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Finite vs. Infinite Traces in Temporal Logics

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1 Introduction

The *first-order temporal language* that we consider in this paper, $T_{\mathcal{U}}\mathcal{QL}$ [10], is obtained by extending the classical first-order language with the *until* temporal operator $\mathcal{U}$. This language can be interpreted over infinite linear structures, often based on the strict linear order of the natural numbers, in which case we speak of *infinite traces* (for more details on syntax and semantics of $T_{\mathcal{U}}\mathcal{QL}$, see [3] and references therein). Decidable fragments of first-order temporal logic [11, 10], and in particular *temporal description logics* [14, 1, 13] (combining linear temporal logic operators with description logics (DLs)) on infinite traces have been extensively investigated as temporal formalisms for knowledge representation.

Besides this semantics defined on infinite linear structures, attention has been devoted also to *finite traces*, which are temporal structures based on time-flows isomorphic to (finite) initial segments of the natural numbers [6, 8, 9]. The finiteness of the time dimension is indeed a natural restriction in many application domains (planning, process modelling, runtime verification, etc.). Moreover, the two semantics behave quite differently, as witnessed by the following examples. The $T_{\mathcal{U}}\mathcal{QL}$ formula

$$\Box^{+}\forall x(P(x)\rightarrow\bigcirc\Box^{+}(\neg P(x)\wedge\exists y R(x,y)\wedge P(y))),$$

which admits only models with an infinite domain of objects [13], is unsatisfiable over finite traces (here $\Box^{+}$ and $\Diamond^{+}$ are the *reflexive* versions of the usual future-time operators *box* $\Box$ and *diamond* $\Diamond$, while $\bigcirc$ is the *strong next* operator). On the other hand, the formula *last*, defined as $\neg\bigcirc\top$ (or, equivalently, $\Box\bot$), is satisfied only at the last instant of a finite trace, and thus it never holds on models with an infinite ascending chain of instants, while $\bigcirc\bot$ cannot be satisfied, neither on finite nor on infinite traces. Therefore, on finite traces, differently from the infinite case, we have that formulas of the form $\bigcirc\varphi$ are *not* equivalent to $\neg\bigcirc\neg\varphi$, and the strong next behaves differently from its *weak* counterpart, $\bullet$ (abbreviating $\neg\bigcirc\neg$), for which it holds that $\bullet\varphi$ is satisfied at a given instant iff $\bigcirc\varphi$ or *last* is satisfied.

Given the differences with the infinite case, the main purpose of our line of research is to establish semantic and syntactic conditions which characterise when the distinction between reasoning on finite and infinite traces can be loosened. Several approaches have been considered to preserve the satisfiability of formulas from the finite to the infinite case, so to reuse algorithms developed for the infinite case [5, 7]. We focus on equivalences between formulas, determining conditions that guarantee when it is preserved from finite to infinite traces, as well as conditions preserving equivalences in the other direction, from infinite to finite traces. This approach opens interesting research directions towards the application of efficient finite traces reasoners [12] to the infinite case.

After a summary of results contained in a paper published at IJCAI 2019 [3], where we also provide a uniform framework for semantic notions to bridge finite and infinite trace semantics, in Section 2 below we present original work on formulas for which satisfiability from finite to infinite traces is preserved. Finally, we report in Section 3 previously obtained [2, 4, 3] complexity results for temporal DLs on finite traces.

2 Finite vs. Infinite Traces

There are examples of temporal formulas, such as $\Box\top$, that are satisfiable both on finite and infinite traces. Others, however, are satisfiable only on finite traces, witness $\Diamond \mathit{last}$, or only on infinite traces, as $\Box^+\bigcirc\top$. It is then natural to ask under which circumstances we can ensure that satisfiability on finite and infinite traces coincide, so that solvers can limit themselves to the construction of a finite trace and avoid the step of building the lasso of an infinite trace. We are also interested in an analogous question concerning equivalences between formulas. For instance, we have that $\Diamond\Box(\varphi \vee \psi)$ and $\Diamond\Box\varphi \vee \Diamond\Box\psi$, which are not equivalent on infinite traces, are equivalent on finite traces [5]. Similarly, formulas $\Box^+\Diamond^+\varphi$ and $\Diamond^+\Box^+\varphi$ are both equivalent to $\Diamond^+(\mathit{last} \wedge \varphi)$ on finite traces [8], but are not equivalent on infinite traces. On the other hand, we have that *last* is satisfiable on finite traces, thus $\bot$ and *last* turn out to be equivalent only on infinite traces. In the following, we report selected (although slightly rephrased) results from [3], where we additionally propose a uniform framework of semantic properties that ensure when formula satisfiability and equivalences between formulas are preserved between the finite and the infinite case. The properties help to understand in which cases the distinction between the two can be harmlessly blurred.

