Identifying Noise Variables in Singular Decisions using Counterfactual Reasoning

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Abstract
In their book, *Noise: A Flaw in Human Judgment*, the authors Daniel Kanheman, Olivier Sibony and Cass R. Sunstein highlight the importance of minimizing bias, i.e. systematic deviation, and noise, i.e. variability, in judgments in order to reduce error. Bias has long been the subject of many discussions but noise is yet to gain the attention it deserves. In this paper, we discuss *noise variables* in decision-making, particularly in unique, non-recurrent or *singular decisions*. For this purpose we introduce and utilize the framework of the Weak Completion Semantics to discuss how noise variables may be identified using counterfactual reasoning.

Keywords
Weak Completion Semantics, Noise, Singular Decisions, Counterfactual Reasoning

1. Noise in Singular Decisions

Bias in decision-making is an idea familiar to many. Bias may be comprehensively defined as a *systematic deviation* to or from a certain decision or choice mainly owing to a person's psychological mechanism. Bias may be considered as an average of error in judgment. Although biases may be favourable in certain contexts, they may be quite unfavourable in some. Deciding on job applicants based on their racial profile, for example, is a social bias that is better avoided. Another example is overconfidence, which is an instance of cognitive bias. There are plenty of articles, talks, and books which have been dedicated to critical discussions on different kinds of biases and how judgement suffers (or not) from it.

A deviation from this norm, the book *Noise: A Flaw in Human Judgment* [1], authored by Daniel Kanheman, Olivier Sibony and Cass R. Sunstein considers another parameter which although often overlooked, plays an important role in errors - noise. According to the authors, *"to understand error in judgment, we must understand both bias and noise"*. Noise, unlike bias, is *variability* in error. It is not a psychological phenomenon but more of a statistical one, which arises from variability or differences across individuals. Put simply, noise can be characterized by different people making different judgments given the same situation. Thus noise has the general property of being recognizable without knowing the intended outcome of a situation or the bias involved. Noise results in variable judgments which are not systematic. For example, the

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same person being interviewed by different interviewers may be rated and assessed differently. This indicates variability and noise. People seeking asylum in the United States are admitted on a system resembling a lottery [2]. This indicates variability and noise. Decisions to grant patents are again variable and thus noisy [3].

When speaking of noise in decision-making, one needs to recognize two kinds of decisions: recurrent and singular. Recurrent decisions are those which we take on a repetitive basis. For example, deciding on the grades or performance of students appearing for an examination. Singular decisions are on the other hand unique and not repeated. For example, deciding on marriage with a particular person, or deciding whether to design a certain kind of mobile application can be considered as singular decisions. Obama's decision to send three thousand healthcare workers and soldiers to West Africa when the Ebola epidemic broke out in 2014 is yet another example of a historically prominent singular decision [1]. Obama had never encountered such a situation before and never had to take such a decision prior to that experience. Thus, he did not have a previously formulated and pre-packaged template of actions or choices he could draw from. Moreover, he did not have the scope to repeat this decision; it was a unique situation which was not recurrent.

Recurrent and singular decisions both suffer from noise and variability. It is however not very easy to identify noise in singular decisions, given their nature. While identifying the presence of noise in the exemplary situation above, one may imagine the following, in lines with [1]: would Obama have taken the same decision had he been surrounded by a different set of advisors? Or if his advisors had a different mindset? What if the situation and its gravity had been presented differently to the president? If we can imagine different alternative outcomes on tweaking these different parameters, we can sense the noise in the system. These parameters are therefore the *noise variables*. The authors of [1] suggest that noise in singular decisions could be identified using counterfactual reasoning. However, this point was not very elaborated upon. Thus, it is the goal of this paper to discuss an example of a singular decision, and using it as a prototypical model illustrate how counterfactual reasoning may be employed to recognize noise variables in the system. We use the Weak Completion Semantics (WCS) framework for our purposes.

