Experimental Research of High-Dimensional Simulation Processes in New Energy Theory

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Abstract. The article is devoted to the fundamental problem solution on an effective digital matrix signal processing development in the framework of research work supported by the Russian Federation Ministry of Science and Higher Education, related to the new materials for a new memory generation using direct recording methods with ultrashort laser pulses to solve problems provided by the end-to-end digital technology "Neurotechnology and Artificial Intelligence". The paper substantiates the need to develop a new energy theory of multidimensional digital representation and conversion of real signals to create fast algorithms which reduce computational complexity and improve the accuracy of signal recovery. Spectral algorithms for simulation of multidimensional discrete deterministic and random bandpass signals are described using the example of two-dimensional discrete basis functions and Fourier and Hartley transformations, considering the energy characteristics and autocorrelation functions of these signals. Communication equations are given for the development of two-dimensional simulation algorithms for signals with a non-axial frequency spectrum. Schemes of algorithms for simulating real-time bandpass signals and software implementation of the algorithm are presented. The developed software for the simulated signal characteristics research is described. These characteristics are the boundary frequencies of the function and the shape of the signal power spectral density.

Keywords: high dimensional simulation algorithms, deterministic and random signals, spectral representation of signals, bandpass signals, Fourier functions, Hartley functions, power spectral density function

1 Introduction

The article is devoted to the fundamental problem solution on an effective multidimensional digital signal processing development in the framework of research work supported by the Russian Federation Ministry of Science and Higher Education, related to the new materials for a new memory generation using direct recording methods with ultrashort laser pulses to solve problems provided by the end-to-end digital technology "Neurotechnology and Artificial Intelligence".

The relevance of the research topic described in this article is due to the need of new scientific methodology development for the synthesis of high-precision and high-performance algorithms for simulating deterministic and random signals of large dimensions within the framework of classical and generalized correlation theory in the spectral domain of harmonic bases, using the original algorithmic relationship of models of pseudo-random and deterministic signals, which will allow to create common simulation models on a single mathematical and software basis, as well as will provide an effective tool for statistical research of real-time systems for various purposes.

The first section of the article justifies the need to develop a new energy theory of digital matrix representation and conversion of real signals to create fast algorithms that reduce computational complexity and improve the accuracy of signal recovery. A step-by-step research plan is described, including experimental verification of theoretical results in comparison of simulation models of signals and systems. The minimization of conversion time and increasing stability and reliability have been considered.

The second section of the article describes spectral algorithms for simulating multi-dimensional discrete deterministic and random bandpass signals on the example of two-dimensional discrete basis functions and Fourier and Hartley transformations, accounting the energy characteristics and autocorrelation functions of these signals. Communication equations are given that allow authors to develop two-dimensional simulation algorithms for signals with a non-axial frequency spectrum.

In the third section of the article, the algorithms for simulating real-time bandpass signals are given.

The fourth section contains a description of the developed software (application), useful for researcher having an opportunity to set and research the characteristics of the simulated signal.

2 The Need to Develop a New Energy Theory of Real-Signal's Digital Matrix Representation and Conversion

There are certain forms of data that are especially well studied and therefore have certain general methods of analysis, such as time series, working with large amounts of data [1]. In other cases, the input data is more complex in shape or size, today we can get data from any source, starting from the genome [2] and ending with the media [3], in these cases more complex data get a more specific approach.

Big data means not only extended in computational space but is also in time. Time data characteristics may render traditional algorithms fundamentally useless. New data could come continuously and it could be not only required to be stored [4], there should be a method of dividing the input data into operational events in real time with further intent of using these events for forecasting [5]. Another level of complication is a multidimensional data. Finance uses a multidimensional dynamic analysis [6], the response timelines to threats in computer networks [7], articulated thinking visualization, all these areas need new approach of research.

One of the problems with multidimensional analysis stems from it being visually unintuitive. While it is possible to imagine the nature behind time series or to locate the connection between real life phenomenon and its two-, three- or even four-dimensional representation, higher numbers of dimensions may often lead to confusion. However, computational power available today does not only allow us to finally work with data as big as it comes but also allows us to disengage from our own biases by placing more work onto the machine. Indeed, such areas as machine learning and neural networks are not limited by executing calculations preassigned by the researcher but can to some extent choose their own mode of actions claiming levels of flexibility previously unattainable.

Modeling and simulation grant us the ability to study any real time processes virtually saving costs for the physical experiments. The usage of higher dimensions contributes to the accuracy delivered by the new algorithms. The usage of energy spectra places the scientific research for these algorithms in the well-researched area of spectral theory [7-10].

Spectral theory provides new approaches of research based on matrix mathematical apparatus that renders the new algorithms prepared for further automatization by the means of tool developed in such areas as machine learning, artificial intelligence, and big data processing.

The authors consider the basics of the theory of spectral simulation of multi-dimensional signals in harmonic bases continuing their research [11- 15]. Properties of two-dimensional harmonic discrete basis functions and transformations necessary for the development of spectral algorithms for simulating two-dimensional signals are considered in this article as well as its experimental software realization.

