How to teach mathematics and experimental sciences? Solving the inquiring *versus* transmission dilemma

Juan D. Godino[1]

[1] Universidad de Granada

jgodino@ugr.es

Abstract. Various theories suggest that the learning of mathematics and experimental sciences should be based on a constructivist pedagogy, oriented towards the students' investigation of problem-situations, and assigning the teacher a facilitator role. At the opposite extreme, other theories defend a more leading role by the teacher, which would imply the explicit transmission of knowledge. After a synthesis of these instructional models, in this paper, we argue that the optimization of learning requires an intermediate position between both extremes, by recognizing the complex dialectic between the student's inquiry and the teacher's transmission of knowledge. We based on anthropological and semiotic assumptions about the nature of mathematical and scientific objects, as well as on assumptions related to the structure of human cognition.

Keywords: didactical models, constructivism, objectivism, onto-semiotic approach

1 Introduction

The question of how to teach mathematics and sciences may appear as impertinent after the long time this activity has been done and the huge amount of available pedagogical and didactic research (English & Kirshner, 2015; Lederman & Abell, 2014). At this point, there should be clear ideas on how a teacher should proceed to plan and develop the teaching of a given mathematical or scientific knowledge. However, the dilemma between directly transmitting knowledge, or facilitating the students' inquiry so that they discover and build themselves that knowledge remains unclear (Zhang, 2016).

After presenting the problem in a more detailed way and summarising some background, in this paper, I include some ideas about the nature and the ontological and semiotic complexity of mathematical knowledge, which are also applicable to the experimental sciences concepts and principles. Next, I describe the characteristics of a mixed, investigative - transmissive didactic model, based on the assumptions from the Onto-semiotic Approach to mathematical knowledge and instruction (Godino, Batanero & Font, 2007; 2019), which assumes the local optimization of teaching and learning mathematics and sciences processes.

2 Constructivism versus objectivism

The family of instructional theories called "Inquiry-Based Education" (IBE), "Inquiry-Based Learning" (IBL), and "Problem-Based Learning" (PBL), postulates inquiry-based learning with little guidance from the teacher (Artigue & Blomhøj, 2013). The different varieties of constructivism share, among others, the assumptions that learning is an active process, that knowledge is built instead of innate or passively absorbed and that in order to achieve effective learning it is necessary to approach students with significant, open and challenging problems (Ernest, 1994; Fox, 2001).

> “The arguments that human beings are active agents constructing knowledge by themselves have made people believe that instructional activities should encourage learners to construct knowledge through their own participations. This constructivist view plays an important role in science teaching and learning and has become a dominant teaching paradigm.” (Zhang, 2016, p. 897).

The recommendations for implementing a teaching and learning of mathematics and sciences based on inquiry have been playing a significant role in the curricular orientations of various countries, in projects, research centers and reform initiatives. Linn, Clark and Slotta (2003) define inquiry-based science learning as follows:

> “We define inquiry as engaging students in the intentional process of diagnosing problems, critiquing experiments, distinguishing alternatives, planning investigations, revising views, researching conjectures, searching for information, constructing models, debating with peers, communicating to diverse audiences, and forming coherent arguments” (p. 518).

In the pedagogical models assuming constructivist principles, the teacher's role is developing a learning environment with which the student interacts autonomously. This means that the teacher has to select some learning tasks and ensure that the student has the cognitive and material resources needed to get involved in solving the problems. In addition, the teacher has to create a cognitive scaffolding, a "choice architecture" that supports and promotes the construction of knowledge by the students themselves. In some way, the aim is implementing a “paternalistic libertarian” pedagogy in the sense of the Thaler and Sunstein (2008) "nudge theory", based on the design of interventions of the "nudge" type. “A nudge, as we will use this term, is any aspect of the architecture of choice that modifies the people behaviour in a predictable manner without prohibiting any option or significantly changing their economic incentives” (Thaler and Sunstein, 208, p. 6).