We start by observing that, for first-order formulas (FOL) without temporal operators but interpreted on traces, there is no distinction between reasoning over finite or infinite traces.

Theorem 1 (Cf. [3])**.** *Formulas without temporal operators (FOL) are equivalent (resp., satisfiable) on finite traces if, and only if, they are equivalent (resp., satisfiable) on infinite traces.*

As a further step, we study cases where diamond and box operators (reflexive or not) are allowed and consider the problem of *equivalences* between formulas. We provide classes of formulas whose syntactic structure ensures that being equivalent on finite traces implies being equivalent also on infinite traces, or, vice-versa, formulas for which, given their construction, it is guaranteed that equivalences on infinite traces are preserved for the finite case. In the following, given $T_{\mathcal{U}}\mathcal{QL}$ formulas φ, ψ, we write $\varphi \equiv_{\mathfrak{F}} \psi$ (resp., $\varphi \equiv_{\mathfrak{I}} \psi$) if φ is equivalent to ψ on finite (resp., infinite) traces.

Let $\Diamond^+$*-formulas* φ, ψ be constructed according to the following grammar, where P is a predicate:

$$\Diamond^+\varphi \mid \varphi \vee \psi \mid \varphi \wedge \psi \mid \exists x\varphi \mid P(\vec{\tau}) \mid \neg P(\vec{\tau}).$$

Moreover, we call $\Diamond$*-formulas* the formulas generated by allowing further $\Diamond\varphi$ in the grammar rule for $\Diamond^+$-formulas, while $\Diamond^+\forall$*-formulas* are obtained by allowing also $\forall x\varphi$ in the grammar rule for $\Diamond^+$-formulas. We have the following result for the formulas so constructed.

Theorem 2 (Cf. [3])**.** *The following holds:*

1. *for all $\Diamond^+$-formulas φ, ψ, $\varphi \equiv_{\mathfrak{F}} \psi$ if and only if $\varphi \equiv_{\mathfrak{I}} \psi$;*
2. *for all $\Diamond^+\forall$-formulas φ, ψ, $\varphi \equiv_{\mathfrak{I}} \psi$ implies $\varphi \equiv_{\mathfrak{F}} \psi$;*
3. *for all $\Diamond$-formulas φ, ψ, $\varphi \equiv_{\mathfrak{F}} \psi$ implies $\varphi \equiv_{\mathfrak{I}} \psi$.*

In [3], we provide examples showing that the results of Theorem 2 are tight, meaning that it is not possible to extend the grammar rule for $\Diamond^+$-formulas with $\forall x\varphi$, and we cannot extend the grammar rule for $\Diamond^+\forall$-formulas with $\Diamond\varphi$.

We define now the $\Box^+$*-formulas* φ, ψ, constructed according to the rule (with P being a predicate):

$$\Box^+\varphi \mid \varphi \vee \psi \mid \varphi \wedge \psi \mid \forall x\varphi \mid P(\vec{\tau}) \mid \neg P(\vec{\tau}).$$

The set generated by further allowing $\Box\varphi$ in the construction of $\Box^+$-formulas is the set of $\Box$*-formulas*, and we call $\Box^+\exists$*-formulas* those obtained by allowing $\exists x\varphi$ in the grammar rule for $\Box^+$-formulas.

Theorem 3 (Cf. [3])**.** *The following holds:*

1. *for all $\Box^+$-formulas φ, ψ, $\varphi \equiv_{\mathfrak{F}} \psi$ if and only if $\varphi \equiv_{\mathfrak{I}} \psi$;*
2. *for all $\Box^+\exists$-formulas φ, ψ, $\varphi \equiv_{\mathfrak{I}} \psi$ implies $\varphi \equiv_{\mathfrak{F}} \psi$;*
3. *for all $\Box$-formulas φ, ψ, $\varphi \equiv_{\mathfrak{F}} \psi$ implies $\varphi \equiv_{\mathfrak{I}} \psi$.*

Again, it can be seen in [3] that the results of Theorem 3 are tight, in that we cannot extend the grammar rule for $\Box^+$-formulas with $\exists x\varphi$, and it is not possible to extend the grammar rule for $\Box^+\exists$-formulas with $\Box\varphi$.