3 First Steps to the New Theory: Spectrum Simulation Algorithms for High Dimensional Discrete Determine and Random Bandpass Signals

3.1 Two-Dimensional Discrete Basis Functions for Fourier's and Hartley's Transformation

The two-dimensional trigonometric functions are the basis of the two-dimensional harmonic ones:

$$\cos\left[2\pi\left(\frac{k_1 i_1}{N_1}+\frac{k_2 i_2}{N_2}\right)\right];\ \sin\left[2\pi\left(\frac{k_1 i_1}{N_1}+\frac{k_2 i_2}{N_2}\right)\right], \tag{1}$$

where N1 and N2 – elements of common domain for defining discrete functions N1×N2; k1 and k2 – elements of two-dimensional function's number; i1 and i2 – elements of function's argument; at that $k_1, i_1 \in [0, N_1)$; $k_2, i_2 \in [0, N_2)$.

Using functions (2.1.1) three two dimensional complete basic systems can be created.

The first basis system is been created by alternating the even functions:

$$\cos\left[2\pi\left(\frac{k_1 i_1}{N_1}+\frac{k_2 i_2}{N_2}\right)\right],$$

and by alternating odd functions:

$$\sin\left[2\pi\left(\frac{k_1 i_1}{N_1}+\frac{k_2 i_2}{N_2}\right)\right],$$

where the number of even functions in it is greater than the number of odd functions. The corresponding spectra will have alternating even (marked E) and odd (marked O) coefficients as well, presented as follows:

$$X_E(k_1,k_2)=\frac{1}{P^E_{K1,K2}N_1N_2}\sum_{i_1=0}^{N_1-1}\sum_{i_2=0}^{N_2-1}x(i_1\,,i_2)cos\left[2\pi\left(\frac{K_1 i_1}{N_1}+\frac{K_2 i_2}{N_2}\right)\right]; \qquad (2)$$

$$X_O(k_1,k_2)=\frac{1}{P^O_{K1,K2}N_1N_2}\sum_{i_1=0}^{N_1-1}\sum_{i_2=0}^{N_2-1}x(i_1\,,i_2)sin\left[2\pi\left(\frac{K_1 i_1}{N_1}+\frac{K_2 i_2}{N_2}\right)\right], \qquad (3)$$

where:

$$P^E_{K1,K2}=\frac{1}{N_1N_2}\sum_{i_1=0}^{N_1-1}\sum_{i_2=0}^{N_2-1}cos^2\left[2\pi\left(\frac{K_1 i_1}{N_1}+\frac{K_2 i_2}{N_2}\right)\right]; \qquad (4)$$

$$P^O_{K1,K2}=\frac{1}{N_1N_2}\sum_{i_1=0}^{N_1-1}\sum_{i_2=0}^{N_2-1}sin^2\left[2\pi\left(\frac{K_1 i_1}{N_1}+\frac{K_2 i_2}{N_2}\right)\right]; \qquad (5)$$

are even and odd elements of the basis functions' power; $x(i_1,i_2)$ – two-dimensional determine signal, defined at the system of N1×N2 points, as well as $k_1\in\left[0,\frac{N_1}{2}\right)$; $k_1\in\left[0,\frac{N_2}{2}\right)$.

A discrete two-dimensional Fourier series (inverse DFT) in a trigonometric basis has the following form:

$$x(i_1,i_2)=X_E(0{,}0)+\sum_{k_1=0}^{\frac{N_1}{2}-1}\sum_{k_2=0}^{\frac{N_2}{2}-1}\left\{X_E(k_1,k_2)cos\left[2\pi\left(\frac{K_1 i_1}{N_1}+\frac{K_2 i_2}{N_2}\right)\right]\right\}+$$
$$+X_E\left(\frac{N_1}{2},\frac{N_2}{2}\right)cos[\pi(i_1,i_2)]\,.x(i_1,i_2)=X_E(0{,}0). \qquad (6)$$

The couple of DFTs (2), (3) and discrete two-dimensional Fourier series (6) establish mathematically one-to-one correspondence between discrete functions $x(i_1,i_2)$ and $X(k_1,k_2)$. Their physical equality is illustrated by Parseval's two-dimensional equation:

$$\frac{1}{N_1N_2}\sum_{i_1=0}^{N_1-1}\sum_{i_2=0}^{N_2-1}x^2(i_1,i_2)=\;x_E^2(0{,}0)+X_E^2\left(\frac{N_1}{2},\frac{N_2}{2}\right)+$$
$$+\sum_{k_1=0}^{\frac{N_1}{2}-1}\sum_{k_2=0}^{\frac{N_2}{2}-1}\left[P^E_{K1,K2}x_E^2(k_1,k_2)+P^O_{K1,K2}x_O^2(k_1,k_2)\right],$$

the second basis system, which is a two-dimensional analog of the DEF system, is formed from the corresponding trigonometric functions and is equal to:

$$exp\left[j2\pi\left(\frac{K_1 i_1}{N_1}+\frac{K_2 i_2}{N_2}\right)\right]=cos\left[2\pi\left(\frac{K_1 i_1}{N_1}+\frac{K_2 i_2}{N_2}\right)\right]+jsin\left[2\pi\left(\frac{K_1 i_1}{N_1}+\right.\right.$$
$$\left.\left.+\frac{K_2 i_2}{N_2}\right)\right];k_1,i_1\in[0,N_1);\;k_2,i_2\in[0,N_2). \qquad (7)$$