In mathematical learning, the use of situations - problems (applications to daily life, other fields of knowledge, or problems internal to the discipline itself) is considered essential, so that students make sense of the conceptual structures that make up Mathematics as a cultural reality. These problems constitute the starting point of mathematical practice, since the problem solving activity, its formulation, communication and justification are considered key in developing the ability to face the solution of non-routine problems. This is the main objective of the “problem solving” tradition (Schoenfeld, 1992), whose emphasis is the identification of heuristics and metacognitive strategies. It is also the main aim of other theoretical models such as the Theory of Didactical Situations (TDS) (Brousseau, 2002), and the Realistic Mathematical Education (RME) (Freudenthal, 1973; 1991).

There are also positions contrary to constructivism, as is the case of Mayer (2004) or Kirschner, Sweller and Clark (2006), which justify, through a wide range of investigations, the greater effectiveness of instructional models in which the teacher, and the transmission of knowledge, have a predominant role. These postures are also related to objectivist philosophical positions (Jonassen, 1991), and to direct instruction or lesson-based pedagogy (Boghossian, 2006).

Sweller, Kirschner and Clark (2007) state that the last half century empirical research on this problem provides overwhelming and clear evidence that a minimum guide during instruction is significantly less effective and efficient than a guide specifically designed to support the cognitive processes necessary for learning. Similar results are reflected in the meta-analysis performed by Alfieri, Brooks, Aldrich and Tenenbaum (2011).

For objectivism, particularly in its behavioural version, knowledge is publicly observable and learning consists in the acquisition of that knowledge through the interaction between stimuli and responses. Frequently, the conditioning form used to achieve desirable verbal behaviours is direct instruction. Cognitive reasons can be provided in favour of applying a didactic model based on the transmission of knowledge (objectivism) versus models based on autonomous construction (constructivism). Kirschner et al. (2006) point out that constructivist positions, with minimally guided instruction, contradict the architecture of human cognition and impose a heavy cognitive burden that prevents learning:

> “We are skilful in an area because our long-term memory contains huge amounts of information concerning the area. That information permits us to quickly recognize the characteristics of a situation and indicates to us, often unconsciously, what to do and when to do it” (Kirschner, et al., 2006, p. 76).

Other reasons contradicting constructivist positions come from cultural psychology. According to Harris (2012):

> “Accounts of cognitive development have often portrayed children as independent scientists who gather first-hand data and form theories about the natural world. I argue that this metaphor is inappropriate for children’s cultural learning. In that domain, children are better seen as anthropologists who attend to, engage with, and learn from members of their culture” (Harris, 2012, p. 259).

The metaphor of the child as a natural scientist, so durable and powerful, is useful when used to describe how children make sense of the universal regularities of the natural world, regularities that they can observe themselves, regardless of their cultural environment. However, the metaphor is misleading when used to explain cognitive development. Children are born in a cultural world that mediates their encounters with the physical and biological world. To access this cultural world, children need a socially oriented learning mode (learning through participant observation). "The mastery of normative regularities calls for cultural learning" (Harris, 2012, p. 261).

The debate between direct teaching, linked to objectivist positions on mathematical and scientific knowledge, which defends a central role of the teacher in guiding learning, and a minimally guided teaching, usually referring to the constructivist-type teaching model, is not clearly solved in the research literature. Hmelo-Silver et al. (2007) argue that PBL and IBL "are not minimally guided instructional approaches, but provide extensive support and guidance to facilitate student learning" (p.91). Supporters of problem-based learning and inquiry focus their arguments on the amount of guidance and the situation in which such guidance is provided. They consider that the guide given contains an extensive body of support and being immersed in real-life situations helps students make sense of the scientific content.

