*

the conclusion to be chosen randomly.[3] The interested reader is referred to [15] for more details.

FOL and the other three cognitive theories make different predictions. In particular, each theory provides at least one prediction which is correct with respect to classical FOL and provides an additional prediction which is false with respect to classical FOL. Currently, the best overall results are achieved by the Verbal Models Theory which predicts 84% of the participants responses, closely followed by the Mental Model Theory with 83%, whereas PSYCOP predicts 77% of the participants responses.

2 Weak Completion Semantics

The general notation, which we will use in the paper, is based on [16,10].

2.1 Logic Programs

We assume the reader to be familiar with logic and logic programming, but recall basic notions and notations. A *(logic) program* is a finite set of (program) clauses of the form $A \leftarrow \top,\ A \leftarrow \bot$ or $A \leftarrow B_1 \wedge \ldots \wedge B_n,\ n > 0$, where A is an atom, B_i, $1 \leq i \leq n$, are literals and $\top$ and $\bot$ denote truth and falsehood, respectively. A is called *head* and $\top$, $\bot$ as well as $B_1 \wedge \ldots \wedge B_n$ are called *body* of the corresponding clause. Clauses of the form $A \leftarrow \top$ and $A \leftarrow \bot$[4] are called

[3] The threshold for the percentage to be significant is determined as follows: Given that there are nine different conclusion possibilities, the chance that a conclusion has been chosen randomly is $1/9 = 11.1\%$. A binomial test shows that if a conclusion is drawn in more than 16% of the cases by the participants it is unlikely that is has been chosen by just random guesses. The statistical analysis is elaborately explained in [15].

[4] We consider weak completion semantics and, hence, a clause of the form $A \leftarrow \bot$ is turned into $A \leftrightarrow \bot$ provided that this is the only clause where A is the head of.

F	$\neg F$
$\top$	$\bot$
$\bot$	$\top$
U	U

$\wedge$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	U	$\bot$
$\bot$	$\bot$	$\bot$	$\bot$

$\vee$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	$\top$	U	U
$\bot$	$\top$	U	$\bot$

$\leftarrow$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	U	$\top$	$\top$
$\bot$	$\bot$	U	$\top$

$\leftrightarrow$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	$\top$	U
$\bot$	$\bot$	U	$\top$

Table 4. The truth tables for the connectives under Ł-logic. $\top$, $\bot$, and U denote *true*, *false*, and *unknown*, respectively.

positive and *negative facts*, respectively. We restrict terms to be constants and variables only, i.e., we consider *data logic programs*. Throughout this paper, $\mathcal{P}$ denotes a program. We assume for each $\mathcal{P}$ that the alphabet consists precisely of the symbols occurring in $\mathcal{P}$ and that non-propositional programs contain at least one constant.

$\mathsf{g}\mathcal{P}$ denotes the set of all ground instances of clauses occurring in $\mathcal{P}$, where a ground instance of clause C is obtained from C by replacing each variable occurring in C by a term not containing any variables. A ground atom A is *defined* in $\mathsf{g}\mathcal{P}$ iff $\mathsf{g}\mathcal{P}$ contains a clause whose head is A; otherwise A is said to be *undefined*. $\mathit{def}(\mathcal{S}, \mathcal{P}) = \{A \leftarrow \mathit{Body} \in \mathsf{g}\mathcal{P} \mid A \in \mathcal{S} \vee \neg A \in \mathcal{S}\}$ is called *definition* of $\mathcal{S}$ in $\mathcal{P}$, where $\mathcal{S}$ is a set of ground literals. $\mathcal{S}$ is said to be *consistent* iff it does not contain a pair of complementary literals. The set of atoms occurring in $\mathsf{g}\mathcal{P}$ is denoted as $\mathsf{atoms}(\mathcal{P})$.