Finally, in the rest of this section (that contains original material), we investigate which fragments of first-order temporal logic have the property that *satisfiability* on finite traces is preserved on infinite traces. As already mentioned, in these cases, reasoners can be more efficient and avoid the steps to construct the lasso that shows that an infinite trace satisfying a given formula exists.

We show that satisfiability from finite to infinite traces is preserved for a class of formulas containing both $\Diamond^+$ and $\Box^+$. Given a finite trace $\mathfrak{F} = (\Delta^{\mathfrak{F}}, (\mathcal{I}_n)_{n\in[0,l]})$, we denote by $\mathfrak{F}^\omega$ the infinite trace that results from extending $\mathfrak{F}$ with $(\mathcal{I}_l)^\omega$—an infinite repetition of the last instant $\mathcal{I}_l$ of $\mathfrak{F}$. We say that φ is F_ω if for all finite traces $\mathfrak{F}$ and all assignments $\mathfrak{a}$, it satisfies:

$$\mathfrak{F} \models^{\mathfrak{a}} \varphi \Leftrightarrow \mathfrak{F}^\omega \models^{\mathfrak{a}} \varphi.$$

Clearly, for any F_ω formula φ, satisfiability is preserved from finite to infinite traces. Syntactically, we have that this property is satisfied by a class of formulas constructed as follows. Let $\Diamond^+\Box^+$-formulas φ, ψ be constructed according to the following rule (with P predicate symbol):

$$\Diamond^+\varphi \mid \Box^+\varphi \mid \varphi \lor \psi \mid \varphi \land \psi \mid \exists x\varphi \mid \forall x\varphi \mid P(\vec{\tau}) \mid \neg P(\vec{\tau}).$$

The next theorem shows that the language generated by the grammar rule for $\Diamond^+\Box^+$-formulas contains only formulas whose satisfiability on finite traces implies satisfiability on infinite traces.

Theorem 4. *Any $\Diamond^+\Box^+$-formula is F_ω.*

Proof. Our theorem is a consequence of the following claims, where we write $\mathfrak{F}^n$ for the suffix of a finite trace $\mathfrak{F}$ starting from time point n.

Claim 1. *For all finite traces $\mathfrak{F}$, all assignments $\mathfrak{a}$, and all $\Diamond^+\Box^+$-formulas φ, if $\mathfrak{F} \models^{\mathfrak{a}} \varphi$ then $\mathfrak{F}^\omega \models^{\mathfrak{a}} \varphi$.*

Proof of Claim 1. We show that $\mathfrak{F} \models^{\mathfrak{a}} \varphi$ implies $\mathfrak{F}^\omega \models^{\mathfrak{a}} \varphi$. The proof is by induction. For the base case, it is straightforward to see that the statement holds for all formulas of the form $P(\vec{\tau})$ and $\neg P(\vec{\tau})$, with P predicate symbol. Assume now that the claim holds for φ and there is a finite trace $\mathfrak{F}$ and an assignment $\mathfrak{a}$ such that $\mathfrak{F} \models \varphi'$. We argue that $\mathfrak{F}^\omega \models^{\mathfrak{a}} \varphi'$, where φ' is as follows.