For Zhang (2016), the tension between these two instructional models does not consist in whether one or another would participate in presenting more or less guidance or support to the students, but between explicitly presenting the solutions to the learners or letting them discover these solutions. "For the advocates of direct instruction, explicitly presenting solutions and demonstrating the process to achieve solutions are essential guidance." (p. 908). Pretending that students discover, explore and find solutions, as structured in IBE, eliminates the need to present such solutions. In constructivist positions, although a certain dose of transmission of information from the teacher to the student is admitted, it is still essential to hide a part of the content. On the contrary for supporters of direct instruction, who assume the theory of cognitive load with emphasis on the examples worked, providing solutions is considered essential. In the next section we introduce a new key in the discussion of didactic models based on constructivism (inquiry) and objectivism (transmission). It consists in recognizing the onto-semiotic complexity of mathematical and scientific knowledge (Godino et al, 2007; Font, Godino and Gallardo, 2013), which must be taken into account in instructional processes intending to achieve the objective of optimizing student learning. By assuming anthropological, semiotic and pragmatic assumptions about mathematical knowledge, is concluded that an essential part of the knowledge that students have to learn are the conceptual, propositional, procedural rules, agreed within the mathematical or scientific community of practices. To solve the problems that constitute the educational objective, students use their previous knowledge, a central part of which are rules, which must be available to understand and address the task. Pretending that students discover those rules is non sense, but also the objective is to find the solutions, which in turn are rules, which must be part of their cognitive heritage to solve new problems. The assumptions of an educational-instructional model that solve the dilemma between inquiry and transmission are obtained by taking into account the onto-semiotic complexity of mathematical and scientific knowledge, while recognizing the central role of problem solving as a rationale for the contents.

3 Onto-semiotic complexity of mathematical knowledge

The onto-semiotic, epistemological and cognitive assumptions of the Onto-semiotic Approach to Mathematical Knowledge and Instruction (OSA) (Godino et al., 2007) serve as the basis for an educational-instructional proposal. Although this modelling of knowledge has been developed and applied for the case of mathematics, it is also relevant for the central core (concepts and principles) of scientific knowledge.

The OSA recognizes a key role to the transmission of knowledge (contextualized and meaningful for the student) in the mathematics, teaching and learning processes although problem solving and inquiring have also an important part in the learning process. Instruction have to take into account the cultural/regulatory nature of the mathematical objects involved in the mathematical practices, whose competent realization by the students is intended. This competence cannot be considered as acquired if it is meaningless to the students and, therefore, it they should be intelligible and meaningful to them. Thus, students should be able to use mathematical objects in their own contexts with autonomy. But, according to OSA, due to the onto-semiotic complexity of mathematical knowledge, this autonomy should not necessarily be acquired in the first encounter with the object or in the determination of some of the senses attributed to it; for example, it can be achieved in a mathematical application practice.

How to learn something depends on what you have to learn. According to the OSA the student must appropriate the institutional mathematical practices and the objects and processes involved in the resolution of situations-problems whose learning is intended (Fig. 1).

An essential component of these practices are conceptual, propositional, procedural and argumentative objects whose nature is normative (Font, Godino and Gallardo, 2013), and which have emerged in a historical and cultural process oriented towards generalization, formalization and maximizing the efficiency of mathematical work. It does not seem necessary or possible that students discover autonomously the cultural conventions that ultimately determine these objects.