2.2 Three-Valued Łukasiewicz Logic

We consider the three-valued Łukasiewicz logic (Ł-logic, [17]), for which the corresponding truth values are $\top$, $\bot$ and U, which mean *true*, *false* and *unknown*, respectively. A *three-valued interpretation* I is a mapping from formulas to a set of truth values $\{\top, \bot, \mathsf{U}\}$. The truth value of a given formula under I is determined according to the truth tables in Table 4. We represent an interpretation as a pair $I = \langle I^\top, I^\bot \rangle$ of disjoint sets of atoms, where $I^\top$ is the set of all atoms that are mapped to $\top$ by I, and $I^\bot$ is the set of all atoms that are mapped to $\bot$ by I. Atoms which do not occur in $I^\top \cup I^\bot$ are mapped to U. Let $I = \langle I^\top, I^\bot \rangle$ and $J = \langle J^\top, J^\bot \rangle$ be two interpretations: $I \subseteq J$ iff $I^\top \subseteq J^\top$ and $I^\bot \subseteq J^\bot$. $I(F) = \top$ means that a formula F is mapped to true under I. $\mathcal{M}$ is a *model* of $\mathcal{P}$ if it is an interpretation, which maps each clause occurring in $\mathsf{g}\mathcal{P}$ to $\top$. I is the *least model* of $\mathcal{P}$ iff for any other model J of $\mathcal{P}$ it holds that $I \subseteq J$.

2.3 Least Models under the Weak Completion

For a given $\mathcal{P}$, consider the following transformation:

1. For each ground atom A which is defined in $\mathcal{P}$, replace all clauses of the form $A \leftarrow \mathit{Body}_1, \ldots, A \leftarrow \mathit{Body}_m$ occurring in $\mathsf{g}\mathcal{P}$ by $A \leftarrow \mathit{Body}_1 \vee \ldots \vee \mathit{Body}_m$.
2. Replace all occurrences of $\leftarrow$ by $\leftrightarrow$.

The obtained ground program is called *weak completion* of $\mathcal{P}$ or $wc\mathcal{P}$.[5]

It has been shown in [12] that logic programs as well as their weak completions admit a least model under Ł-logic. Moreover, the least Ł-model of $wc\mathcal{P}$ can be obtained as the least fixed point of the following semantic operator, which is due to Stenning and van Lambalgen [23]: Let $I = \langle I^\top, I^\bot \rangle$ be an interpretation. $\Phi_\mathcal{P}(I) = \langle J^\top, J^\bot \rangle$, where

$$\begin{aligned} J^\top &= \{A \mid A \leftarrow \mathit{Body} \in \mathit{def}(A,\mathcal{P}) \text{ and } \mathit{Body} \text{ is } \mathit{true} \text{ under } \langle I^\top, I^\bot \rangle\} \\ J^\bot &= \{A \mid \mathit{def}(A,\mathcal{P}) \neq \emptyset \text{ and} \\ &\qquad\qquad \mathit{Body} \text{ is } \mathit{false} \text{ under } \langle I^\top, I^\bot \rangle \text{ for all } A \leftarrow \mathit{Body} \in \mathit{def}(A,\mathcal{P})\} \end{aligned}$$

Weak completion semantics (WCS) is the approach to consider weakly completed logic programs and to reason with respect to the least Ł-models of these programs. We write $\mathcal{P} \models_{wcs} F$ iff formula F holds in the least Ł-model of $wc\mathcal{P}$. In the remainder of this paper, $\mathcal{M}_\mathcal{P}$ denotes the least Ł-model of $wc\mathcal{P}$.

The correspondence between weak completion semantics and well-founded semantics [24] for tight programs, i.e. those without positive cycles, is shown in [8].

2.4 Integrity Constraints

A set of *integrity constraints* $\mathcal{IC}$ consists of clauses of the form $\mathsf{U} \leftarrow \mathit{Body}$, where *Body* is a conjunction of literals and U denotes the unknown.[6] Hence, an interpretation maps an integrity constraint to $\top$ iff *Body* is either mapped to U or $\bot$. This understanding is similar to the definition of the integrity constraints for the well-founded semantics in [20]. Given an interpretation I and a set of integrity constraints $\mathcal{IC}$, I *satisfies* $\mathcal{IC}$ iff all clauses in $\mathcal{IC}$ are true under I.

3 Reasoning Towards an Appropriate Logical Form

We will apply four principles in developing a logical form for the representation of syllogisms.