- For $\varphi' = \Diamond^+\varphi$: by assumption $\mathfrak{F} \models^{\mathfrak{a}} \Diamond^+\varphi$. This means that there is $n \in [0, l]$ such that $\mathfrak{F}, n \models^{\mathfrak{a}} \varphi$. In other words, $\mathfrak{F}^n \models^{\mathfrak{a}} \varphi$. By the inductive hypothesis, if $\mathfrak{F}^n \models^{\mathfrak{a}} \varphi$ then $(\mathfrak{F}^n)^\omega \models^{\mathfrak{a}} \varphi$. So $\mathfrak{F}^\omega \models^{\mathfrak{a}} \Diamond^+\varphi$.
- For $\varphi' = \Box^+\varphi$: by assumption $\mathfrak{F} \models^{\mathfrak{a}} \Box^+\varphi$. This means that for all $n \in [0, l]$ we have that $\mathfrak{F}, n \models^{\mathfrak{a}} \varphi$. In other words, $\mathfrak{F}^n \models^{\mathfrak{a}} \varphi$ for all $n \in [0, l]$. By the inductive hypothesis, for all such n, if $\mathfrak{F}^n \models^{\mathfrak{a}} \varphi$ then $(\mathfrak{F}^n)^\omega \models^{\mathfrak{a}} \varphi$. So $\mathfrak{F}^\omega \models^{\mathfrak{a}} \Box^+\varphi$.
- For $\varphi' = \exists x\varphi$: by assumption $\mathfrak{F} \models^{\mathfrak{a}} \exists x\varphi$. This means that there is $d \in \Delta$ such that $\mathfrak{F} \models^{\mathfrak{a}[x\mapsto d]} \varphi$. By the inductive hypothesis, $\mathfrak{F}^\omega \models^{\mathfrak{a}[x\mapsto d]} \varphi$. Then, by the semantics of $\exists$, $\mathfrak{F}^\omega \models^{\mathfrak{a}} \exists x\varphi$.
- For $\varphi' = \forall x\varphi$: by assumption $\mathfrak{F} \models^{\mathfrak{a}} \forall x\varphi$. This means that for all $d \in \Delta$ we have that $\mathfrak{F} \models^{\mathfrak{a}[x\mapsto d]} \varphi$. By the inductive hypothesis, $\mathfrak{F}^\omega \models^{\mathfrak{a}[x\mapsto d]} \varphi$ for all $d \in \Delta$. Then, by the semantics of $\forall$, $\mathfrak{F}^\omega \models^{\mathfrak{a}} \forall x\varphi$.
- The remaining cases can be proved by a straightforward application of the inductive hypothesis.

Claim 2. *For all finite traces $\mathfrak{F}$, all assignments $\mathfrak{a}$, and all $\Diamond^+\Box^+$-formulas φ, if $\mathfrak{F}^\omega \models^{\mathfrak{a}} \varphi$ then $\mathfrak{F} \models^{\mathfrak{a}} \varphi$.*

Proof of Claim 2. We show that $\mathfrak{F}^\omega \models^{\mathfrak{a}} \varphi$ implies $\mathfrak{F} \models^{\mathfrak{a}} \varphi$. The proof is by induction. For the base case, it is again easy to check that the statement holds for all formulas of the form $P(\vec{\tau})$ and $\neg P(\vec{\tau})$, with P predicate symbol. Assume that the claim holds for φ and there is a finite trace $\mathfrak{F}$ and an assignment $\mathfrak{a}$ such that $\mathfrak{F}^\omega \models \varphi'$. We argue that $\mathfrak{F} \models^{\mathfrak{a}} \varphi'$, where φ' is as follows.

