Analysis of the mathematical Errors of first cycle students of the National University of Piura

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Abstract. University higher education is not exempt from the manifestation of mathematical errors in learning. The study is non-experimental, descriptive and intended to analyze the mathematical mistakes made by 80 university students of the first cycle of the National University of Piura in the 2014-II and 2015-I semester. The results show a high percentage of errors in cognitive processes in terms of operations: with complex numbers, linear and quadratic equations, with polynomial expressions; translations of the graphic representation of the real line and representation of everyday language in formal language; revealing that most commit themselves due to absences and inaccuracies in the construction of prior knowledge, for not understanding the semantics of mathematical concepts. It is noted that 50% of the students obtained a grade less than or equal to 7.78.

Keywords: Education · Mathematical error · Cognitive process

1 Introduction

Rousseau, David and Werner cited by [6], point out four ways in which the error may arise: the result of great inadequate conceptions of fundamental aspects of mathematics; result of the correct and credible application of an identifiable systematized imperfect procedure; for using imperfect procedures and having inappropriate conceptions; Finally, by using original non-formal methods invented by the teacher to carry out the tasks and solve problems.

However, as stated by [6], most researchers and specialists agree to consider as general characteristics of the mistakes made by students: errors arise spontaneously in class or much earlier, which surprises the teacher; they are also persistent, particular and difficult to overcome because they require a reorganization of the knowledge of the individual; Students are not aware of the error, because they do not question what seems obvious and do not consider the meaning of the concepts, rules or symbols with which they work. Some errors are even generated

in the student's understanding or processing of the information provided by the teacher.

According to [11], identify the mistakes made by students in a partial exam, then analyze from the perspective of the different records of semiotic representation and also notice a strong regularity in their production. They largely consider that errors come from insufficient work with different records of representations of a mathematical object.

Also, [1] presented a broader approach to the possibilities of using error analysis in learning-teaching processes, considers that errors are analyzed with two fundamental objectives: eliminate them or explore their potential.

Although the error may have different origins, researchers have established several categories for the mathematical errors that students make, such as those proposed by [2] where he observed binary reversals, errors induced by language or notation, errors when recovering a scheme above, errors produced by improper representation and rules that produce erroneous criteria. While [9] offers the following taxonomy of errors due to: language difficulties; difficulties in obtaining spatial information; poor learning of facts, skills and previous concepts; incorrect associations or rigidity of thought; out of perseverance; of association; of interference; assimilation; of negative transfer of previous tasks and due to the application of irrelevant rules or strategies.

On the other hand, [10] makes an empirical classification of errors based on a constructive analysis of student solutions by experts. It proposes six categories: misused data; incorrect interpretation of language; logically invalid inferences; deformed theorems or definitions; lack of verification in the solution and technical errors.

Generally it tends to be considered as the presence of an inadequate cognitive scheme in students and not only as a result of a specific lack of knowledge. Errors do not appear by chance, but arise in a coherent conceptual framework, based on previously acquired knowledge and any instructional process is potentially error-generating, due to different causes, some of which inevitably occur. It should also be taken into account that students' opportunities to learn Mathematics depend on the environment, the type of tasks and the discourse in which they participate, depending on what they learn from how they participate in mathematical activities, without neglecting the attitudes they have towards this science

When we talk about mathematical learning, we talk about acquiring new knowledge, often based on previous knowledge. Mathematics has a compulsory curricular space in Regular Basic Education and is present at least in the first year of studies in university higher education, according to the curriculum of the faculties of the National University of Piura.

In the case of the National University of Piura, it is evident that the mistakes made by the students influence the learning of the contents programmed in the respective general training courses in Mathematics, observing significant percentages of withdrawn, disapproved or with a regular final average, that is, eleven and twelve in the twentieth evaluation system. Therefore, mathematical

errors are a constant concern for teachers of the Department of Mathematics of the National University of Piura, given the results obtained at the end of the cycle in the promotional records.

In this reflection on the learning of the students of the first one it has been observed: low scores in the objective test of access to the National University of Piura in recent years; not all vacancies offered are filled; as well as a marked heterogeneity among the students of different educational centers in our region. On the other hand, high failure rates and withdrawals in courses in the area of Mathematics that require algebraic processes and an attitude of rejection towards Mathematics courses for not finding a relationship with their professional development. Also, in the programmed academic activities repetitive mathematical errors, a sign of the serious deficiencies they have in their learning in the regular basic education stage, although some consider them lack of study or attention by the various distractors.