3.1 Licenses for Inferences

Stenning and van Lambalgen [23] propose to formalize conditionals in human reasoning not by inferences straight away, but rather by *licenses for inferences*. For example, the conditional *if* $p(X)$ *then* $q(X)$ is represented by the program

$$\{q(X) \leftarrow p(X) \wedge \neg ab(X),\ ab(X) \leftarrow \bot\},$$

which states that $q(X)$ *holds if* $p(X)$ *and* $\neg ab(X)$ *hold* and $\neg ab(X)$ *holds*, where $\neg ab(X)$ means that *nothing is abnormal for* X *with respect to this clause.*

[5] Note that undefined atoms are not identified with $\bot$ as in the completion of $\mathcal{P}$ [1].

[6] Formally, we need to extend the alphabet by this symbol.

3.2 Existential Import and Gricean Implicature

Humans do understand quantifiers differently due to a pragmatic understanding of language. For instance, in natural language we normally do not quantify over things that do not exist. Consequently, *for all* implies *there exists*. This appears to be in line with human reasoning and has been called the *Gricean Implicature* [9]. Several theories like the theory of mental models [14] or mental logic [22] assume that the sets we quantify about are not empty. Likewise, Stenning and van Lambalgen [23] have shown that humans require existential import for a conditional to be true. Furthermore, as mentioned in [15], the quantifier '*some a are b*' often implies that '*some a are not b*', which again can be explained by assuming the *Gricean Implicature*: Someone would not state '*some a are b*' if that person knew that '*all a are b*'. As the person does not say '*all a are b*' but instead '*some a are b*', we have to assume that '*not all a are b*', which in turn implies '*some a are not b*'.

3.3 Negation by Transformation

Logic programs do not allow negative literals as heads of clauses. In order to represent a negative conclusion $\neg p(X)$ an auxiliary formula $p'(X)$ is used together with a clause $p(X) \leftarrow \neg p'(X)$ and the integrity constraint $\mathsf{U} \leftarrow p(X) \wedge p'(X)$. This is a widely used technique in logic programming. Together with the principle discussed in Section 3.1, the additional clause becomes $p(X) \leftarrow \neg p'(X) \wedge \neg ab_{npp}(X)$, and its weak completion is $p(X) \leftrightarrow \neg p'(X) \wedge \neg ab_{npp}(X)$, stating that p' is the negation of p iff nothing abnormal is known with respect to that clause. The integrity constraint states that an object cannot belong to both, p and p'. We call this principle *negation by transformation*.

3.4 Unknown Generalization

Humans seem to distinguish between '*some y are z*' and '*some z are y*', as the results reported in [15] show. However, if we would represent '*some y are z*' by $\exists X(y(X) \wedge z(X))$ then this is semantically equivalent to $\exists X(z(X) \wedge y(X))$ in FOL because conjunction is commutative. Likewise, humans seem to distinguish between '*some y are z*' and '*all y are z*', as we have already explained in Section 3.2. Accordingly, if we only observe that an object o belongs to y and z then we do not want to conclude both, '*some y are z*' and '*all y are z*'.

In order to prevent such unwanted conclusions we introduce the following principle: if we know that '*some y are z*' then there must not only be an object o_1 which belongs to y and z (by Gricean implicature) but there must be another object o_2 which belongs to y and for which it is unknown whether it belongs to z. We call this principle *unknown generalization*.

4 Representing the Syllogisms

Based on the principles presented in the previous section, we can now represent the syllogisms by logic programs. The programs will be specified using the predicates y and z and depending on the figures shown in Table 2: yz must be replaced by ab, ba, cb or bc.

4.1 All (A)

'*All y are z*' is represented by the program $\mathcal{P}_{Ayz}$ which consists of the following clauses:

$$\begin{array}{l} z(X) \leftarrow y(X) \wedge \neg ab_{yz}(X) \\ ab_{yz}(X) \leftarrow \bot \\ y(o) \leftarrow \top \end{array}$$