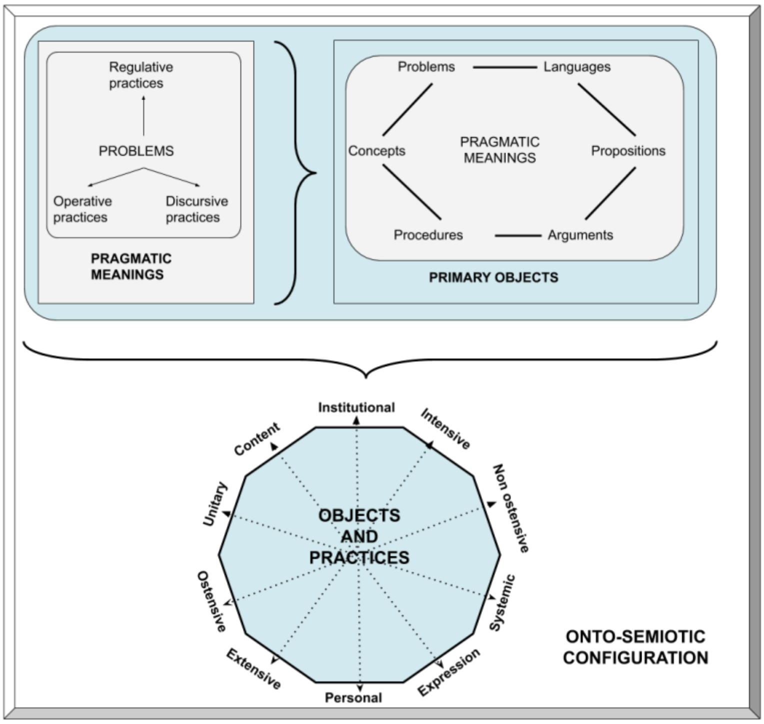

Fig. 1. Pragmatic meanings and onto-semiotic configuration (Godino, et al, 2017)

In an instructional process, the student's realization of mathematical practices linked to the solution of some problematic tasks puts into play a conglomerate of objects and processes whose nature, from an institutional point of view, is essentially normative (Font, et al., 2013). In the OSA mathematical ontology, according to Wittgenstein's philosophy of mathematics (Baker and Hacker, 1985; Bloor, 1983; Wittgenstein, 1953, 1978), the concepts, propositions and procedures are conceived as grammatical rules of the languages used to describe our worlds. They do neither describe properties of objects that have some kind of existence independent of the people who build or invent them, nor of the languages by which they are expressed. From this perspective, mathematical truth is nothing more than an agreement with the result of following a rule that is part of a language game that is put into operation in certain social practices. It is not an agreement of arbitrary opinions, it is an accord of practices subject to rules.

The realization of the mathematical practices involves the intervention of previous objects to understand the demands of the situation - problem and to be able to implement a starting strategy. Such objects, their rules and conditions of application, must be available in the subject's working memory. Although it is possible to individually seek such knowledge in the workspace, there is not always enough time or the student does not succeed in finding that knowledge. Therefore, the teacher and classmates provide invaluable support to avoid frustration and abandonment.

4 A mixed inquiry – transmissive instructional model

In Godino et al. (2006) some theoretical tools for the analysis of mathematical instruction processes are developed, by taking into account the previously developed onto-semiotic model for mathematical knowledge. In particular, the notions of didactic configuration and didactic suitability, serve as a basis to define a mixed didactic model that articulates the processes of inquiry and transmission of knowledge, related in a dialectical way in different types of didactic configurations.

A didactic configuration is any segment of didactic activity put into play when approaching the study of a problem, concept, procedure or proposition, as a part of the instruction process of a topic, which requires the implementation of a didactical trajectory (articulated sequence of didactic configurations). It implies, therefore, taking into account the teacher’s and student’s roles, the resources used and the interactions with the context. In fact, there are different types of didactic configurations, depending on the interaction patterns, and the management of the institutionalization and personalization of knowledge. According to the students’ previous knowledge and whether it is a first encounter with the object, or an exercise, application, institutionalization and evaluation moment, the didactic configurations can be of dialogical, collaborative, personal, magisterial, or a combination of these types (Figure 2). The optimization of the learning process through the didactic trajectories may involve a combination of different types of didactic configurations. This optimization, that is, the realization of a suitable didactic activity, has a strongly local character, so that the didactic models, either student-centred (constructivist), or teacher-centred and content (objectivist), are partial visions that drastically reduce the complexity of the educational-instructional process.