The system of these functions is complete orthonormal and multiplicative. The couple of discrete transformations for this system has following form:

$$x(i_1, i_2) = \sum_{k_1=0}^{\frac{N_1}{2}-1} \sum_{k_2=0}^{\frac{N_2}{2}-1} \left\{X(k_1, k_2) exp\left[j2\pi\left(\frac{K_1 i_1}{N_1} + \frac{K_2 i_2}{N_2}\right)\right]\right\}, \quad (8)$$
$$i_1 \in [0, N_1);\ i_2 \in [0, N_2),$$

$$X_F(k_1, k_2) = \frac{1}{N_1 N_2} \sum_{i_1=0}^{N_1-1} \sum_{i_2=0}^{N_2-1} \left\{x(i_1, i_2) exp\left[j2\pi\left(\frac{K_1 i_1}{N_1} + \frac{K_2 i_2}{N_2}\right)\right]\right\}, \quad (9)$$
$$i_1 \in [0, N_1);\ i_2 \in [0, N_2).$$

The Parseval's two-dimensional equation is show below:

$$\frac{1}{N_1 N_2} \sum_{i_1=0}^{N_1-1} \sum_{i_2=0}^{N_2-1} x^2(i_1, i_2) = \sum_{k_1=0}^{\frac{N_1}{2}-1} \sum_{k_2=0}^{\frac{N_2}{2}-1} X_F(k_1, k_2)\, X_F^*(k_1, k_2), \quad (10)$$
$$i_1 \in [0, N_1);\ i_2 \in [0, N_2).$$

The third basis system is formed by adding trigonometric functions:

$$cos\left[2\pi\left(\frac{K_1 i_1}{N_1} + \frac{K_2 i_2}{N_2}\right)\right] = cos\left[2\pi\left(\frac{K_1 i_1}{N_1} + \frac{K_2 i_2}{N_2}\right)\right] + sin\left[2\pi\left(\frac{K_1 i_1}{N_1} + \frac{K_2 i_2}{N_2}\right)\right], \quad (11)$$

and is a two-dimensional Hartley functions' modification. This system is orthonormal and obeys a pair of discrete two-dimensional transformations:

$$X_H(k_1, k_2) = \frac{1}{N_1 N_2} \sum_{i_1=0}^{N_1-1} \sum_{i_2=0}^{N_2-1} \left\{x(i_1, i_2) cos\left[2\pi\left(\frac{K_1 i_1}{N_1} + \frac{K_2 i_2}{N_2}\right)\right]\right\}. \quad (12)$$

$$x(i_1, i_2) = \sum_{i_1=0}^{N_1-1} \sum_{i_2=0}^{N_2-1} \left\{X_H(k_1, k_2) cos\left[j2\pi\left(\frac{K_1 i_1}{N_1} + \frac{K_2 i_2}{N_2}\right)\right]\right\}, \quad (13)$$

the given Fourier transformations are oriented to the determined signals. However, they will also apply to random signals $y(i_1, i_2)$ with spectra $Y(k_1, k_2)$.

3.2 Energy Characteristics of Two-Dimensional Signals and their Relation to Fourier Coefficients

The energy properties of deterministic two-dimensional signals, as well as their one-dimensional counterparts, are characterized using SPDF (spectral power density function)

$$S(\omega_1, \omega_2) = \lim_{\substack{T_1 \to \infty \\ T_2 \to \infty}} \left[\frac{1}{T_1 T_2} |X_F(\omega_1, \omega_2)|^2\right],$$

where $X(\omega_1, \omega_2)$ is a continuous spectrum defined on an infinite interval of two-dimensional values of frequency:

$$X(\omega_1, \omega_2) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x(t_1, t_2)\, exp[-j(\omega_1 t_1 + \omega_2 t_2)] dt_1\, dt_2.$$

For finite signals $x(t_1, t_2), t_1 \in [0, T_1), t_2 \in [0, T_2)$

$$S(\omega_1, \omega_2) = \frac{1}{T_1 T_2} |X_F(\omega_1, \omega_2)|^2 \tag{14}$$

with

$$X_F(\omega_1, \omega_2) = \int_0^{T_1} \int_0^{T_2} x(t_1, t_2)\, exp[-j(\omega_1 t_1 + \omega_2 t_2)] dt_1\, dt_2. \tag{15}$$