The first two clauses are obtained by applying the principle of using licenses for inferences. The last clause follows by the principle of Gricean implicature, where o is the object which is assumed to exist for y. The least Ł-model for $wc\mathcal{P}_{Ayz}$ is

$$\langle\{y(o), z(o)\}, \{ab_{yz}(o)\}\rangle.$$

4.2 No (E)

'*No y are z*' is represented by the program $\mathcal{P}_{Eyz}$ which consists of the following clauses:

$$\begin{array}{l} z'(X) \leftarrow y(X) \wedge \neg ab_{ynz}(X) \\ ab_{ynz}(X) \leftarrow \bot \\ z(X) \leftarrow \neg z'(X) \wedge \neg ab_{nzz}(X) \\ y(o) \leftarrow \top \\ ab_{nzz}(o) \leftarrow \bot \end{array}$$

In addition we need the integrity constraint

$$\mathsf{U} \leftarrow z(X) \wedge z'(X).$$

The first two clauses are obtained by applying the principle of using licenses for inferences, where z' is an auxiliary predicate symbol used to denote the negation of z. This auxiliary predicate is formally related to z by the third clause applying the principle of negation by transformation. In addition, this principle enforces the integrity constraint. The fourth clause of $\mathcal{P}_{Eyz}$ follows by the principle of Gricean implicature.

The last clause cannot be generalized to all X, because otherwise we allow conclusions by double negation. For example, consider the case where there is some o' for which $y(o')$ is false due to some clause being part of another premise. In this case, the first clause will enforce the falsehood of $z'(o')$. Now, if $ab_{nzz}(X) \leftarrow \bot$ would hold, then $ab_{nzz}(o')$ would be false and, consequently, by the third clause, $z(o')$ would be true. In other words, $z(o')$ follows by the negation of $y(o')$, which in turn is responsible for the negation of $z'(o')$. Even though this might be logically reasonable, the empirical results indicate that participants do not infer conclusions based on double negation. Therefore, we decided to restrict $ab_{nzz}(o) \leftarrow \bot$ to the objects occurring in $\mathcal{P}_{Eyz}$.

The least Ł-model for $wc\mathcal{P}_{Eyz}$ is

$$\langle\{y(o), z'(o)\}, \{ab_{ynx}(o), ab_{nzz}(o), z(o)\}\rangle.$$

4.3 Some (I)

'*Some y are z*' is represented by the program $\mathcal{P}_{Iyz}$ which consists of the following clauses:

$$\begin{array}{l} z(X) \leftarrow y(X) \wedge \neg ab_{yz}(X) \\ ab_{yz}(o_1) \leftarrow \bot \\ y(o_1) \leftarrow \top \\ y(o_2) \leftarrow \top \end{array}$$

The first two clauses are again obtained by the principle of using licenses for inferences. However, the abnormality predicate is restricted to the object o_1, which is assumed to exist by the principle of Gricean implicature (see third clause). The fourth clause is obtained by the principle of unknown generalization. The least Ł-model of $wc\mathcal{P}_{Iyz}$ is

$$\langle \{y(o_1), y(o_2), z(o_1)\}, \{ab_{yz}(o_1)\}.$$

Note that nothing is stated in $\mathcal{P}_{Iyz}$ about $ab_{yz}(o_2)$. Accordingly, $z(o_2)$ stays unknown in the least Ł-model.

4.4 Some Are Not (O)

'*Some y are not z*' is represented by the program $\mathcal{P}_{Oyz}$ which consists of the following clauses:

$$\begin{array}{l} z'(X) \leftarrow y(X) \wedge \neg ab_{ynz}(X) \\ ab_{ynz}(o_1) \leftarrow \bot \\ z(X) \leftarrow \neg z'(X) \wedge \neg ab_{nzz}(X) \\ y(o_1) \leftarrow \top \\ y(o_2) \leftarrow \top \\ ab_{nzz}(o_1) \leftarrow \bot \\ ab_{nzz}(o_2) \leftarrow \bot \end{array}$$

In addition, we need the integrity constraint

$$U \leftarrow z(X) \wedge z'(X).$$

The first four clauses as well as the integrity constraints are derived as in the program $\mathcal{P}_{Eyz}$ except that object o_1 is used instead of o and ab_{ynz} is restricted to o_1 like in $\mathcal{P}_{Iyz}$. The fifth clause of $\mathcal{P}_{Oyz}$ is obtained by the principle of unknown generalization. The last two clauses are again not generalized to all objects for the same reason as previously discussed in Section 5.2 for the representation of E. The least Ł-model for $wc\mathcal{P}_{Oyz}$ is

$$\langle \{y(o_1), y(o_2), z'(o_1)\}, \{ab_{ynz}(o_1), ab_{nzz}(o_1), ab_{nzz}(o_2), z(o_1)\}\rangle.$$

5 Entailment of Syllogisms

We can now combine the proposed representations with respect to the figures in Table 2. In doing so, we replace yz by ab, ba, cb or bc. In addition, we may need to rename objects such that different objects are referred to in the representations of different syllogisms. Thereafter, we compute the least Ł-model of the obtained programs and check which syllogisms hold in this model.