- For $\varphi' = \Diamond^+\varphi$: by assumption $\mathfrak{F}^\omega \models^\mathfrak{a} \Diamond^+\varphi$. This means that there is $n \in [0,\infty)$ such that $\mathfrak{F}^\omega, n \models^\mathfrak{a} \varphi$. In other words, $\mathfrak{F}^{\omega,n} \models^\mathfrak{a} \varphi$, where $\mathfrak{F}^{\omega,n}$ is the suffix of $\mathfrak{F}^\omega$ starting from time point n. If $n \geq l$ (the last time point of $\mathfrak{F}$) then, by definition of $\mathfrak{F}^\omega$, since $\mathfrak{F}^{\omega,n} = \mathfrak{F}^{\omega,l}$, we have that $\mathfrak{F}^{\omega,l} \models^\mathfrak{a} \varphi$. Let $\mathfrak{F}^l$ be the finite trace with only the last time point of $\mathfrak{F}$. As $\mathfrak{F}^{\omega,l} = (\mathfrak{F}^l)^\omega$, we have that $(\mathfrak{F}^l)^\omega \models^\mathfrak{a} \varphi$. By the inductive hypothesis, $\mathfrak{F}^l \models^\mathfrak{a} \varphi$. So $\mathfrak{F} \models^\mathfrak{a} \Diamond^+\varphi$. If $n < l$ then, by the inductive hypothesis, $\mathfrak{F}^n \models \varphi$, where $\mathfrak{F}^n$ is the suffix of $\mathfrak{F}$ starting from time point n. So $\mathfrak{F} \models^\mathfrak{a} \Diamond^+\varphi$.
- For $\varphi' = \Box^+\varphi$: by assumption $\mathfrak{F}^\omega \models^\mathfrak{a} \Box^+\varphi$. This means that for all $n \in [0,\infty)$ we have that $\mathfrak{F}^\omega, n \models^\mathfrak{a} \varphi$. In other words, $\mathfrak{F}^{\omega,n} \models^\mathfrak{a} \varphi$ for all $n \in [0,\infty)$, where $\mathfrak{F}^{\omega,n}$ is the suffix of $\mathfrak{F}^\omega$ starting from time point n. For all $n \in [0,l]$ (recall that l is the last time point of $\mathfrak{F}$), we have that $\mathfrak{F}^{\omega,n} = (\mathfrak{F}^n)^\omega$ and so, $(\mathfrak{F}^n)^\omega \models^\mathfrak{a} \varphi$. By applying the inductive hypothesis on all $n \in [0,l]$, we conclude that $\mathfrak{F}^n \models^\mathfrak{a} \varphi$ for all such n. In other words, $\mathfrak{F}, n \models^\mathfrak{a} \varphi$ for all $n \in [0,l]$. This means that $\mathfrak{F} \models^\mathfrak{a} \Box^+\varphi$.
- The remaining cases can be proved by a straightforward application of the inductive hypothesis. □

3 Complexity Results

We briefly mention here the complexity results recently obtained for different fragments of $T_\mathcal{U}\mathcal{QL}$ [2, 4]. In [2], we consider the fragment with a single free variable, unary and binary predicates, and temporal operators applied just to unary predicates (contained in the so-called *monodic* fragment [10]), in particular the description logic (DL) $T_\mathcal{U}\mathcal{ALC}$—the temporal extension of the DL $\mathcal{ALC}$ with the until temporal operator on concepts. A $T_\mathcal{U}\mathcal{ALC}$ *concept* is an expression of the form:

$$C, D ::= A \mid \neg C \mid C \sqcap D \mid \exists R.C \mid C\,\mathcal{U}\,D,$$

where A is a concept name (unary predicate) and R is a role name (binary predicate). A $T_\mathcal{U}\mathcal{ALC}$ *axiom* is either a *concept inclusion (CI)* of the form $C \sqsubseteq D$, or an *assertion* of the form $A(a)$ or $R(a,b)$. Formulas in $T_\mathcal{U}\mathcal{ALC}$ have the form: $\varphi, \psi ::= \alpha \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi\,\mathcal{U}\,\psi$, where α is a $T_\mathcal{U}\mathcal{ALC}$ axiom. We studied the complexity of checking satisfiability of formulas in: $T_\mathcal{U}\mathcal{ALC}$ interpreted over finite traces; $T_\mathcal{U}\mathcal{ALC}$ interpreted over k-bounded traces, i.e., finite traces with at most k (given in binary) instants; and in $T_\mathcal{U}(\mathrm{g})\mathcal{ALC}$, the restriction of $T_\mathcal{U}\mathcal{ALC}$ *to global CIs* (*GCIs*), i.e., with formulas only of the form $\Box^+(\mathcal{T}) \wedge \phi$, where $\mathcal{T}$ is a conjunction of CIs (now true at all time points) and ϕ does not contain CIs, interpreted over k-bounded traces. We obtained the following.

Theorem 5 (Cf. [2])**.** *Formula satisfiability in $T_\mathcal{U}\mathcal{ALC}$ on finite traces is* ExpSpace*-complete, while it reduces to* NExpTime*-complete in $T_\mathcal{U}\mathcal{ALC}$ on k-bounded traces, and to* ExpTime*-complete in $T_\mathcal{U}(g)\mathcal{ALC}$ on k-bounded traces.*

In [4], we consider temporal extensions of *DL-Lite*, i.e., the logic $T_\mathcal{U}$*DL-Lite*$^\mathcal{N}_{bool}$ with roles and concepts so defined:

$$R ::= L \mid L^- \mid G \mid G^-, \qquad\qquad C ::= \bot \mid A \mid \geq qR \mid \neg C \mid C_1 \sqcap C_2 \mid C_1\,\mathcal{U}\,C_2$$

where now roles can be either *local* (L, varying in time) or *global* (G, not varying in time), and can have *inverses* (L^-, G^-). Formulas are as before, with either CIs or GCIs. We consider various FO fragments (*core, krom, horn*) and the case where just $\Box, \bigcirc$ are in front of concepts. We obtain the following.