The WCS is a formal three-valued, computational, non-monotonic cognitive theory. Till date it has been used to adequately model the suppression task by [4] as shown by [5], disjunctive reasoning as shown by [6], human syllogistic reasoning as shown by [7], and its belief bias as discussed in [8]. It has also been used to model the four inference tasks associated with reasoning about a conditional sentence, namely affirmation of the antecedent, affirmation of the consequent in [9], denial of the antecedent in [10] and denial of the consequent in [11].

Counterfactual reasoning, as the name suggest, is reasoning with imagined alternatives which are contrary to facts. That is, how people reflect on past events which have occurred, and imagine possibilities like, *"What if ... had happened? Would the outcome have been any different? What if ... had not happened?"*. People can imagine alternatives to something that has happened, by either deleting or undoing some aspect of it which had happened in the past reality, or by adding some new aspects to their simulation of reality [12]. To use an example from the field of explainable AI, discussed in [12], suppose a human tries to understand why an autonomous vehicle swerved into a wall to avoid hitting a pedestrian (thus injuring its passenger) and not hit the brakes instead. One might wonder, *"if the car detected the pedestrian earlier and braked, the passenger would not have been injured"* or *"if the car had not swerved* *towards the wall then the passenger would not have been injured"*. The former is called an *additive* counterfactual as it adds or considers new information for the reasoning process, and the latter is called *subtractive* because it deletes facts for the reasoning process.

Summing up, during decision-making or judgment it is advisable that both noise and bias within a system be addressed and minimized as much as possible. In order to reduce noise, one needs to identify it first. Our aim is to propose one such modelling method using the WCS to identify noise variables within a given system. The paper is thus organized as follows: In Section 2 we formally introduce the WCS. A classification of conditional sentences relevant to the goal of this paper is given in Section 3. An example of a singular decision is presented in Section 4. Identification of noise variables in this particular system using the WCS is demonstrated in Section 5. Finally, in Section 6 we conclude and outline further possible research.

2. The Weak Completion Semantics

We assume the reader to be familiar with logic and logic programming as presented in e.g. [13] and [14]. Let $\top$, $\bot$, and U be truth constants denoting *true*, *false*, and *unknown*, respectively. A *(logic) program* is a finite set of clauses of the form $B \leftarrow body$, where B is an atom (also called a $head$) and *body* is $\top$, or $\bot$, or a finite, non-empty conjunction of literals. Clauses of the form $B \leftarrow \top$, $B \leftarrow \bot$, and $B \leftarrow L_1, \ldots, L_n$ are called *facts*, *assumptions*, and *rules*, respectively, where L_i, $1 \leq i \leq n$, are literals. We restrict our attention to propositional programs although the WCS extends to first-order programs as well [15].

Throughout this paper, $\mathcal{P}$ will denote a program. An atom B is *defined* in $\mathcal{P}$ if and only if $\mathcal{P}$ contains a clause of the form $B \leftarrow body$. As an example consider the program

$$\mathcal{P} = \{C \leftarrow A \wedge \neg ab,\ ab \leftarrow \bot\},$$

where A, C, and ab are atoms. C and ab are defined, whereas A is undefined. ab is an abnormality predicate which is assumed to be false. In the WCS, this program represents the conditional sentence *if A then C*. In their everyday lives humans are often required to reason in situations where the information of all factors affecting the situation might not be complete. They still reason, unless new information which needs consideration comes to light. The abnormality predicate in the program serves the purpose of this (default) assumption, as was suggested in [16]. Let S be a consistent subset of literals occurring in P, i.e. S does not contain both an atom and its negation. Then, $defs(\mathcal{P},\ S) = \{B \leftarrow body \in \mathcal{P} \mid B \in S \text{ or } \neg B \in S\}$. E.g. let $\mathcal{P}_0 = \{a \leftarrow \top,\ c \leftarrow b,\ d \leftarrow \bot\}$ and $S = \{a,\ \neg c\}$. Then, $defs(\mathcal{P}_0,\ S) = \{a \leftarrow \top,\ c \leftarrow b\}$.