These complications in student learning seem to be related to a series of weaknesses in the understanding of concepts and in the way of focusing on Algebra and Arithmetic, which immediately result in an erroneous way of treating their learning. In most cases, students memorize without understanding the rules and calculation procedures and apply them automatically, which leads them to make the same mistakes persistently.

If we, as teachers, want to achieve high levels of meaningful learning with our students, we must be very careful to help them recognize and correct their mistakes in their learning [7].

Due to the above, the following research question is asked: What types of mathematical mistakes do students make in the mathematics area courses of the first year of study?

In this work they have proposed as a general objective: to analyze the mathematical mistakes made by the students who enter the professional schools of Zootechnics (2014- II), Statistics (2015-I) and Accounting and Financial Sciences (2015-I-PROEDUNP Paita) in learning the mathematics of the first cycle of studies at the National University of Piura. This analysis leads us to identify, understand their cognitive processes and interpret their mistakes, to categorize them: due to the non-identification of the semantics of the mathematical fact; due to incorrect deductions or associations; due to the recovery of previous knowledge; due to occasional erroneous or accidental calculations due to inaccuracies in the construction of prior knowledge and due to lack of prior knowledge. To develop the work, the survey has been used as a method of data collection, creating a specific performance questionnaire. The data collected were analyzed descriptively according to the response given for classification according to the typology suggested by Marcel Pochulu. Subsequently, the results obtained will be contrasted with other research papers, to establish conclusions and suggestions for future research work.

2 Methodology

2.1 Desing

The research work is substantive, because it intends to analyze the mistakes that students make in Mathematics courses during the year studies at the National University of Piura. The research design that corresponds to the study carried out is non-experimental, descriptive and is located in the line of error analysis. Likewise, the study developed was of a transversal nature, since the application of the instrument that allowed the collection of the corresponding information was carried out in a single moment.

2.2 Sample

The total sample selected was 80 non-probabilistically selected students, composed of the students who attended the courses on the first day of school: Mathematics I (group 12, section 2-semester II-2014, twenty-two students of the twenty-two enrolled) opened by the professional school of Zootechnics and Veterinary Medicine of the Faculty of Zootechnics; Mathematics I (PROEDUNP-Paita, semester I-2015, thirty-one students of the forty-four enrolled) opened by the professional school of the Faculty of Accounting and Financial Sciences; as well as, Mathematical analysis I (group 02, section 3, semester I-2015, twenty-seven thirty-year-old students).

3 Instruments

In the first phase, regular basic education teachers were interviewed, who were in the Mathematics specialization program aimed at secondary level teachers (Convention No. 105-2012-2014-MINEDU-UNP), to provide information on mistakes they observe in their students in regular basic education. In a second phase, university professors were asked to teach first-year courses of the Department of Mathematics through an informal interview that will indicate that mathematical errors indicated by regular basic education teachers had observed in university students in the first year of studies and wondered if they did something to correct the mistakes presented by the students.

In view of the above, the instrument was structured as follows: Item I: arithmetic operations with complex numbers, with a total of twenty-one sub items; Item II: First degree equations with one variable, with a total of three sub items; Point III: operations with polynomials, with a total of 9 subpoints; Item IV: change of semiotic record, with a sub-item; Item V: second degree equations with a variable, with a sub-item; Item VI: equation, with a sub-item.

In each of the items, students were asked to solve each sub-item by developing the complete operating procedure to respond. The objective in the performance questionnaire was to identify the mistakes that students make and analyze their cognitive processes and be able to perform the respective categorization of errors.

The respect for the reliability of the items of the instrument was applied using two techniques to present dichotomous responses: Kuder and Richardson 20 obtaining 0.876 and Cronbach's alpha with 0.869; being highly reliable and reliable respectively.

The validation of the instrument was confirmed, in reference to the congruence of the items, the breadth of the content, the wording of the items, the clarity, precision and relevance, the evaluation of excellent, given by the evaluating specialist.