At the discrete points of the frequency range

$$\begin{gathered} \omega_1 = k_1 \Delta\omega_1 = \frac{2\pi}{T_1} k_1 \text{ и } \omega_2 = k_2 \Delta\omega_2 = \frac{2\pi}{T_2} k_2, \\ S(k_1 \Delta\omega_1 + k_2 \Delta\omega_2) = \frac{1}{T_1 T_2} |X_F(k_1 \Delta\omega_1,\, k_2 \Delta\omega_2)|^2 = \\ = \frac{1}{T_1 T_2} [x_{FE}^2(k_1 \Delta\omega_1,\, k_2 \Delta\omega_2) + x_{FO}^2(k_1 \Delta\omega_1,\, k_2 \Delta\omega_2)], \end{gathered} \tag{16}$$

where

$$X_{FE}(k_1 \Delta\omega_1,\, k_2 \Delta\omega_2) = \int_0^{T_1} \int_0^{T_2} x(t_1, t_2)\, cos(k_1 \Delta\omega_1 +\, k_2 \Delta\omega_2) dt_1\, dt_2,$$

$$X_{FO}(k_1 \Delta\omega_1,\, k_2 \Delta\omega_2) = \int_0^{T_1} \int_0^{T_2} x(t_1, t_2)\, sin(k_1 \Delta\omega_1 +\, k_2 \Delta\omega_2) dt_1\, dt_2.$$

In the basis of complex exponential functions, the spectral coefficients are equal to

$$\begin{gathered} X_F(k_1,\, k_2) = \frac{1}{T_1 T_2} \int_0^{T_1} \int_0^{T_2} x(t_1, t_2)\, exp[-j2\pi(k_1 t_1 | T_1 + k_2 t_2 | T_2)] dt_1\, dt_2 = \\ = X_{FE}(k_1,\, k_2) - jX_{FO}(k_1,\, k_2). \end{gathered}$$

The values $X_{FE}(k_1\Delta\omega_1,\, k_2\Delta\omega_2)$, $X_{FO}(k_1\Delta\omega_1,\, k_2\Delta\omega_2)$ and $X_{FE}(k_1,\, k_2)$, $X_{FO}(k_1,\, k_2)$ determine the even and odd components of the corresponding complex spectra. Comparing them with each other, a record of a discrete spectral density is been created in the form of

$$S(k_1 \frac{2\pi}{T_1} + k_2 \frac{2\pi}{T_2}) = S(k_1, k_2) = T_1 T_2 \left[X_{FE}{}^2(k_1,\, k_2) + X_{FO}{}^2(k_1,\, k_2) \right]. \tag{17}$$

However, one equation (17) is not sufficient to determine even and odd components of the spectrum. Therefore, we introduce a phase two-dimensional density for modeling $\psi(k_1 \frac{2\pi}{T_1}, k_2 \frac{2\pi}{T_2})$, by setting it in the form

$$\lambda_{k_1,k_2} = tan\left[\psi(k_1\frac{2\pi}{T_1}, k_2\frac{2\pi}{T_2})\right] = \frac{X_{FO}(k_1, k_2)}{X_{FE}(k_1, k_2)}. \quad (18)$$

Then get

$$X_{FO}(k_1,\ k_2) = \lambda_{k_1,k_2}X_{FE}(k_1,\ k_2);\ \lambda_{0,0} = 0;\ \lambda_{N_1,N_2} = 0;\ \lambda_{k_1,k_2} \neq 0. \quad (19)$$

Solving the equations (18) and (19), we get new equations for the relationship between the Fourier coefficients and the spectral density power function (SDPF):

$$X_{FE}(0,0) = \sqrt{\frac{S(0,0)}{T_1T_2}};\ X_{FE}(k_1,\ k_2) = \sqrt{\frac{S(k_1\frac{2\pi}{T_1}, k_2\frac{2\pi}{T_2})}{T_1T_2(1+\lambda_{k_1,k_2}{}^2)}} \quad (20)$$

$$X_{FO}(k_1,\ k_2) = \lambda_{k_1,k_2}X_{FE}(k_1,\ k_2);\ k_1 \in [0, N_1);\ k_2 \in [0, N_2)\,. \quad (21)$$

These coupling equations allow to develop two-dimensional simulation algorithms for signals with a non-axial frequency spectrum.

Given non-axial signals are a special case of bandpass signals, they are described in additional materials enclosed with the paper (zip file), so further we consider only two-dimensional bandpass signals whose SPDF and ACF are easily associated with Fourier coefficients, replacing in equations (20) and (21) the definition intervals [0, N_1) and [0, N_2) with [N_1L, N_1R] and [N_2L,N_2R], respectively. Here in a following sub-section 2.3 there are obtained equations for two-dimensional signals' autocorrelation functions.

3.3 The Two-Dimensional Signals' Autocorrelation Functions

Suggested spectral algorithms of imitation may be illustrated with the help of a signal with a two-dimensional spectral density having the shape of a parallelepiped. Such spectral density is visualized on the figure 1, this spectral density is technically the spectral density of a bandpass two-dimensional white noise with the intensity of S_0.