Let us now consider the following transformation: (1) For all defined atoms B occurring in a program $\mathcal{P}$, replace all clauses of the form $B \leftarrow body_1,\ B \leftarrow body_2,\ \ldots$ by $B \leftarrow body_1 \vee body_2 \vee \ldots$. (2) Replace all occurrences of $\leftarrow$ by $\leftrightarrow$. The resulting set of equivalences is called the *weak completion* of $\mathcal{P}$, denoted by $wc(\mathcal{P})$. It differs from the program completion defined in [17] in that undefined atoms in the weakly completed program are not mapped to false, but to unknown instead. Reconsidering the previous program $\mathcal{P}_0$, the reader may note that while the undefined atom b is mapped to false under the completion of $\mathcal{P}_0$, it is mapped to unknown under its weak completion. Weak completion is necessary for the WCS framework to adequately model the suppression task (and other reasoning tasks) as demonstrated in [5].

As shown in [18], each weakly completed program admits a least model under the three-

Table 1
The truth tables for the Łukasiewicz logic. As shown in the gray cells, $\mathsf{U} \leftarrow \mathsf{U} = \mathsf{U} \leftrightarrow \mathsf{U} = \top$.

F	$\neg F$
$\top$	$\bot$
$\bot$	$\top$
U	U

$\wedge$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	U	$\bot$
$\bot$	$\bot$	$\bot$	$\bot$

$\vee$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	$\top$	U	U
$\bot$	$\top$	U	$\bot$

$\leftarrow$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	U	$\top$	$\top$
$\bot$	$\bot$	U	$\top$

$\leftrightarrow$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	$\top$	U
$\bot$	$\bot$	U	$\top$

valued Łukasiewicz logic [19] (see Table 1). This model will be denoted by $\mathcal{M}_{wc(\mathcal{P})}$. It can be computed as the least fixed point of a semantic operator introduced in [20]. Let $\mathcal{P}$ be a program and I be a three-valued interpretation represented by the pair $\langle I^\top, I^\bot \rangle$, where $I^\top$ and $I^\bot$ are the sets of atoms mapped to true and false by I, respectively, and atoms which are not listed are mapped to unknown. We define $\Phi_\mathcal{P}\, I = \langle J^\top, J^\bot \rangle$,[1] where

$$J^\top = \{B \mid \text{there exists } B \leftarrow body \in \mathcal{P} \text{ and } I\ body = \top\},$$
$$J^\bot = \{B \mid \text{there exists } B \leftarrow body \in \mathcal{P} \text{ and for all } B \leftarrow body \in \mathcal{P} \text{ one finds } I\ body = \bot\}.$$