However, we have not yet defined when a syllogism holds given a program $\mathcal{P}$. These definitions will be developed in this section.

One should observe that [15] does not contain a formal definition for the entailment of syllogisms. They use first-order theory as a normative theory, i.e., they test if the conclusions drawn by the participants are correct with respect to a first-order representation of a syllogism. In the following an entailment regarding the weak completion semantics is presented, where yz is to be replaced by ab, ba, cb, bc, ac, or ca.

5.1 All (A)

$\mathcal{P} \models Ayz$ iff there exists an object o such that $\mathcal{P} \models_{wcs} y(o)$ and for all objects o we find that if $\mathcal{P} \models_{wcs} y(o)$ then $\mathcal{P} \models_{wcs} z(o) \wedge \neg ab_y z(o)$.

The existence of an object o belonging to y is due to the principle of Gricean implicature. Moreover, all objects belonging to y must belong to z. The requirement that $\neg ab_{yz}(o)$ is also entailed is a technical one which is based on the principle of licences for inferences. We may omit these abnormalities to obtain the following alternative definition.

$\mathcal{P} \models' Ayz$ iff there exists an object o such that $\mathcal{P} \models_{wcs} y(o)$ and for all objects o we find that if $\mathcal{P} \models_{wcs} y(o)$ then $\mathcal{P} \models_{wcs} z(o)$.

5.2 No (E)

$\mathcal{P} \models Eyz$ iff there exists an object o such that $\mathcal{P} \models_{wcs} y(o)$ and for all objects o we find that if $\mathcal{P} \models_{wcs} y(o)$ then $\mathcal{P} \models_{wcs} z'(o) \wedge \neg z(o) \wedge \neg ab_{ynz}(o) \wedge \neg ab_{nzz}(o)$.

The existence of an object o belonging to y is again due to the principle of Gricean implicature. Moreover, all objects o belonging to y must not belong to z and, hence, $\neg z(o)$ must be entailed. The remaining entailed formulas are due to the principles of negation by transformation and of licenses for inferences. We may omit these formulas to obtain the following alternative definition.

$\mathcal{P} \models' Eyz$ iff there exists an object o such that $\mathcal{P} \models_{wcs} y(o)$ and for all objects o we find that if $\mathcal{P} \models_{wcs} y(o)$ then $\mathcal{P} \models_{wcs} \neg z(o)$.

5.3 Some (I)

$\mathcal{P} \models Iyz$ iff there exists an object o_1 such that $\mathcal{P} \models_{wcs} y(o_1) \wedge z(o_1) \wedge \neg ab_{yz}(o_1)$ and there exists an object o_2 such that $\mathcal{P} \models_{wcs} y(o_2)$ and $\mathcal{P} \not\models_{wcs} z(o_2) \wedge \neg ab_{yz}(o_2)$.

The existence of an object o_1 belonging to y is again due to the principle of Gricean implicature. This object o_1 must also belong to z. In addition, there must be another object o_2 belonging to y for which it is unknown whether it belongs to z due to the principle of unknown generalization. The abnormality predicates $\neg ab_{yz}(o_1)$ and $\neg ab_{yz}(o_2)$ are again due to the principle of licenses for inferences. We may omit these abnormalities to obtain the following alternative definition.

$\mathcal{P} \models' Iyz$ iff there exists an object o_1 such that $\mathcal{P} \models_{wcs} y(o_1) \wedge z(o_1)$ and there exists an object o_2 such that $\mathcal{P} \models_{wcs} y(o_2)$ and $\mathcal{P} \not\models_{wcs} z(o_2)$.