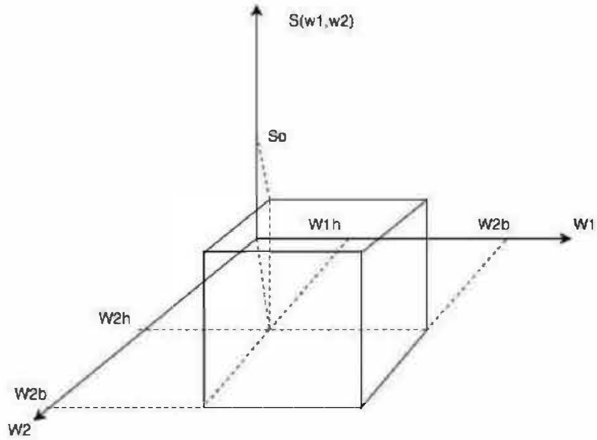

Fig 1. Functional scheme of the two-dimensional spectral density's visualization.

The equation for random two-dimension signal's calculation with form shown at the Fig. 1 as follows:
for the even N_1 and N_2:

$$y(i_1, i_2) = Y_F(N_{1R}, N_{2R}) exp\left[j2\pi\left(\frac{N_{1R}i_1}{N_1} + \frac{N_{2R}i_2}{N_2}\right)\right] +$$
$$+\sum_{k_1=N_{1L}}^{N_{1R}} \sum_{k_2=N_{2L}}^{N_{2R}} \left\{Y_F(k_1, k_2) exp\left[j2\pi\left(\frac{k_1 i_1}{N_1} + \frac{k_2 i_2}{N_2}\right)\right] +\right.$$
$$+Y_F(N_{1R} + N_{1L} - 1 - k_1, N_{2R} + N_{2L} - 1 - k_2) exp\left[-j2\pi\left(\frac{(N_{1R}+N_{1L}-1-k_1)i_1}{N_1} +\right.\right.$$
$$\left.\left.\left.+\frac{(N_{2R}+N_{2L}-1-k_2)i_2}{N_2}\right)\right]\right\};$$

for the odd N_1 and N_2:

$$y(i_1, i_2) = Y_F \sum_{k_1=N_{1L}}^{N_{1R}} \sum_{k_2=N_{2L}}^{N_{2R}} \left\{Y_F(k_1, k_2) exp\left[j2\pi\left(\frac{k_1 i_1}{N_1} + \frac{k_2 i_2}{N_2}\right)\right] +\right.$$
$$+Y_F(N_{1R} + N_{1L} - k_1, N_{2R} + N_{2L} - -k_2) exp\left[-j2\pi\left(\frac{(N_{1R}+N_{1L}-k_1)i_1}{N_1} +\right.\right.$$
$$\left.\left.\left.+\frac{(N_{2R}+N_{2L}-k_2)i_2}{N_2}\right)\right]\right\}.$$

Even and odd Fourier coefficients and the right boundary Fourier coefficients are defined as follows:

$$X_{FE}(k_1, k_2) = \sqrt{\frac{S_0}{T_1 T_2 (1+\lambda_{k_1,k_2}^2)}};\ X_{FO}(k_1, k_2) = \lambda_{k_1,k_2} X_{FE}(k_1, k_2);$$
$$X_{FE}(N_{1R}, N_{2R}) = \sqrt{\frac{S_0}{T_1 T_2}};\ k_1 \in [N_{1L}, N_{1R});\ \ k_2 \in [N_{2L}, N_{2R}).$$

where

$$S_0 = \frac{T_1 T_2}{2(N_{1R}-N_{1L})(N_{2R}-N_{2L})}.$$

Considering $\lambda_{k_1,k_2} = 1$ with all k_1, k_2 apart from $k_1 = N_{1R}, k_2 = N_{2R}$, where $\lambda_{N_{1R},N_{2R}} = 0$, the coefficients are defined differently:

$$X_{FE}(k_1, k_2) = X_{FE}(k_1, k_2) = \frac{1}{\sqrt{4(N_{1R}-N_{1L})(N_{2R}-N_{2L})}};$$
$$X_{FE}(N_{1R}, N_{2R}) = \frac{1}{\sqrt{2(N_{1R}-N_{1L})(N_{2R}-N_{2L})}};\ k_1 \in [N_{1L}, N_{1R});\ \ k_2 \in [N_{2L}, N_{2R}).$$

Algorithmic ACF is useful for evaluating the quality of random signal's simulation.

$$R_A(i_1, i_2, j_1, j_2) = M\big(y(i_1, i_2), y(j_1, j_2)\big) = R_A(m_1, m_2),$$

where $m_1 = i_2 - i_1;\ m_2 = j_2 - j_1$.

4 Experimental Research: Software Realization of the Random Bandpass Signal Simulation Algorithm with Complex Basis

4.1 Experimental Setup

Accuracy can be measured by comparing with theoretical values for a specific signal, and speed can depend on the mathematical form of the algorithm – formulas are computed faster than complex algorithms using matrices. Mathematical equations provide low memory size requirements as well. A possible disadvantage of the described algorithms is the need for impressive mathematical training before programming.

The output data of the software is the theoretical, algorithmic, and experimental autocorrelation functions, as well as the errors, as the difference between autocorrelation functions, and the simulated signal itself.