Following [21] we consider an *abductive framework* $\langle \mathcal{P}, \mathcal{A}_\mathcal{P}, \mathcal{IC}, \models_{wcs} \rangle$, where $\mathcal{P}$ is a program, $\mathcal{A}_\mathcal{P} = \{B \leftarrow \top \mid B \text{ is undefined in } \mathcal{P}\} \cup \{B \leftarrow \bot \mid B \text{ is undefined in } \mathcal{P}\}$ is the *set of abducibles*, $\mathcal{IC}$ is a finite set of *integrity constraints* of the form $\bot \leftarrow body$ or $\mathsf{U} \leftarrow body$,[2] and $\mathcal{M}_{wc(\mathcal{P})} \models_{wcs} F$ iff $\mathcal{M}_{wc(\mathcal{P})}$ maps the formula F to true. Let $\mathcal{O}$ be an *observation*, i.e., a finite set of literals each of which does not follow from $\mathcal{M}_{wc(\mathcal{P})}$. We apply abduction to explain $\mathcal{O}$, where $\mathcal{O}$ is called *explainable* in the abductive framework $\langle \mathcal{P}, \mathcal{A}_\mathcal{P}, \mathcal{IC}, \models_{wcs} \rangle$ if and only if there exists a non-empty $\mathcal{X} \subseteq \mathcal{A}_\mathcal{P}$ called an *explanation* such that $\mathcal{M}_{wc(\mathcal{P}\cup\mathcal{X})} \models_{wcs} L$ for all $L \in \mathcal{O}$ and $\mathcal{M}_{wc(\mathcal{P}\cup\mathcal{X})}$ satisfies $\mathcal{IC}$. We have assumed that the set of explanations is non-empty as otherwise the observation already follows from the weak completion of the program. Formula F *follows credulously* from $\mathcal{P}$ and $\mathcal{O}$ if and only if there exists an explanation $\mathcal{X}$ for $\mathcal{O}$ such that $\mathcal{M}_{wc(\mathcal{P}\cup\mathcal{X})} \models_{wcs} F$. F *follows skeptically* from $\mathcal{P}$ and $\mathcal{O}$ if and only if $\mathcal{O}$ can be explained and for all explanations $\mathcal{X}$ for $\mathcal{O}$ we find $\mathcal{M}_{wc(\mathcal{P}\cup\mathcal{X})} \models_{wcs} F$. The latter is an application of the so-called *Gricean implicature* [22]: humans normally do not quantify over things which do not exist. Meaning, (unlike classical logic) *all* explanations for an observation $\mathcal{O}$ may only be taken into account to skeptically decide on a formula F, when $\mathcal{O}$ is explainable and these so-called explanations exist in the first place. If a formula F does not follow skeptically from $\mathcal{P}$ and $\mathcal{O}$, we conclude that *nothing follows*. Furthermore, one should also observe that if an observation $\mathcal{O}$ cannot be explained, then *nothing follows* credulously as well as skeptically. In all examples discussed in this paper the set of integrity constraints is empty; they are not relevant to the goal of this paper. However they are needed in other applications of the WCS like human disjunctive reasoning [6]. For the purposes of our current goal we utilize a so-called revision operator rev, defined in [23]. Let us consider a consistent set of literals S, whose elements in the general case may be mapped to false under the least model of $wc(\mathcal{P})$. We then define the revision of $\mathcal{P}$ with respect to S as, $rev(\mathcal{P}, S) = (\mathcal{P} \setminus \mathit{defs}(\mathcal{P},\ S)) \cup \{A \leftarrow \top \mid A \in S\} \cup \{A \leftarrow \bot \mid \neg A \in S\}$.

[1]Whenever we apply a unary operator like $\Phi_\mathcal{P}$ to an argument like I, we omit the parenthesis and write $\Phi_\mathcal{P}\, I$ instead. Likewise, we write $I\ body$ instead of $I(body)$.

[2]Please note that assumptions like $A \leftarrow \bot$ are weakly completed. Their truth value is propagated via Φ. Moreover, they can be overridden by another clause of the form $A \leftarrow body$. On the other hand, the constraint $\bot \leftarrow A$ cannot be part of the program. It's only purpose is to eliminate models, where A is true.

Overall, given premises, general knowledge, and observations, *reasoning in the WCS* is modelled in six steps:

1. Reasoning towards a logic program $\mathcal{P}$ following [20].
2. Weakly completing the program, leading to $wc(\mathcal{P})$.
3. Computing the least model $\mathcal{M}_{wc(\mathcal{P})}$ of $wc(\mathcal{P})$ under the three-valued Łukasiewicz logic.
4. Reasoning with respect to $\mathcal{M}_{wc(\mathcal{P})}$.
5. If observations cannot be explained, then applying skeptical abduction.
6. Searching for counterexamples [10].

