Statistical Sense in the Information Society

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Abstract. We are currently attending a paradoxical situation in the teaching of statistics. Being a topic that appears throughout the curriculum in practically all courses from Primary Education to University and even in postgraduate education, didactic research describes numerous errors in its interpretation. There are also numerous criticisms of the way in which statistics is used or interpreted in the media, politics and research. In this paper we analyse this situation and suggest that current teaching does not develop the statistical sense of students. We describe the components of statistical sense and offer some suggestions to improve of this sense in students and professionals.

Keywords: Statistical literacy, Statistical reasoning, Teaching statistics.

1 Introduction

Statistics is a fundamental tool in scientific research, especially in the human sciences that deal with non-deterministic phenomena, which are modelled by probability distributions. The abundance of statistical information in the media and on the Internet, promoted by technology and by the increasing use of statistics in the workplace and in all the disciplines has led to what we know as the information society, where the citizen is frequently involved in making decisions in an uncertain environment about various issues. As discussed by Engel (2019), misinformation and ignorance in statistics threats our way of life in a complex world and are a risk for democracy.

Politicians and administrators inform citizens about the evolution of different economic and social indicators, and consequently they need to confront arguments based on statistical information almost every day. Batanero and Borovcnik (2016) offer examples of such indicators related to variables that influence the Life expectancy in different countries, while Engel (2019) analyses another situation dealing with income inequality. Both topics frequently appear in the media, together with different recommendations to act or judge the evolution of the variables involved. Other similar reports concern migration, pollution, crime, human rights, education, gender inequality, production or public expenditures (Gal, 2019). Science also uses its statistical models to make predictions about the future of our planet or to warn of the need to change some of our habits to safeguard this future or to improve our health. To critically evaluate these arguments, we need to understand the way in which the data are produced and analysed, and how the conclusions in these reports are obtained.

Are citizens prepared to understand these messages and the decisions of their leaders or to collaborate themselves with changes in their behaviour? In a previous paper (Batanero, 2013), I pointed to the paradox that, in spite of the wide teaching of statistics at all curricular levels, statistics is probably the worst understood and applied part of mathematics. An example is the long-standing controversy around the use of statistical tests in empirical research in Psychology, Education and other sciences (see, for example, Batanero, 2000; Harlow, Mulaik & Steiger,2016; Kline, 2013; Morrison & Henkel, 2006; Schneider, 2018; Wilkinson, 1999).

In fact, errors and difficulties do not only appear in advanced statistical topics. There is a wide research describing these misunderstanding in elementary themes, such as for example, statistical graphs (Arteaga, Batanero, Contreras, & Cañadas, 2012), averages (Cruz & Garrett, 2006) or correlation and regression (Estepa & Sánchez-Cobo, 2001).

Moreover, since statistical data are inherently messy, and interpretations of statistical analyses require understanding, which dependent on assumptions and context, people often rely in their own false intuitions or preconceptions about these data, instead of using sound statistical knowledge or reasoning (McGilliwray & Pereira-Mendoza, 2011).

2 Current problems in the teaching of statistics

The above problems are not explained by lack of statistical training. Statistics is studied along all the school life, from primary to high school, and afterwards at university and un postgraduate courses. In fact, the theme has been present in secondary and high school for decades even when today we attend any suggestions to renew its teaching (e.g., CCSSI, 2010; MECD, 2014; 2015).

The teaching of statistics to all citizens has been justified because statistics thinking is a component of critical reasoning as well as for the same of the instrumental role of statistics in different disciplines (Franklin et al., 2007; Gal, 2002). Another reason is that improving statistical reasoning prepares students for life, since we are surrounding by randomness since we are born.

However, the last reason is often forgotten and statistics is taught in a too formal way using non-realistic examples and a consequence is that students develop distaste for the topic. Due to the need to complete a too wide program in a short time, many teachers present only definition of concepts and procedures for solving statistical problems. Consequently, students not acquire a deep understanding of the underlying concepts.

The situation is also explained by the fact that the frontier between advanced and elementary statistics is fuzzy, since the school curricula in secondary and high school include advanced ideas, such as sampling, random variable or confidence intervals in many countries. (Artigue, Batanero, & Kent, 2007). Moreover, the work with some statistical methods require and advanced level of algebraic reasoning; more specifically levels 4 ad 5 in the six levels hierarchy proposed by Godino, Neto, Wilhelmi, Aké, Etchegaray and Lasa (2015) are characterised by parameters and operation with parameters. But parameters appear in statistics not only in the study of inference, but also to specify the density function for random variables with different theoretical distributions, such as normal or binomial distributions. To make the situation more complex, both in high school and university these topics are studied by social sciences students who generally do not have a strong algebraic background.