4 Results

The data collected in the performance test were according to the typological description of the errors proposed by Marcel Pochulu:

Table 1. Description of the type of errors, in the items that require treatment of arithmetic operations.

Items	Due to the non-identification of the semantics of the mathematical fact	Due to incorrect deductions or associations	Due to the recovery of previous knowledge	Due to erroneous or accidental calculations	Due to inaccuracies in the construction of prior knowledge	Absence of prior knowledge	More than one type of error
I-1	01.25%	00.00%	02.50%	05.00%	00.00%	00.00%	00.00%
I-2	00.00%	00.00%	16.25%	00.00%	01.25%	00.00%	01.25%
I-3	16.25%	17.50%	00.00%	00.00%	03.75%	00.00%	01.25%
I-4	13.75%	15.00%	00.00%	00.00%	07.50%	02.50%	00.00%
I-5	01.25%	00.00%	57.50%	02.50%	00.00%	03.75%	00.00%
I-6	00.00%	00.00%	08.75%	10.00%	01.25%	36.25%	00.00%
I-7	00.00%	00.00%	00.00%	00.00%	32.5%	16.25%	00.00%
I-8	00.00%	00.00%	00.00%	00.00%	06.25%	00.00%	00.00%
I-9	00.00%	00.00%	00.00%	00.00%	38.75%	02.50%	00.00%
I-10	00.00%	00.00%	00.00%	00.00%	10.00%	02.50%	00.00%
I-11	06.25%	00.00%	00.00%	00.00%	22.50%	43.75%	05.00%
I-12	63.75%	00.00%	00.00%	00.00%	02.50%	16.25%	00.00%
I-13	00.00%	31.25%	00.00%	00.00%	47.50%	05.00%	00.00%
I-14	13.75%	00.00%	00.00%	00.00%	16.25%	46.25%	00.00%
I-15	17.50%	18.75%	00.00%	00.00%	00.00%	42.50%	00.00%
I-16	00.00%	30.75%	00.00%	10.00%	12.50%	02.50%	00.00%
I-17	00.00%	10.00%	07.50%	00.00%	16.25%	20.00%	26.25%
I-18	00.00%	00.00%	12.50%	05.00%	10.00%	26.50%	01.25%
I-19	22.50%	00.00%	00.00%	01.25%	55.00%	10.00%	00.00%
I-20	00.00%	00.00%	00.00%	00.00%	30.00%	42.50%	00.00%
I-21	00.00%	00.00%	00.00%	00.00%	33.75%	52.50%	00.00%
Prom	07.44%	04.58%	05.00%	01.61%	16.55%	17.69%	01.67%

The results show us that in the items in which arithmetic treatments are requested from students, the typological errors that occur most frequently are: due to inaccuracies in the construction of prior knowledge with 16.55% and due to lack of prior knowledge with a 17.69%.

Table 2. Description of the type of errors, in the items that require treatment of algebraic (II-III) operations.

Items	Due to the non-identification of the semantics of the mathematical fact	Due to incorrect deductions or associations	Due to the recovery of previous knowledge	Due to erroneous or accidental calculations	Due to inaccuracies in the construction of prior knowledge	Absence of prior knowledge	More than one type of error
II-1	05.00%	01.25%	00.00%	06.25%	00.00%	40.00%	11.25%
II-2	03.75%	02.50%	00.00%	18.75%	00.00%	27.50%	01.25%
II-3	00.00%	01.25%	10.00%	03.75%	07.50%	36.25%	02.50%
III-1	45.00%	00.00%	00.00%	00.00%	00.00%	20.00%	15.00%
III-2	31.25%	00.00%	00.00%	00.00%	05.00%	12.50%	00.00%
III-3	00.00%	01.25%	00.00%	00.00%	00.00%	05.00%	25.00%
III-4	08.75%	32.50%	00.00%	00.00%	13.75%	16.25%	00.00%
III-5	03.75%	17.50%	00.00%	00.00%	07.50%	01.25%	00.00%
III-6	00.00%	26.25%	00.00%	00.00%	00.00%	11.25%	01.25%
III-7	12.50%	00.00%	00.00%	00.00%	00.00%	08.75%	00.00%
III-8	35.00%	00.00%	00.00%	00.00%	00.00%	10.00%	00.00%
III-9	23.75%	00.00%	00.00%	00.00%	20.00%	33.75%	00.00%
Prom	14.06%	06.88%	00.83%	02.40%	04.48%	18.54%	04.69%

The results show us that in the items in which students are requested algebraic treatments, the typological errors that occur most frequently are: due to the non-identification of the semantics of the mathematical fact with 14.06% and due to lack of prior knowledge with 18.54%.