The developed application allows us to simulate signals based on specified characteristics with the ability to simulate one- and two-dimensional signals. The following case proves the efficiency of the algorithm for one-dimensional signals' simulation. Two-dimensional signal simulation output will be available in future articles.

4.2 Functional Scheme

The Fig. 2 shows the functional scheme describing the basic path of work with the algorithm. The input contains borders limiting the band of the signal being generated, the time period, the number of discretization steps. These characteristics, also known as input characteristics, allow us to display the spectral density diagram.

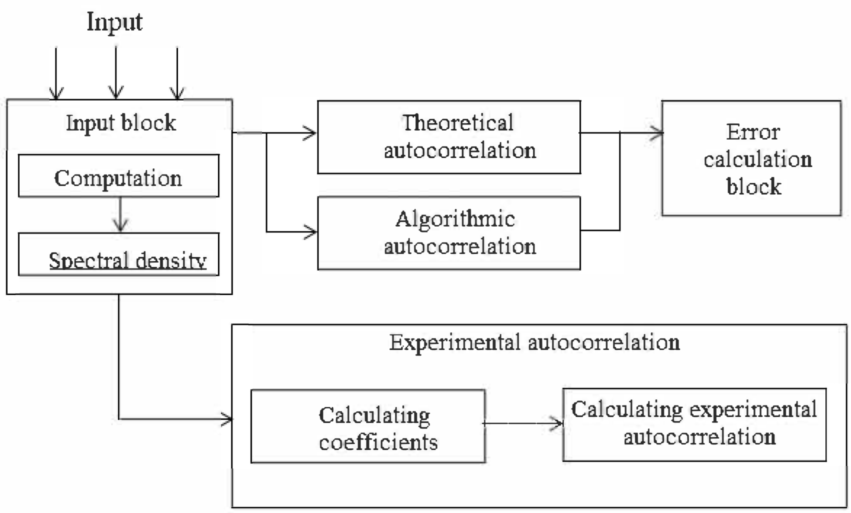

Fig 2. Functional scheme of the algorithm.

The spectral density is used to compute the theoretical and algorithmic autocorrelations used to evaluate the quality of the imitation, it is also used to imitate the signal according to the set input characteristics and to form its experimental autocorrelation.

4.3 The Algorithm

The set of steps undertaken to generate the signal, its energetic characteristic and to estimate the quality of the imitation process is shown on the Fig. 3. Every step of the algorithm implements mathematical formulas described earlier.

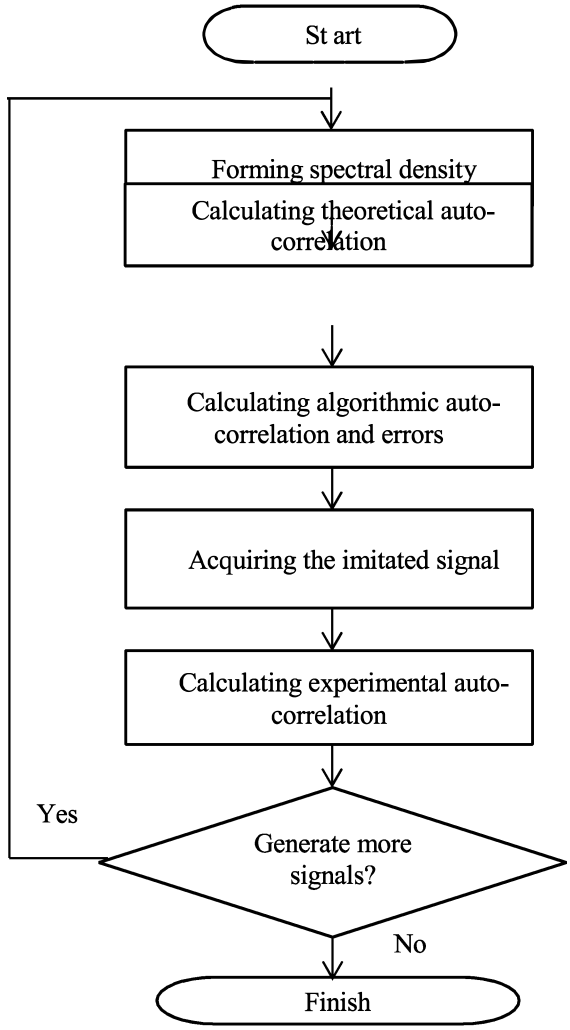

Fig 3. Flowchart of the random signal imitation algorithm.

4.4 Software Implementation

The software implementation has been done using Lazarus IDE, supporting Free Pascal programming language. The chosen IDE provides all the necessary mathematical functions and instruments allowing to build desired interfaces. The workflow in the program mirrors shown at the figure 3. The user sets the input characteristics which are used to picture the spectral density, later the autocorrelation, the signal and the error graph are calculated and visualized.

4.5 Computational Complexity for the Complex Based Algorithm

Computational complexity of the method used to calculate theoretical is O(M). Computational complexities for calculating algorithmic and experimental are both O(M*(N2-N1)).