Another source of difficulty is the mixture of theoretical ideas and personal conceptions when people apply stochastic reasoning. Independence, for example, can be mathematically reduced to the multiplicative rule. This definition, however, neither includes all the causality perceptions that students often relate to independence nor does it always serve to help decide if independence applies in a particular experiment.

There are many calls to change the situation and try to decrease the level of formalization, in order to help student making sense of statistics (e.g., Shaughnessy, Chance, & Kranendork, 2009). In Batanero and Borovcnik (2016) we recommend to use contexts familiar to the students, build on students' previous knowledge and introduce the concepts making students understand how statistical concepts help solve the given task. We also present examples of meaningful contexts and problems that can be used to make sense of the different concepts and methods to the students. Other suggestions include working with real data, adequate use of technology and using assessment to improve the students' learning. All of these suggestions are directed to increase the students' statistical sense, which is analysed below.

3 What is statistical sense?

Suggestions for making sense of mathematics appeared early in relation to ideas of numeracy, that described ability to cope with every day mathematical demands and as a way of understanding, which was opposed to routine learning. Soon the idea of number sense appeared, as a main outcome of school mathematics.

> Number sense refers to a person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful strategies for handling numbers and operations It reflects an inclination and an ability to use numbers and quantitative methods as a means of communicating, processing and interpreting information (McIntosh, Reys & Reys, 1992).

We found no similar ideas about statistical sense, although in Shaughnessy, Chance, & Kranendork (2009) we found recommendations about helping students making sense of statistics. When comparing with the above definition of number sense, in our point of view statistical sense should refer to general understanding of statistical concepts and ability to use this knowledge in a flexible way in solving problems, that is in statistical reasoning and thinking. It also includes the ability to use statistics to communicate, process and interpret statistical information. This statistical sense would be needed for everyday decision-making situations (e.g., taking an insurance policy, voting, evaluating risks of accident, interpreting coincidences, etc.).

Ideas related to statistical sense can be found in the ideas of statistical literacy, reasoning and thinking, which are viewed by some authors (e.g. Garfield & Ben-Zvi, 2008) as three different levels of statistical knowledge. We, instead, prefer to include these three ideas in what we call *statistical sense,* and distinguish, on the one hand, statistical literacy as a set of knowledge and dispositions, and statistical thinking and reasoning as the ability and processes needed to solve statistical problems. Below we describe these components of statistical sense.

1.1 Statistical literacy

The need of statistical knowledge for every citizen in a democratic society based on information has been widely recognized (Gal, 2002; Franklin et al, 2007) and led to the introduction of the term *statistical literacy*, which was defined by Wallman in the following terms:

> The ability to understand and critically evaluate statistical results that permeate daily life, coupled with the ability to appreciate the contributions that statistical thinking can make in public and private, professional and personal decisions (Wallman, 1993, p.1).

While literacy means the ability to find, read, interpret, analyse and evaluate written materials (and to detect possible errors or biases in this information), to be statistically literate, people need a basic understanding of statistics. This includes knowing what statistical terms and symbols mean; being able to read statistical graphs and data; understanding the basic logic of statistics; understanding and critically evaluating statistical results that appear in daily life and a positive attitude towards statistics (Gal, 2002).

Statistical literacy describes the set of statistical competences needed to manage in our society and include general reading and writing literacy, mathematical and statistical knowledge, and attitude (Watson, 2006). The authors of GAISE project (Franklin et al., 2007) suggest that statistics literacy is the final goal of statistics education, and that an investment in statistical literacy by a country will be reflected in that country´s economy and development.

A statistical literate citizen should acquire a minimum level of understanding of the fundamental statistical ideas (Burrill & Biehler, 2011). These ideas commonly appear in statistical situations, can be taught with different levels of formalization along the curricula and have played a main role in the development of statistics as a science:

1. *Data:* While data are also used in mathematics, in statistics the context of data is fundamental to interpret the results of statistical analyses and for this reason the understanding of the context is central to statistical literacy and for this reason we should bring realistic contexts, in addition to bringing statistical ideas to the classroom. (Gal, 2019). Moreover, there is a wider type of data in the work with statistics when compare with mathematics, because of the use of categorical variables. This mean the possibility of mathematising a wider range of situations that cannot be worked with mathematical methods.
2. *Variation:* although variation also appear in the study of functions, statistics deal with random variation. A main goal of statistics is identifying and measuring variability to predict, explain or control various phenomena.