Table 3. Description of the type of errors, in the items that require change of semiotic records.

Items	Due to the non-identification of the semantics of the mathematical fact	Due to incorrect deductions or associations	Due to the recovery of previous knowledge	Due to erroneous or accidental calculations	Due to inaccuracies in the construction of prior knowledge	Absence of prior knowledge	More than one type of error
IV	00.00%	00.00%	00.00%	00.00%	00.00%	85.00%	00.00%
V	10.00%	00.00%	0.00%	01.25%	11.25%	48.75%	01.25%
VI	00.00%	00.00%	0.00%	00.00%	40.00%	58.75%	00.00%
Prom	03.33%	00.00%	0.00%	00.42%	17.08%	64.17%	0.42%

We can observe that more than 50% of students make mathematical mistakes due to lack of knowledge when going from one semiotic record to another.

Table 4. Description of the typology of average errors of the research instrument items.

Items	Due to the non-identification of the semantics of the mathematical fact	Due to incorrect deductions or associations	Due to the recovery of previous knowledge	Due to erroneous or accidental calculations	Due to inaccuracies in the construction of prior knowledge	Absence of prior knowledge	More than one type of error
Prom	09.31%	04.97%	03.19%	01.77%	12.57%	21.85%	02.57%

We can observe in table 1 that the type of error that has the highest incidence is the absence of prior knowledge with a percentage arithmetic average of 21.85% and possible errors due to inaccuracies in the construction of prior knowledge with a percentage average of 12.57% and the one with the lowest incidence percentage is due to erroneous or accidental calculations with an average percentage of 1.77%.

5 Discussion

Next we discuss the results obtained against other research works of the same nature:

Regarding the research work of [3] titled: Errors and difficulties of Mexican students of first university course in the resolution of algebraic tasks executed

at the University of Granada, similarities have been found regarding the technological tests of scientific cutting careers that we can demonstrate that schools Zootechnical professionals with 7.5259 and 10.4944, the professional school of Statistics obtained an average of disapproval, when these schools were expected to perform better. In both research papers, it is concluded that students present errors in their Mathematics training in the previous levels of studies.

On the research work of [12], the mathematical analysis of an algebraic error in students and professors, executed at the Universidad Panamericana Campus Guadalajara-Mexico, can be observed that there are similar results because the students presented several erroneous cognitive processes that require different strategies to be overcome. The university professors of the National University of Piura do not pay interest to the mistakes made by the students and do not give them help to overcome these errors, which can be reflected in a significant percentage of disapproved and withdrawn in the promotional records.

On the research work of [8], Errors and difficulties in Mathematics: Analysis of causes and work suggestions. National University of Villa María. Buenos Aires - Argentina, the same conclusions have been reached that the errors are persistent from the previous levels of training and that from the total number of students who participated in the study, they made some type of error in the resolution of the elements of the instrument . research.

It should be noted that regarding the research work of [11] entitled: Recurring errors in the learning of linear algebra and semiotic representation records executed at the University of Buenos Aires-Argentina. We were able to verify through the instrument and its answers provided by the students the statement of the researcher, who warns that students have errors that come from insufficient work with different records of representations: verbal, numerical, algebraic, graphic, among others. As for example we can show it in item IV with 85% of the total indica sample that I did not know when I could not interpret the graph, take it to algebraic and numerical, even in item VI where students could not pass the Written verbal representation to the algebraic representation with 58.75% and 40.00% only partially resolved.