In the student's first encounter with a specific meaning of an object, a dialogic - collaborative configuration, where the teacher and students work together to solve problems that put knowledge O at stake in a critical way can optimize learning. The first encounter should therefore be supported by an expert intervention by the teacher, so that the teaching-learning process could thus achieve greater epistemic and ecological suitability (Godino, Font, Wilhelmi & Castro, 2009). When the rules and the circumstances of application that characterize the object of learning O are understood, it is possible to tend towards higher levels of cognitive and affective suitability, proposing to deepen the study of O (situations of exercising and application), through didactic configurations that progressively attribute greater autonomy to the student (Fig. 2).

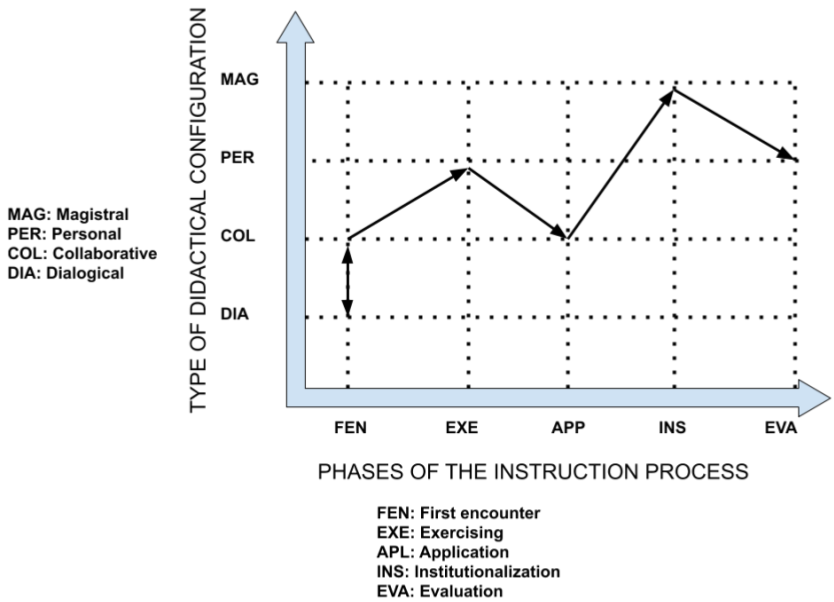

Fig. 2. A mixed inquiry – transmissive instructional model

In summary, within the OSA framework, it is assumed that the types of didactic configurations that promote learning can vary depending on the types of knowledge sought, the students' initial state of knowledge, the context and circumstances of the instructional process. When it comes to learning new and complex content, the transmission of knowledge at specific times, already by the teacher, and by the leading student within the work teams, can be crucial in the learning process. That transmission can be meaningful when students are participating in the activity and working collaboratively. The didactic configuration tool helps to understand the dynamics and complexity of the interactions between the content, the teacher, students and the context. The optimization of learning can take place locally through a mixed model that articulates the transmission of knowledge, inquiry and collaboration, a model managed by criteria of didactic suitability (Godino et al, 2007; Breda, Font and Pino-Fan, 2018) interpreted and adapted to the context by the teacher.

5 Final reflections

In this work we have complemented the cognitive arguments of Kirschner et al. (2006) in favour of models based on the transmission of knowledge with reasons of onto-semiotic nature for the case of mathematical learning and science, especially in the moments of students' "first encounter" with the intended content: what they have to learn are, in a large dose, epistemic / cultural rules, the circumstances of their application and the conditions required for its relevant application. The learners start from known rules (concepts, propositions and procedures) and produces others, which must be shared and compatible with those already established in the mathematical culture. Such rules have to be stored in the subject's long-term memory and put into operation in a timely manner in the short-term memory.

The postulate of constructivist learning with little guidance from the teacher can lead to instructional processes with low cognitive and affective suitability for real subjects, and with low ecological suitability (context adaptation) by not taking into account the onto-semiotic complexity of mathematical knowledge or the potential development zone (Vygotsky, 1993) of the subjects involved.