Contrary to mathematics that provides exact results, in statistics we often deal with approximation and randomness.

3. *Distribution:* a pure statistical idea that describes the behaviour of a variable in a collective is the distribution, which is characterized by its central tendency and spread. Instead of working with each isolated value of the variable, we work with the whole distribution and a goal of statistics is predicting the model for the distribution of the variables in the study. Often these variables can be modelled by general types of distributions, such as the normal, binomial, uniform or exponential distribution.
4. *Representation:* the values of a distribution can be condensed in a graph or a table. These graphical or tabular representation of a distribution are used as tools that help discovering patterns hidden in the raw data by a process of transnumeration (Wild & Pfannkuch, 1999).
5. *Association, correlation and regression:* that help modelling relations between two variables and expand the idea of functional dependence to random variables. There are different models that can be fitted to bivariate data (e.g. lineal, multivariate, etc.) and there is also different strength in the relation between the variables from independence to perfect correlation.
6. *Probability:* statistics would be useless without probability that provides the mathematical models underlying statistical analysis and in particular inference. At the same time, statistical data are needed to implement the frequentist and Bayesian views of probability.
7. *Sampling and inference:* The main value of statistics is the power of using results of analyses performed in samples to expand the conclusions to the populations from which the samples were collected. Although methods such as confidence intervals or statistical tests are complex, elementary ideas of inferences can be understood by the general citizen.

1.2 Statistical thinking and reasoning

In addition to knowledge of concepts, statistical sense requires ability to apply this knowledge and adequate way of reasoning and thinking. Statistical thinking has been described by many authors, including a long discussion about how should we distinguish statistical thinking, reasoning and literacy (e.g. in Garfield & Ben-Zvi, 2008). The term statistical thinking will not be distinguished in the paper, and will be used with the broad meaning of solving problems in which variation and/or uncertainty is present.

We understand that statistical thinking appears each time a person is faced and try to solve a statistical problem and that statistical reasoning is concerned with the arguments used by the person to justify his or her solutions to the problem or else to refute or agree with a statement based on statistical data.

A relevant model to describe what is involved in thinking and reasoning in statistics is the model proposed by Wild and Pfannkuch (1999) that describes statistical thinking in empirical research during the process of solving real problems and considered the following components:

1. *The statistical investigation cycle:* which involve the phases of defining a statistical problem, planning for the solution, collecting the data, analysing the data and obtaining a conclusion about the problem. Many teachers use this PPCAD cycle as a framework for proposing activities or projects that help their students to develop their statistical thinking and reasoning.
2. *Fundamental types of thinking.* These are five typical ways in which a statistical reason: a) Realizing what data are needed for a particular research or problem; b) Trasnumeration, which consist in changing the way in which the data are represented to obtain new information not visible in the raw data; c) Perceiving the variation and identifying the sources of variation, such as the sampling process, the variable being studied, or error in measurement; d) Integrating statistical data and analysis with the context from which the problem has arisen; and e) Selecting and using the appropriate statistical models.
3. *The interrogative cycle.* Along problem solving in statistics, there is a constant, recursive generic process of interrogation, which may start at any step of the process. In this interrogative cycle the solver tries to generate possibilities, search for causes and explanations, and check ideas and approaches for the solution to the problem. The ideas may can from the problem, the data or the statistical analysis. Other components of this process are interpretation and criticisms of data, representations or solutions.
4. *Dispositions.* In addition to the above components, statistical thinking requires a series of dispositions, such as engagement and perseverance, curiosity, imagination and scepticism.

4 Making sense of statistics

Taking into account the above components, making sense of statistics involve the work with statistical projects and real data that students can collect themselves of download from the many data servers available on the Internet. In Batanero, Díaz, Contreras and Roa (2013) we analysed one of these projects to identify the different fundamental ideas and types of statistical thinking that could be exercised when working with the said project.

Other examples that depend on the statistical content we want the students to work with are given in Batanero and Borovcnik (2016). Let's consider, the example used to introduce exploratory data analysis: What does it means to be a typical student? The project starts by requesting the students to collect physical measures of themselves, such as for example, as height and weight, arm span, shoe size. Once the students have collected the data, they are asked to describe the physical measures of an ideal or typical student. To get response to this task the students have to summarize the data, select an adequate graph to represent the variables and analyse the main differences in physical measures in boys and girls to finally describe the typical student.

In this project the students work with qualitative (e.g. gender), discrete numerical (e.g. shoe size) and continuous variables (height, weight, arm spam) and use different types of graphs, such as pie chart, bar graphs, histograms or dot plots to represent the different variables.