5.4 Some Are Not (O)

$\mathcal{P} \models Oyz$ iff there exists an object o_1 such that $\mathcal{P} \models_{wcs} y(o_1) \wedge z'(o_1) \wedge \neg z(o_1) \wedge \neg ab_{ynz}(o_1) \wedge \neg ab_{nzz}(o_1)$ and there exists an object o_2 such that $\mathcal{P} \models_{wcs} y(o_2)$ and $\mathcal{P} \not\models_{wcs} z'(o_2) \wedge \neg z(o_2) \wedge \neg ab_y nz(o_2) \wedge \neg ab_{nzz}(o_2)$.

This case combines E and I. First of all, there must be an object o_1 which belongs to y and does not belong to z. Moreover, there must be another object o_2 which belongs to y and for which it is unknown whether it belongs to z. If we omit the abnormalities and the auxiliary predicate z' the following alternative definition is obtained.

$\mathcal{P} \models' Oyz$ iff there exists an object o_1 such that $\mathcal{P} \models_{wcs} y(o_1) \wedge \neg z(o_1)$ and there exists an object o_2 such that $\mathcal{P} \models_{wcs} y(o_2)$ and $\mathcal{P} \not\models_{wcs} \neg z(o_2)$.

5.5 No Valid Conclusion (NVC)

When we can not conclude any of the previous moods, then we derive from $\mathcal{P}$ that no valid conclusion holds.

6 Predictions by the Weak Completion Semantics

Combining the syllogisms representation and entailment rules explained before we accomplished an average of 85% accuracy in our predictions. In 9 cases we have a perfect match with the answers given by the participants. In 29 cases the match is 89% and in 21 cases the match is 78%. The five cases left, have a match of 67%.

First, we will explain how the accuracy of the predictions is computed in general. After that we will show the logic program representation of three syllogisms with different predictions accuracy, and compare their results with the data from human experiments taken from [15]. In those experiments people were asked to infer conclusions about the predicates a and c from a syllogism built according to the figures in Table 2. Therefore, in our predictions we only consider entailment of syllogisms between those two predicates, a and c.

6.1 Accuracy of Predictions

We have nine different answer possibilities for each of the 64 syllogisms:

Aac, Eac, Iac, Oac, Aca, Eca, Ica, Oca and NVC.

For every syllogism, we define a list of length 9 for the predictions of the weak completion semantics, where the first element represents Aac, the second element represents Eac, and so forth. When Aac is predicted under the weak completion semantics for a given syllogism, then the value of the first element of this list is a 1, otherwise it is a 0, and the same holds for the other eight elements in the list (representing the other eight answer possibilities). Analogously, for every syllogism we define a list of the participants' conclusions of length 9 containing either 1 or 0 for all nine answer possibilities, depending on whether the majority concluded Aac, Eac, and so forth. For each syllogism we then simply compare each element of both lists as follows, where i is the ith element of both lists:

$$\textsc{comp}(i) = \begin{cases} 1 & \text{if both lists have the same value for the } i\text{th element} \\ 0 & \text{otherwise} \end{cases}$$

The matching percentage of this syllogism is then computed by $\sum_{i=1}^{9} \textsc{comp}(i)/9$. Note that the percentage of the match does not only take in account when the weak completion semantics correctly predicts a conclusion, but also whenever it correctly rejected a conclusion. The average percentage of accuracy is then simply the average of the matching percentage of all 64 syllogisms.

6.2 OA4 - Perfect Match (1.00)

The syllogism OA4 is obtained by combining the last and the first mood in Table 1 according to the figure 4 in Table 2. It can be read as:

First Premise	Oba	'*Some b are not a*'
Second Premise	Abc	'*All b are c*'