Thus, computational complexity reaches only O(M*(N2-N1)), which is closer to linear time complexity rather than to O(M^2) – such result may be deemed sufficient.

5 Comparison Between Theoretical and Experimental Results

In the following example characteristics of the signal under generation are: ωR = 6π; ωL = 3π; T = 2 s; N = 51; n = 4. Fig. 4 displays the spectral density printed according to the characteristics and generated theoretical and algorithmic autocorrelations.

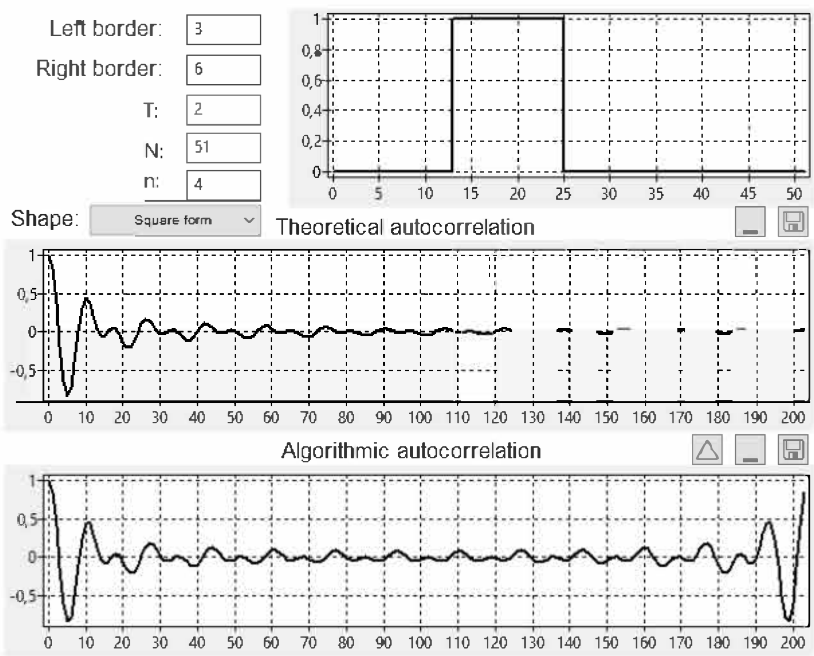

Fig 4. Signal's spectral density and theoretical and algorithmic autocorrelations.

In the case of the random signal one set of characteristics can describe different signals. Three signals with characteristics ωR = 6π; ωL = 3π; T = 2 s; N = 51; n = 4 are shown on the figure 5. Three different generated experimental correlations are shown on the Fig. 6.

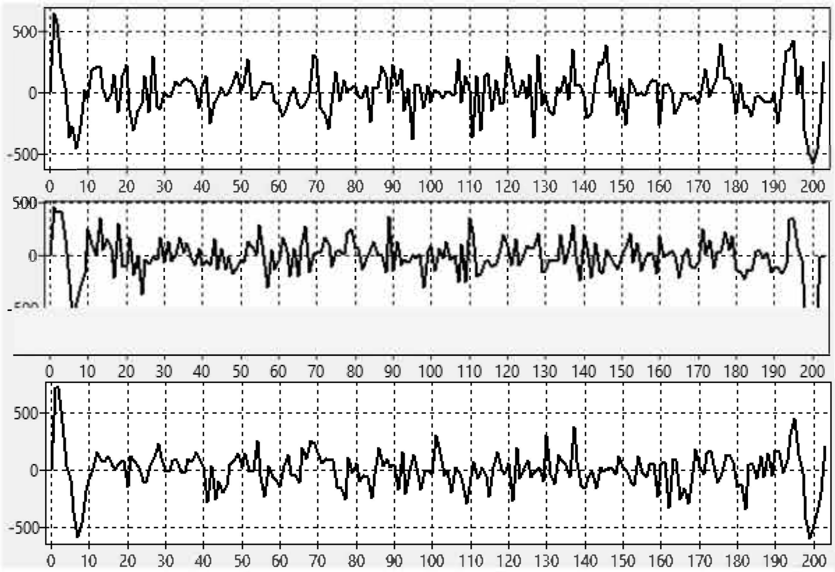

Fig 5. Generated random signals.