3. A Classification of Conditional Sentences

In this section we introduce a classification of conditionals and their antecedents which we consider relevant for the identification of noise variables in a system. Pragmatics, life experiences and cultural differences play a deciding role in how different individuals comprehend or classify both. Following [24] we identify two types of conditional sentences: *obligational* and *factual*. We call a conditional sentence obligational if the truth of the consequent appears to be obligatory when given that the antecedent is true. For each obligational conditional there are two initial possibilities humans comprehending the conditional think about. The first possibility is the conjunction of the antecedent and the consequent, which is permitted in the viewpoint of the individual. The second possibility is the conjunction of the antecedent and the negation of the consequent, which is forbidden. Exceptions are possible but unlikely. Let us consider for example, *if Maria drinks alcoholic beverages in pubs then she must be over 19 years of age*. In many countries the law demands that a person may only drink alcohol publicly when they are above a certain age group (e.g. 19 years). This implies that for an individual from such a background, *Maria is drinking alcoholic beverages in a pub* and *she is older than 19 years* is a permitted possibility, whereas *Maria is drinking alcoholic beverages in a pub* and *she is not older than 19 years* is a forbidden one. Hence, this particular conditional is an obligational one. As another example consider, *if plants get water then they grow*. Here, *plants getting water* and *plants growing* is a permitted possibility. But as many plant enthusiasts would know, *plants getting water* and *plants not growing* is also possible. There are many factors, such as lack of light, overwatering, pest infestation, etc. which may hinder a plant's growth. Conditionals such as these, where the (truth of the) consequent is not obligatory given the antecedent, we call factual conditionals. In particular, in such a case the truth of the antecedent is deemed *inconsequential* to that of the consequent by the individual in question.

After the above discussion a question that may naturally arise is, what happens when the antecedent of a conditional sentence is not satisfied? To that end, the antecedent A of a conditional sentence *if* A *then* C can be classified as *necessary* with respect to the consequent C, if and only if C cannot be true *unless* A is true. This implies that if A does not hold, C cannot either. For example, in case of the conditional *if plants get water then they grow*, the antecedent *plants get water* is a necessary one for the consequent *plants will grow*. If a plant is not watered at all, it will very likely die. This does not imply however, that the antecedent need always be a precondition for the consequent, per se. On the other hand, the antecedent A of a conditional sentence *if* A *then* C is said to be *non-necessary* with respect to the consequent C, if C can be true despite the falsity of A. This implies, if A does not hold, C may or may not hold. In the conditional, *if Maria drinks alcoholic beverages in pubs then she must be over 19 years of age*, the falsity of *drinking alcoholic beverages in a pub* is inconsequential to the truth of the consequent *older than 19 years*. There are plenty of adults (over 19 years) who do not drink alcohol. The antecedent of the conditional sentence is therefore called non-necessary.

In order to account for the aforementioned classifications in the WCS, given a clause $\{C \leftarrow A \wedge \neg ab\}$ in a program $\mathcal{P}$, the previous definition of the set of abducibles $\mathcal{A}_{\mathcal{P}}$ (see Section 2) can be extended to include $\mathcal{A}_{\mathcal{P}}^{nn} = \{C \leftarrow \top\}$ when the antecedent is deemed non-necessary, and to include $\mathcal{A}_{\mathcal{P}}^{f} = \{ab \leftarrow \top\}$ when the conditional is deemed factual. An interested reader may find this discussed in a lot more detail in [10].

4. A Singular Decision: Dr. Snow and the Broad Street Pump

As an example of a singular decision let us consider the following excerpt from the life of the father of epidemiology, Dr. John Snow, synthesized from [25, 26].

The human race has been threatened by a multitude of diseases, among which Cholera alone has claimed the lives of many over the years. Its patients suffer from mild to severe symptoms, the latter including intense diarrhoea and dehydration which may cause the person's skin to turn bluish-grey and even be life-threatening. This symptom gives Cholera its colloquial nickname, "the blue death". Although Cholera still poses a problem in certain parts of the world, sewage and water treatment systems have helped curb it in many parts of the globe. This was not however always the case. England for example suffered high Cholera outbreaks in 1831, 1848 and 1854. No effective treatment had yet been in sight, and people did not yet have the knowledge that bacteria transmitted the disease. It was a common (mis)conception that Cholera spread through "miasmas" or toxic gases from open graves, garbage dumps, sewers etc.