The distributions can be summarized further by using the measures of location, centre, and spread (mean, median, mode, range standard deviation), and select which of them is most appropriated for each variable in order to describe the typical student. To study differences between girls and boys they compare the five-number summary and boxplots of measures in boys and girls. In case we are dealing with university students they can also perform statistical tests to decide if the differences that are visible in the various graphs and summaries are statistically significant.

In relation to correlation and regression, students can be asked to make a scatter plot of arm spam as a function of height, where they can observe that the taller a person is, the larger is his or her arm span. The relationship is approximately linear and direct and with a reasonable sample size, the students can fit a regression line to these data. The activity can also be used to remind the students the elementary properties of the linear function. More details about the specific questions, graphs and calculations needed in the project and given in Batanero and Borovnick (2016). This project is useful to achieve the following learning goals:

- Making students understand the main differences between qualitative and numerical variables, as well as about the main types of graphs and summaries that are adequate for each of these types of variables.
- Becoming familiar with the concept of distribution, the difference between absolute, relative and cumulative frequencies and understanding the usefulness of each of these types of frequencies.
- Making sense of measure of location such as mean, median and mode and being able to select the most adequate measure for each type of variable and shape of the distribution.
- Understanding variability in the data and making sense to each different measures of spread.
- Interpreting covariation and discriminating different types of covariation (direct, inverse, lineal or no). Realizing the usefulness of the line of best fit as a summary of a bivariate data.

The brief summary of this project suggests the many possibilities of expansion by just including other variables or research questions. A project is a simplified investigation that can be carried out by the students, help improving their attitudes towards the topic and understand and make sense of the different concepts and procedures we try to teach them. The computations needed to complete the project can be facilitated by the widely available software. For example, we recently count with the platform CODAP, a user-friendly statistical software freely available online at https://codap.concord.org/. Moreover, there is also a simulator available, that help exploring probability problems and models, as well as many examples of data sets and projects that can be used in the classrooms.

5 Final reflections

Statistics is today taught to almost all students in the different educational levels, but errors and misinterpretation of statistics published in the media and used in professional work is pervasive. In this paper we have suggested the need to change the approach in teaching, by focusing on conceptual understanding and making sense of statistics. The explanation for the poor learning of statistics is that teachers often forget the nature of statistics, which was described by Cabria (1994) in the following way:

> Statistics studies the behaviour of the so-called collective phenomena. It is characterized by information about a collective or universe, which constitutes its material object; a specific type of reasoning, the statistical method, which is its formal object and predictions for future involving an uncertain atmosphere, which constitute its final object or cause (Cabriá, 1994, p. 22).

The nature of statistics requires then to couple its study with that of probability, in considering settings where chance is present and at the same time should consider both statistical literacy and thinking. We also discussed the relevance of context to statistical literacy; as suggested by Gal (2019) most statistical methods emerged from the need to solve problems in disciplines different from mathematics. It is therefore necessary to find adequate contexts that serve to make sense of different statistical methods. This is very easy today given the abundance of real data available on Internet.

Working with statistical projects and investigation is a didactic method recommended by many statistical educators to better educate statistical literate citizens. There are many examples of such projects available, since different institutions organize statistical projects competitions for schools. Some examples are the Best cooperative project award (http://iase-web.org/islp/) promoted by the International Association for Statistical Education as a part of the International Statistical Literacy Project or the competition organized by the Canarias Statistical Office (http://www3.gobiernodecanarias.org/istac/webescolar/index.php), which also provides didactic materials in its web page.

This project work can be reinforced with other activities, in particular those based on simulations and microworlds that are easily available on the web. Many resources are today available to increase statistical literacy and reasoning, for example, those listed at the Civic-Stat project (http://iase-web.org/islp/pcs).

All of this involve a previous preparation of teachers, since the impact of curricular changes directly depends on teachers' willingness to and interest in teaching the given topic. Because of a routine learning when they were students. some teachers may not value statistics, feel scared with the idea or introducing statistical projects in the classroom or consider themselves not well prepared to teach statistics with this approach (Groth & Meletiou-Mavrotheris, 2018). Souza, Lopes, and Pfannkuch (2015) indicate that teachers may not be aware of the possibilities that statistical projects offer or the way they can be used to make sense of reality. According to McGilliwray and Pereira-Mendoza (2011), the use of data investigation projects is ideal to develop both statistical and didactic knowledge for teachers; therefore, it is important to immerse teachers in a reasoning and learning environment centred on this type of work.

We hope this paper help interesting these teachers to work with projects and encourage them to reinforce the teaching of statistics in the classrooms in order that all the students acquire an adequate sense of statistics.

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