> "Children cannot discover the properties and regularities of the cultural world via their own independent exploration. They can only do that through interaction and dialogue with others. Children's trust in testimony, their ability to ask questions, their deference toward the use opaque tools and symbols, and their selection among informants all attest to the fact that nature has prepared them for such cultural learning" (Harris, 2012, p. 267).

I believe that learning optimization implies a dialectical and complex combination between the teacher's roles as an instructor (transmitter) and facilitator (manager) and the student's roles as a knowledge builder and active receiver of meaningful information. The need for this mixed model is reinforced by the need to adapt the educational project to temporary restrictions and the diversity of learning modes and rhythms in large groups of students. "Given the myriad of potential design situations, the designer's "best" approach may not ever be identical to any previous approach, but will truly "depend upon the context" (Ertmer and Newby, 1993, p. 62).

Hudson, Miller & Butler (2006) justify the implementation of mixed instructional models that adapt and mix explicit instruction (teacher-centred) with instruction based on problem solving (student-centred) because of the need to make curricular adaptations given the diversity of students' abilities. Steele (2005) comes to similar conclusions, for whom, "The best teaching often integrates ideas of constructivist and behavioural principles" (p. 3).

The teaching of mathematics and experimental sciences, should start and focus on the use of situations-problems, as a strategy to make sense of the techniques and theories studied, to propitiate exploratory moments of mathematical activity and develop research skills. However, configurations of mathematical objects (concepts, propositions, procedures, arguments) intervene in mathematical and scientific practice (Font, et al., 2013), which must be recognized by the teacher to plan their study. Such objects must be progressively dominated by students if we wish they progress towards successive advanced levels of knowledge and competence.

References

1. Alfieri, L., Brooks P. J., Aldrich, N. J. & Tenenbaum, H. R. (2011). Does discovery-based instruction enhance learning? *Journal of Educational Psychology, 103*(1), 1-18.
2. Artigue, M., & Blomhøj, M. (2013). Conceptualizing inquiry-based education in mathematics. *ZDM Mathematics Education 45*, 797–810.
3. Baker, G. P. & Hacker, P. M. S. (1985). *Wittgenstein. Rules, grammar and necessity. An analytical commentary on the Philosophical Investigations*. Glasgow: Basil Blackwell.
4. Bloor, D. (1983). Wittgenstein. *A social theory of knowledge.* London: The Macmillan Press.
5. Boghossian, P. (2006). Behaviorism, constructivism, and Socratic pedagogy. *Educational Philosophy and Theory, 38*(6), 713-722.
6. Breda, A., Font, V. & Pino-Fan, L.R. (2018). Criterios valorativos y normativos en la Didáctica de las Matemáticas: el caso del constructo idoneidad didáctica. *Bolema, 32* (60), p. 255 - 278.
7. Brousseau, B. (2002). *Theory of didactical situations in mathematics.* Dordrecht: Kluwer A. P.
8. English, L. D., & Kirshner, D. (Eds.). (2015). *Handbook of international research in mathematics education*. Routledge.
9. Ertmer P. A., & Newby T. J. (1993). Behaviorism, cognitivism, constructivism: comparing critical features from an instructional design perspective. *Performance Improvement Quarterly, 6*(4), 50–72. https://doi.org/10.1111/j.1937-8327.1993.tb00605.x
10. Ernest, P. (1994). Varieties of constructivism: Their metaphors, epistemologies, and pedagogical implications. *Hiroshima Journal of Mathematics Education, 2*, 1-14.
11. Font, V., Godino, J. D. & Gallardo, J. (2013). The emergence of objects from mathematical practices. *Educational Studies in Mathematics, 82*, 97–124.
12. Fox, R. (2001). Constructivism examined. *Oxford Review of Education, 27* (1), 23-35.
13. Freudenthal, H. (1973). *Mathematics as an educational task*. Dordrecht: Reidel.
14. Freudenthal, H. (1991). *Revisiting mathematics education. China lectures.* Dordrecht: Kluwer.
15. Godino, J. D. Batanero, C. & Font, V. (2007). The onto-semiotic approach to research in mathematics education. *ZDM. The International Journal on Mathematics Education, 39* (1-2), 127-135.
16. Godino, J. D., Batanero, C. & Font, V. (2019). The onto-semiotic approach: implications for the prescriptive character of didactics. *For the Learning of Mathematics, 39* (1), 37-42.
17. Godino, J. D., Beltrán-Pellicer, P., Burgos, M. & Giacomone, B. (2017). Significados pragmáticos y configuraciones ontosemióticas en el estudio de la proporcionalidad. En J. M. Contreras, P. Arteaga, G. R.Cañadas, M. M. Gea, B. Giacomone y M. M. López-Martín (Eds.), *Actas del Segundo Congreso International Virtual sobre el Enfoque Ontosemiótico del Conocimiento y la Instrucción Matemáticos*.
18. Godino, J. D., Contreras, A. & Font, V. (2006). Análisis de procesos de instrucción basado en el enfoque ontológico-semiótico de la cognición matemática. *Recherches en Didactiques des Mathematiques, 26* (1), 39-88.
19. Godino, J. D., Font, V., Wilhelmi, M. R. & Castro, C. de (2009). Aproximación a la dimensión normativa en Didáctica de la Matemática desde un enfoque ontosemiótico. *Enseñanza de las Ciencias, 27*(1), 59–76.