During the Cholera outbreak in 1848, Dr. John Snow, who had received his medical degree in London was a practising physician there. The outbreak led Snow to examine many Cholera patients and become heavily involved in the study of the disease. It is also noteworthy that during the 1831 outbreak, young Snow who aimed to be a physician had been working as an apprentice to a physician who treated Cholera patients. These experiences ultimately led Snow to publish a book on specific conclusions he had reached about the nature of the disease which was unfortunately rejected by most of his colleagues in the medical community. This did not discourage Snow however, and in 1854 when Cholera broke out once again Snow conducted his own investigation. His investigations showed that most death cases were approximately concentrated around a hand pump in Broad Street. Furthermore, numerous (fruitful) interviews with the families of the deceased indicated that the victims had consumed water from the aforementioned hand pump. Convinced that the hand pump's contaminated water played an important role in the epidemic, Snow appeared for an interview with the Board of Guardians of the St. James parish. The board paid heed to Snow's advice and had the hand pump removed the following day. This helped control the epidemic to a large extent, the repercussions of which echoed through time.

The decision taken by the Board of Guardians to remove the Broad Street pump was a singular one because it was a unique decision with unique circumstances that was never repeated. This singular decision, however, was not entirely free from noise. In other words, had certain

Table 2
Background conditional statements and their corresponding program clauses.

	Conditional Statements	Program Clauses
(1)	Because Dr. Snow had an interview with the Board of Guardians, the board removed the Broad Street hand pump.	$rem \leftarrow int \wedge \neg ab_{int}$, $ab_{int} \leftarrow \bot$
(2)	Because Dr. Snow was convinced of the Broad Street pump playing an important role in the epidemic, he had an interview with the Board of Guardians.	$int \leftarrow conv \wedge \neg ab_{conv}$, $ab_{conv} \leftarrow \bot$
(3)	Because Dr. Snow had previous experience with Cholera patients, he was convinced of the Broad Street pump playing an important role in the epidemic.	$conv \leftarrow exp \wedge \neg ab_{exp}$, $ab_{exp} \leftarrow \bot$
(4)	Because Dr. Snow's investigations were fruitful, he was convinced of the Broad Street pump playing an important role in the epidemic.	$conv \leftarrow inv \wedge \neg ab_{inv}$, $ab_{inv} \leftarrow \bot$
(5)	Because Dr. Snow was practising in London during the 1848 outbreak, he had previous experience with Cholera patients.	$exp \leftarrow prac \wedge \neg ab_{prac}$, $ab_{prac} \leftarrow \bot$
(6)	Because Dr. Snow had been an apprentice during the 1831 outbreak, he had previous experience with Cholera patients.	$exp \leftarrow app \wedge \neg ab_{app}$, $ab_{app} \leftarrow \bot$
(7)	Because John received his medical degree in London, he was practising there during the 1848 outbreak.	$prac \leftarrow deg \wedge \neg ab_{deg}$, $ab_{deg} \leftarrow \bot$

parameters in the background story been changed, the singular decision might have had a different outcome. Identification of the noise, or more specifically the noise variables can be brought about by reasoning counterfactually about the singular decision. How this can be modelled using the WCS framework will be discussed in the next section.

5. Identifying Noise Variables using Counterfactual Reasoning

5.1. Representing Background Knowledge using Causal Conditionals

Table 2 lists the various conditional statements which may be used to comprehensively summarize the background knowledge presented in Section 4. For the convenience of the reader it also includes the corresponding clauses in a logic program representing the said conditionals, which shall be further used during the modelling of noise variables in the next sub-section. The reader may observe that the abnormality predicate in each clause has been assumed to be false. This may be overridden in the subsequent discussion when we take the classification of the conditionals and their antecedents into account while searching for noise variables.

5.2. Modelling Noise Variables using the Weak Completion Semantics

The idea behind the approach that we demonstrate in this paper is to pick up the thread of the singular decision or the first statement, counterfactually gauge which antecedent could have altered the consequent, which in turn hints at the former being a noise variable. Then, go further back in the chain of events to explore the rules of inference which led to the aforementioned antecedent, thus repeating the exercise.