The program $\mathcal{P}_{\mathsf{OA4}}$ representing the two premises is obtained as the union of the programs $\mathcal{P}_{Oba}$ (obtained from $\mathcal{P}_{Oyz}$ by replacing y and z by b and a, respectively) and $\mathcal{P}_{Abc}$ (obtained from $\mathcal{P}_{Ayz}$ by replacing y and z by b and c, respectively). In addition, the constant o occurring in $\mathcal{P}_{Abc}$ has been replaced by o_3. $\mathcal{P}_{\mathsf{OA4}}$ consists of the following clauses:

$$\begin{array}{l}
a'(X) \leftarrow b(X) \wedge \neg ab_{bna}(X) \\
ab_{bna}(o_1) \leftarrow \bot \\
a(X) \leftarrow \neg a'(X) \wedge \neg ab_{naa}(X) \\
b(o_1) \leftarrow \top \\
b(o_2) \leftarrow \top \\
ab_{naa}(o_1) \leftarrow \bot \\
ab_{naa}(o_2) \leftarrow \bot \\
c(X) \leftarrow b(X) \wedge \neg ab_{bc}(X) \\
ab_{bc}(X) \leftarrow \bot \\
b(o_3) \leftarrow \top
\end{array}$$

The least Ł-model of $\mathcal{P}_{\mathsf{OA4}}$ is

$$\begin{aligned}&\langle\ \{b(o_1),\ b(o_2),\ b(o_3),\ ab_{ca}(o_1),\ a'(o_1),\ c(o_1),\ c(o_2),\ c(o_3)\},\\ &\quad \{ab_{bna}(o_1),\ ab_{naa}(o_1),\ ab_{bc}(o_1),\ ab_{bc}(o_2),\ ab_{bc}(o_3),\ a(o_1)\}\ \rangle.\end{aligned}$$

This model entails only the conclusion '*some c are not a*'. We have an object that is in c and not in a ($\mathcal{P}_{\mathsf{OA4}} \models_{wcs} c(o_1)$ and $\mathcal{P}_{\mathsf{OA4}} \models_{wcs} \neg a(o_1)$) and an object that is in c and it is not known if it is not in a ($\mathcal{P}_{\mathsf{OA4}} \models_{wcs} c(o_2)$ and $\mathcal{P}_{\mathsf{OA4}} \not\models_{wcs} \neg a(o_2)$). This prediction matches perfectly with the answers from the participants.

6.3 EA3 - Worst Match (0.67)

We will discuss now the syllogism EA3, one of the syllogisms with the lowest match (67%). EA3 can be read as:

First Premise	Eab	'*No a are b*'
Second Premise	Acb	'*All c are b*'

The program $\mathcal{P}_{\mathsf{EA3}}$ representing the two premises is obtained as the union of the programs $\mathcal{P}_{\mathsf{E}ab}$ (obtained from $\mathcal{P}_{Eyz}$ by replacing y and z by a and b, respectively) and $\mathcal{P}_{\mathsf{A}cb}$ (obtained from $\mathcal{P}_{Eyz}$ by replacing y and z by c and b, respectively). In addition the constant o occurring in $\mathcal{P}_{\mathsf{E}ab}$ and $\mathcal{P}_{\mathsf{A}cb}$ have been replaced by o_1 and o_2, respectively. $\mathcal{P}_{\mathsf{EA3}}$ consists of the following clauses:

$$\begin{aligned}&b'(X) \leftarrow a(X) \wedge \neg ab_{anb}(X)\\ &ab_{anb}(X) \leftarrow \bot\\ &b(X) \leftarrow \neg b'(X) \wedge \neg ab_{nbb}(X)\\ &ab_{nbb}(o_1) \leftarrow \bot\\ &a(o_1) \leftarrow \top\\ &b(X) \leftarrow c(X) \wedge \neg ab_{cb}(X)\\ &ab_{cb}(X) \leftarrow \bot\\ &c(o_2) \leftarrow \top\end{aligned}$$

The least Ł-model of $\mathcal{P}_{\mathsf{EA3}}$ is

$$\begin{aligned}&\langle\ \{a(o_1),\ c(o_2),\ b'(o_1),\ b(o_2)\},\\ &\quad \{ab_{anb}(o_1), ab_{nbb}(o_1),\ ab_{anb}(o_2), ab_{nbb}(o_2),\ ab_{cb}(o_1),\ ab_{cb}(o_2)\}\ \rangle.\end{aligned}$$

This model does not entail any conclusion between a and c, so our prediction is NVC. Participants concluded both Eac and Eca.
Even though none of the entailed conclusions predicted by the weak completion semantics match to the participants' answers, a match of 67 % is computed. The reason is that we also take into account the correct rejections, i.e. the conclusions that are not entailed, as we have explained in Section 6.1