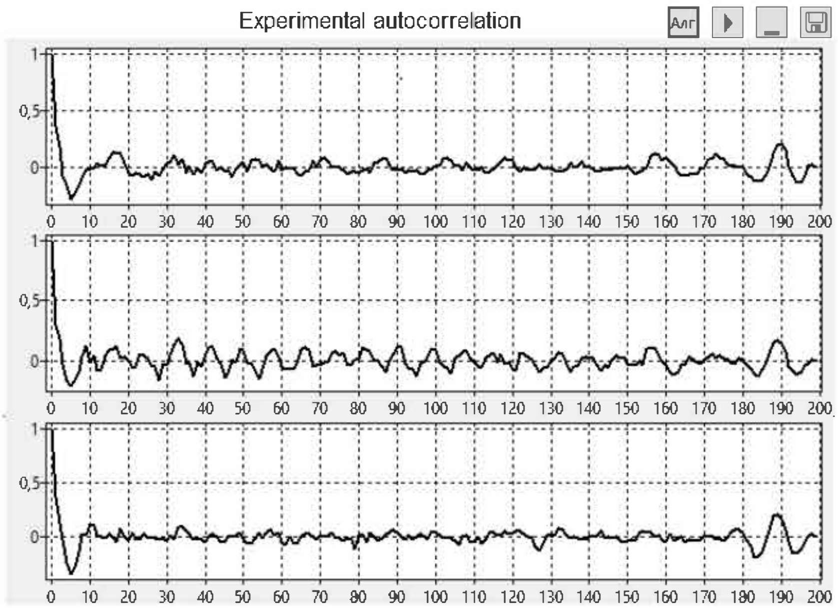

Fig 6. Generated experimental autocorrelation functions.

Graph visualization of errors between autocorrelation functions is shown at the Fig. 7.

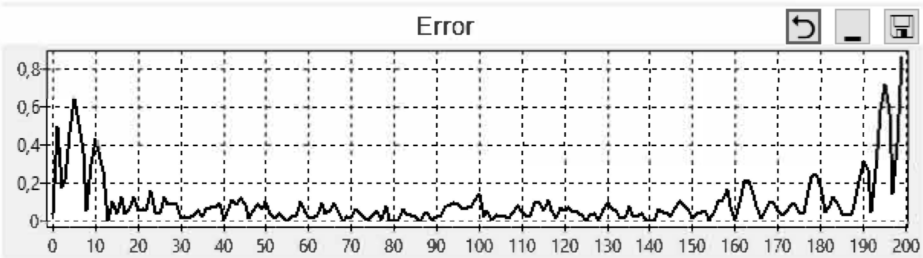

Fig 7. Error graph.

The table 1 contains mean errors calculated with different input characteristics and allows to estimate the quality of the algorithm for different spectral densities.

Table 1 – Errors' testing with different characteristics

Characteristics	Mean errors
Square spectral density method	
$\omega_R = 10\ \pi$; $\omega_L = 2{,}5\ \pi$; $T = 3{,}2$ s; $N = 16$; $n = 2$	0,03335
$\omega_R = 10\ \pi$; $\omega_L = 2{,}5\ \pi$; $T = 3{,}2$ s; $N = 57$; $n = 3$	0,03052
$\omega_R = 6\ \pi$; $\omega_L = 3{,}0\ \pi$; $T = 2{,}0$ s; $N = 51$; $n = 4$	0,04902
$\omega_R = 6\ \pi$; $\omega_L = 3{,}0\ \pi$; $T = 2{,}0$ s; $N = 50$; $n = 4$	0,01060
Right triangle spectral density method	
$\omega_R = 10\ \pi$; $\omega_L = 2{,}5\pi$; $T = 3{,}2$ s; $N = 32$; $n = 2$	0,03335

The acquired mean errors meet the requirements set at the beginning of the project. Factor analysis shown that separate input characteristics do not affect the mean errors directly which was expected from the random signal.

Conclusion

This paper starts new spectra theory development for high-dimensional signal simulation. First steps using two-dimensional simulation proofed new direction of research. The paper shows that the spectral theory provides new approaches of research based on matrix mathematical apparatus. The new algorithms could be prepared for further automatization by the means of tool developed in such areas as machine learning, artificial intelligence, and big data processing.

The method of two-dimensional simulation of signals in a complex basis reduces algorithmizing to the execution of pre-derived mathematical formulas, which reduces the computational complexity and resource intensity of the algorithm, and the use of linear data structures positively affect the scalability of the developed solution. The use of a complex basis that more accurately describes the nature of ongoing processes provides higher simulation accuracy.

The software solution implemented in the Lazarus environment in the Free Pascal language meets the requirements and allows generating deterministic and random signals, as well as evaluating the quality of simulation by displaying the error graph and/or displaying the average error number.

The simulation method in the Hartley basis and the software solution based on it, as in the case of the complex basis, make the algorithm less resource intensive. The software solution is implemented in Microsoft Visual Studio using the C# language. For both bases, both deterministic and random signals can be simulated (at the user's discretion), and the shape of the SPDF signal can be selected: rectangular and rectangular-triangular. The signals obtained meet the expectations for the spectrum, and when compared with theoretical and algorithmic ACF, they show error levels of less than 0.05. In future studies, it is planned to expand the choice of signal forms, as well as to test new methods on a wider range of tasks. It is also planned to create a library of obtained algorithms combined in a single solution that provides simulation in different bases. It is worth noting that formulas are easier to implement in low-level programming languages, which means that the methods listed in section 4 can be used in embedded and microprocessor technology.

Acknowledgments. This work is been financially supported by the Russian Federation Ministry of Science and Higher Education in the framework of the Research Project titled "Component's digital transformation methods' fundamental research for micro- and nanosystems" (Project #0705-2020-0041).

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