Theorem 6 (Cf. [4])**.** *$T_\mathcal{U}$DL-Lite$^\mathcal{N}_{bool}$ and $T_{\Box\bigcirc}$DL-Lite$^\mathcal{N}_{horn}$ formula satisfiability on finite traces is* ExpSpace*-complete. Allowing only GCIs, $T_\mathcal{U}(g)$DL-Lite$^\mathcal{N}_{bool}$, $T_\mathcal{U}(g)$DL-Lite$^\mathcal{N}_{core}$, $T_{\Box\bigcirc}(g)$DL-Lite$^\mathcal{N}_{bool}$ and $T_{\Box\bigcirc}(g)$DL-Lite$^\mathcal{N}_{horn}$ on finite traces are* PSpace*-complete.*

For traces with at most k time points, given in binary as part of the input, the following holds.

Theorem 7 (Cf. [4])**.** *$T_\mathcal{U}$DL-Lite$^\mathcal{N}_{bool}$ and $T_\mathcal{U}$DL-Lite$^\mathcal{N}_{horn}$ formula satisfiability on k-bounded traces are* NExpTime*-complete. Allowing only GCIs, $T_\mathcal{U}(g)$DL-Lite$^\mathcal{N}_{core}$ and $T_\mathcal{U}(g)$DL-Lite$^\mathcal{N}_{bool}$ on k-bounded traces are* PSpace*-complete.*

References

[1] A. Artale and E. Franconi. Temporal description logics. In *Handbook of Temporal Reasoning in Artificial Intelligence*, pages 375–388. Elsevier, 2005.

[2] A. Artale, A. Mazzullo, and A. Ozaki. Temporal description logics over finite traces. In *31st International Workshop on Description Logics, (DL'18)*, 2018.

[3] A. Artale, A. Mazzullo, and A. Ozaki. Do you need infinite time? In *Proc. of the 28th Int. Joint Conference on Artificial Intelligence (IJCAI-19)*, Macao, China, August 10-16, 2019. AAAI Press.

[4] A. Artale, A. Mazzullo, and A. Ozaki. Temporal DL-Lite over finite traces (preliminary results). In *32nd International Workshop on Description Logics, (DL'19)*, Oslo, Norway, 18-21 June, 2019.

[5] A. Bauer and P. Haslum. LTL goal specifications revisited. In *ECAI*, pages 881–886, 2010.

[6] S. Cerrito, M. C. Mayer, and S. Praud. First order linear temporal logic over finite time structures. In *LPAR*, pages 62–76, 1999.

[7] G. De Giacomo, R. De Masellis, and M. Montali. Reasoning on LTL on finite traces: Insensitivity to infiniteness. In *AAAI*, pages 1027–1033, 2014.

[8] G. De Giacomo and M. Y. Vardi. Linear temporal logic and linear dynamic logic on finite traces. In *IJCAI*, pages 854–860, 2013.

[9] V. Fionda and G. Greco. The complexity of LTL on finite traces: Hard and easy fragments. In *AAAI*, pages 971–977, 2016.

[10] D. M. Gabbay, A. Kurucz, F. Wolter, and M. Zakharyaschev. *Many-dimensional Modal Logics: Theory and Applications*, volume 148. Elsevier, 2003.

[11] I. M. Hodkinson, F. Wolter, and M. Zakharyaschev. Decidable fragment of first-order temporal logics. *Annals of Pure and Applied Logic*, 106(1-3):85–134, 2000.

[12] J. Li, L. Zhang, G. Pu, M. Y. Vardi, and J. He. LTL^f Satisfiability Checking. In *ECAI*, pages 513–518, 2014.

[13] C. Lutz, F. Wolter, and M. Zakharyaschev. Temporal description logics: A survey. In *TIME*, pages 3–14, 2008.

[14] F. Wolter and M. Zakharyaschev. Temporalizing description logics. In *FroCoS*, pages 104–109, 1998.