6 Conclusions

A group of teachers from the area of Mathematics of Regular Basic Education of the Department of Piura selected by the Ministry of Education were asked to receive a second specialization. What types of mathematical errors had the students of the last level made frequent¿ With this information, the instrument was designed to identify the common mathematical errors committed by students when performing arithmetic operations in the field of complex numbers, first degree and second degree equations, as well as situations that required a change in semiotic registration. Subsequently, the question was asked of the university professors of the Department of Mathematics of the National University of Piura. Do incoming students make mathematical mistakes in their development of activities programmed in the first year Mathematics courses?, they said

yes, but that do nothing to correct these errors. The evaluation instrument was applied to identify the mathematical errors to the selected university students, and their answers revealing that the students manifest some type of error in their cognitive processes in 78.15% or absence of previous knowledge in 21.85% on average. It was even verified that students show errors or lack of knowledge when going from one semiotic record to another.

The items of the assessment instrument are previous knowledge required for the construction of new mathematical knowledge in the courses programmed in the first year of university studies and if we consider the evaluation obtained from the students in the vigesimal scale of their procedures they obtained an average grade of 8.75, which would imply that the mistakes made by the students are the persistence of their training in the EBR.

There are also correspondences with the conceptions stipulated by Brousseau, David and Werner that were cited by [6]. They point to four types of errors, through which the error appeared in the students' cognitive processes due to an imperfect systematized procedure and inappropriate conceptions, which cannot easily identify the teacher. While Rico (1995) argues that systematic errors are the result of inadequate conceptions of Mathematics, recognizable or not recognizable by the teacher. It was also possible to confirm that there are random or occasional errors, as [6] points out; The errors due to incorrect or accidental calculations were made in a smaller percentage since the development of the items was brief. Based on the foregoing, it is clear that the students show persistence of errors of their regular basic education, which were not overcome in their school education by not taking them due importance, they even manifest and persist in their university academic training evidenced by the results At the end of the semester, the significant percentage of failure and / or withdrawal in the Mathematics courses of the first year of studies provided by the Mathematical Academic Department is worrying, even that in approved students there are errors that continue to persist, but in a smaller percentage.

Students must receive a quality education, where the university professor can establish methodological strategies that allow them to overcome the mathematical errors that they comment on or remedy the absence of previous knowledge. Also to provide students with a range of possibilities to overcome their own deficiencies, thus achieving satisfactory learning in the area of Mathematics, also finding the motivation and articulation of this science with their professional training.

This research opened the possibility of comparative studies on the mistakes made by students entering professional careers in Science and Humanities of the National University of Piura; also, the design of didactic strategies for overcoming said Mathematical errors. The implementation of the zero cycle or the accompaniment to the student is recommended according to their academic needs after study.

References

1. Borasi, R.: Reconceiving Mathematics Instruction. A Focus on Errors. Ablex Publishing Corporation Norwood, New Jersey (1996)
2. Davis, R.: Learning Mathematics. The cognitive Science Approach to Mathematics Education. Croom Helm, Australia (1984)
3. García, J. et al: Errores y dificultades de estudiantes mexicanos de primer curso universitario en la resolución de tareas algebraicas ejecutada en la Universidad de Granada. Spain (2011)
4. Guzmán, M.: Enseñanza de las Ciencias y las Matemáticas. Journal **43**, 18–58 (2007)
5. Godino, J., Batero, C., Font, V.: Fundamentos de la enseñanza y aprendizaje de la Matemática para maestros. Granada University, Spain (2003)
6. Rico, L.: Errores y dificultades en el aprendizaje de las Matemáticas. Grupo Editorial Iberoamérica, Mexico (1995)
7. Novak, J.: Meaningful learning. The Essential Factor for Conceptual change in Limited or Inappopriate Propositional Hierarchies (LIPHs) Leading to Empowerment of Learners. Science Education **86**(4), (2002)
8. Pochulu, M.: Análisis y categorización de errores en el aprendizaje de la matemática en alumnos que ingresan a la Universidad Nacional de Villa María. Argentina (2005)
9. Radatz, H.: Students' errors in the mathematical learning process. For the Learning of Mathematics **1**(1), (1980)
10. Rico, L.: Investigaciones sobre errores de aprendizaje en educación Matemática. Granada University, Spain (1992)
11. Scardigli, M. et al: Errores recurrentes en el aprendizaje del algebra lineal y registros de representación semiótica. Argentina (2005)
12. Wellmann, E. et al: Análisis matemático de un error algebraico en estudiantes y profesores, ejecutado en Universidad Panamericana Campus Guadalajara. Mexico (2010)