Beginning with statement (1) in Table 2, upon considering the nature of the antecedent as discussed in Section 3, one may deem the antecedent to have been necessary for the consequent. In other words, one may not easily imagine a possibility where *Snow had no interview with the* *board* yet *the board decided to remove the hand pump*. Had Snow not approached the board for an interview, the very idea of removing the hand pump may have slipped their attention. The logic program $\mathcal{P}_1$ which may be constructed from statement (1) consists of the following:

$$\{rem \leftarrow int \wedge \neg ab_{int},\ ab_{int} \leftarrow \bot,\ int \leftarrow \top\},$$

where rem denotes *the board removed the hand pump* and int denotes *Snow had an interview with the board.* ab_{int} is an abnormality predicate denoting anything that went wrong with regard to the interview, e.g. one of the board members fell sick on the spot and the meeting was adjourned. As there was no such case it is assumed to be false. The last clause represents a fact. The weak completion of $\mathcal{P}_1$, $wc(\mathcal{P}_1)$, is

$$\{rem \leftrightarrow int \wedge \neg ab_{int},\ ab_{int} \leftrightarrow \bot,\ int \leftrightarrow \top\},$$

which has the least model $\mathcal{M}_{wc(\mathcal{P}_1)} = \langle\{int, rem\},\ \{ab_{int}\}\rangle$.[3] This signifies int and rem are true, while ab_{int} is false. When reasoning with the situation counterfactually, one may use the conditional *if Dr. Snow did not have an interview with the board, then the board would not have removed the hand pump* or, *if* $\neg int$ *then* $\neg rem$. Upon evaluating $\neg int$ under the aforementioned least model, the reader may find that it is false. Hence we call the conditional a subtractive counterfactual conditional, in lines with [12]. Revising $\mathcal{P}_1$ with respect to $\{\neg int\}$ using the revision operator rev (see Section 2) leads us to the revised program $rev(\mathcal{P}_1, \{\neg int\}) = (\mathcal{P}_1 \setminus \{int \leftarrow \top\}) \cup \{int \leftarrow \bot\}$. Its weak completion,

$$\{rem \leftrightarrow int \wedge \neg ab_{int},\ ab_{int} \leftrightarrow \bot, int \leftrightarrow \bot\}$$

admits the least model $\langle\emptyset, \{int, rem, ab_{int}\}\rangle$. In particular, rem which was true is now false. Thus, the variability in the truth of the consequent leads to variability in the outcome, which indicates noise. Consequently, int is a noise variable.

Now we further explore the atom int and hence statement (2). Considering that the antecedent may be deemed necessary for the consequent, any possibility that *Snow himself was not convinced that the hand pump should be removed* yet *had an interview with the board about the same* may be discounted. The logic program $\mathcal{P}_2$ constructed from the statements (1) and (2) is:

$$\{rem \leftarrow int \wedge \neg ab_{int},\ ab_{int} \leftarrow \bot,\ int \leftarrow conv \wedge \neg ab_{conv},\ ab_{conv} \leftarrow \bot,\ conv \leftarrow \top\},$$

where $conv$ denotes *Snow was convinced about the hand pump*, and ab_{conv} is an abnormality predicate assumed to be false. The last clause represents a fact. $wc(\mathcal{P}_2)$ admits the least model $\mathcal{M}_{wc(\mathcal{P}_2)} = \langle\{conv,\ int,\ rem\}, \{ab_{conv},\ ab_{int}\}\rangle$. Reasoning counterfactually, one may imagine the conditional *if John had not been convinced then he would not have asked for an interview* viz. *if* $\neg conv$ *then* $\neg int$. As $\neg conv$ evaluates to false under the aforementioned least model, this is a subtractive counterfactual conditional. Then, $rev(\mathcal{P}_2, \{\neg conv\})$ leads to $(\mathcal{P}_2 \setminus \{conv \leftarrow \top\}) \cup \{conv \leftarrow \bot\}$. Its weak completion admits the least model $\langle\emptyset, \{conv, int, rem, ab_{conv}, ab_{int}\}\rangle$. This indicates that $conv$ is a noise variable.