6.4 AA4 - Partial Match (0.78)

For AA4 we got an accuracy of 78% in our prediction. This syllogism combines two premises in the first mood according to the figure 4. It can be read as:

First Premise	Aba	'*All b are a*'
Second Premise	Abc	'*All b are c*'

The program $\mathcal{P}_{\mathsf{AA4}}$ representing the two premises is obtained as the union of programs $\mathcal{P}_{\mathsf{A}ba}$ (obtained from $\mathcal{P}_{Ayz}$ by replacing y and z by b and a, respectively) and $\mathcal{P}_{\mathsf{A}bc}$ (obtained from $\mathcal{P}_{Ayz}$ by replacing y and z by b and c, respectively). In addition the constant o occurring in $\mathcal{P}_{\mathsf{A}ba}$ and $\mathcal{P}_{\mathsf{Abc}}$ have been replaced by o_1 and o_2, respectively. $\mathcal{P}_{\mathsf{AA4}}$ consists of the following clauses:

$$\begin{array}{l} a(X) \leftarrow b(X) \wedge \neg ab_{ba}(X) \\ b(o_1) \leftarrow \top \\ ab_{ba}(X) \leftarrow \bot \\ c(X) \leftarrow b(X) \wedge \neg ab_{bc}(X) \\ ab_{bc}(X) \leftarrow \bot \\ b(o_2) \leftarrow \top \end{array}$$

The least Ł-model of $\mathcal{P}_{\mathsf{AA4}}$ is

$$\begin{array}{l} \langle\ \{b(o_1),\ b(o_2),\ a(o_1),\ a(o_2), c(o_1),\ c(o_2)\}, \\ \quad \{ab_{ba}(o_1),\ ab_{ba}(o_2),\ ab_{bc}(o_1),\ ab_{bc}(o_2)\}\ \rangle. \end{array}$$

This model entails both '*all a are c*' and '*all c are a*'. We have an object that is in a, and for all objects that are in a ($\mathcal{P} \models_{wcs} a(o_1)$ and $\mathcal{P} \models_{wcs} a(o_2)$) it holds that they are also in c ($\mathcal{P} \models_{wcs} c(o_1)$ and $\mathcal{P} \models_{wcs} c(o_2)$). Analogously this also holds for '*all c are a*'. This prediction matches partially with the answers from participants who concluded Aac and NVC.

7 Results

We discussed three examples, which we formalized under WCS. The results are summarized and compared to FOL, PSYCOP, the Verbal, and the Mental Model Theory in Table 5. The selected examples are typical in the sense that for some syllogisms the conclusions drawn by the participants and WCS are identical, for some syllogisms the conclusions drawn by the participants and WCS overlap, and for some syllogisms the conclusions drawn by the participants and WCS are disjoint. Moreover, WCS differs from the other cognitive theories.

The overall result with respect to the 64 syllogisms under WCS shows that we can predict 85% of the participants responses. Compared to the other cognitive theories, we achieve the best performance, which closely followed by the Verbal Models Theory (84%) and the Mental Model Theory (83%). It seems natural to compare these theories in more detail and see where their similarities are and where they differ.

	Participants	FOL	PSYCOP	Verbal Models	Mental Models	WCS
OA4	Oca	Oca	Oca, Ica, Iac	Ocs, NVC	Oca, Oac, NVC	Oca
EA3	Eac, Eca	Eac, Eca Oac, Oca	Eac, Eca Oac, Oca	NVC, Eca	Eac, Eca	NVC
AA4	Aac, NVC	Iac, Ica	Iac, Ica	NVC, Aca	Aca, Aac, Iac, Ica	Aac, Aca

Table 5. The conclusions drawn by a significant percentage of participants are highlighted in gray and compared to the predictions of the theories FOL, PSYCOP, Verbal, and Mental Models as well as WCS for the syllogisms OA4, EA3, and AA4.

Another aspect that might be interesting to investigate is whether the combinations of the moods influence how the participants perceive the syllogisms. For instance, it seems that participants give different answers when they consider syllogistic premises of the same mood, especially in the cases for AA and EE, than when they consider syllogistic premises of two different moods. However, this observation needs to be further examined, and can possibly give us more insight about the human reasoning process.

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