20. Harris, P. L (2012). The child as anthropologist. *Infancia y Aprendizaje, 35* (3), 259-277.
21. Hmelo-Silver, C. E., Duncan, R. G., & Chinn, C. A. (2007). Scaffolding and achievement in problem-based and inquiry learning: A response to Kirschner, Sweller, and Clark (2006). *Educational Psychologist, 42*(2), 99–107.
22. Hudson P., Miller S. P., & Butler F. (2006). Adapting and merging explicit instruction within reform based mathematics classrooms. *American Secondary Education, 35*(1), 19-32.
23. Jonassen D. H. (1991). Objectivism vs. constructivism: do we need a new philosophical paradigm? *Educacional Technology Research & Development, 39*(3), 5-14.
24. Kirschner P. A., Sweller J., & Clark R. E. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational Psychologist, 41*(2), 75-86.
25. Lederman, N. G., & Abell, S. K. (Eds.). (2014). *Handbook of research on science education*. Routledge.
26. Linn, M. C., Clark, D., & Slotta, J. D. (2003). WISE design for knowledge integration. *Science Education, 87*(4), 517–538.
27. Mayer, R. E. (2004). Should there be a three-strikes rule against pure discovery learning? *American Psychologist, 59* (1), 14 - 19.
28. Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In: D. Grouws (Ed.). *Handbook of research on mathematics teaching and learning* (p. 334-370).. New York: MacMillan.
29. Steele M. M. (2005). Teaching students with learning disabilities: Constructivism of behaviorism? *Current Issues in Education,* 8(10). Retrieved from http://cie.ed.asu.edu/coume8/number10
30. Sweller J., Kirschner P. A., & Clark R. E. (2007). Why minimally guided teaching techniques do not work: A reply to commentaries. *Educational Psychologist, 42*(2), 115-121.
31. Thaler, R. H., & Sunstein, C. R. (2008). *Nudge improving decisions about health, wealth and happiness*. Yale University Press.
32. Zhang, L. (2016). Is inquiry-based science teaching worth the effort? Some thoughts worth considering. *Science Education, 25*, 897-915.
33. Wittgenstein, L. (1953). *Philosophical investigations.* New York, NY: The MacMillan Company.
34. Wittgenstein, L. (1978). *Remarks on the foundations of mathematics* (3rd. ed.). Oxford, England: Basil Blackwell.
35. Vygotsky, L. S. (1993). *Pensamiento y lenguaje.* [Obras escogidas II, pp. 9-287]. Madrid: Visor.