Going further back on the chain of events, we now consider the statements (1) to (4). In case of statements (3) and (4), both antecedents of the conditionals may be considered necessary for the consequent. Hence, in such a case the logic program $\mathcal{P}_3$ is:

[3]This model can be computed as follows. Starting with the interpretation $\langle\emptyset, \emptyset\rangle$, we obtain $\Phi_{\mathcal{P}_1}\langle\emptyset, \emptyset\rangle = \langle\{int\}, \{ab_{int}\}\rangle$ and $\Phi_{\mathcal{P}_1}\langle\{int\}, \{ab_{int}\}\rangle = \langle\{int,\ rem\}, \{ab_{int}\}\rangle = \Phi_{\mathcal{P}_1}\langle\{int,\ rem\}, \{ab_{int}\}\rangle$.

$$\{rem \leftarrow int \wedge \neg ab_{int},\ ab_{int} \leftarrow \bot,\ int \leftarrow conv \wedge \neg ab_{conv},\ ab_{conv} \leftarrow \bot,$$
$$conv \leftarrow exp \wedge \neg ab_{exp},\ conv \leftarrow inv \wedge \neg ab_{inv},$$
$$ab_{exp} \leftarrow \bot,\ ab_{exp} \leftarrow \neg inv,\ ab_{inv} \leftarrow \bot,\ ab_{inv} \leftarrow \neg exp,\ exp \leftarrow \top,\ inv \leftarrow \top\},$$

where exp denotes *Snow had a lot of experience with Cholera patients* and inv denotes *Snow's investigations proved fruitful.* ab_{exp} and ab_{inv} are abnormality predicates. The last two clauses represent facts. One may observe that we have now added the clauses $ab_{exp} \leftarrow \neg inv$ and $ab_{inv} \leftarrow \neg exp$ and we attempt to clarify this in what follows. As the antecedents of both the conditionals *if* exp *then* $conv$ and *if* inv *then* $conv$ have been deemed necessary for the consequent, the possibility that one of the antecedents is false but the consequent is true is discounted. *Not having enough experience with Cholera patients could have prevented Snow from being convinced about the removal of the hand pump even if his investigations had proved fruitful.* Likewise, *an unproductive investigation could have prevented Snow from being convinced about the pump, despite him having enough experience.* Thus given the two conditionals *if* exp *then* $conv$ and *if* inv *then* $conv$, where both the antecedents are deemed to be necessary, we characterize $\neg inv$ as an abnormality with respect to the former conditional and $\neg exp$ as an abnormality with respect to the latter. Hence the additional clauses $ab_{exp} \leftarrow \neg inv$ and $ab_{inv} \leftarrow \neg exp$. Such a characterization is along the lines of so-called *enabling* relations as discussed in [27]. Using this we have also modelled experiments involving additional arguments in the suppression task [4, 5]. The reader may note that in $wc(\mathcal{P}_3)$, the assumptions $ab_{inv} \leftarrow \bot$ and $ab_{exp} \leftarrow \bot$ are overridden by $ab_{inv} \leftarrow \neg exp$ and $ab_{exp} \leftarrow \neg inv$, respectively. $wc(\mathcal{P}_3)$ has the least model $\mathcal{M}_{wc(\mathcal{P}_3)} = \langle\{exp,\ inv,\ conv,\ int,\ rem\}, \{ab_{exp},\ ab_{inv},\ ab_{conv},\ ab_{int}\}\rangle$ where inv and exp are true, and so is rem. In particular no least models of $wc(\mathcal{P}_3)$ where inv, exp or both are false but $conv$ is true are constructed. Reasoning with the situation counterfactually now gives rise to the conditionals, *if Snow had no experience with Cholera patients then he would not have been convinced*, viz. *if* $\neg exp$ *then* $\neg conv$, and *if his investigations were unsuccessful then he would not have been convinced* viz. *if* $\neg inv$ *then* $\neg conv$. Upon evaluation of $\neg exp$ and $\neg inv$ with respect to $\mathcal{M}_{wc(\mathcal{P}_3)}$ the reader may find that they are both false. Hence these are subtractive counterfactual conditionals. So, $rev(\mathcal{P}_3, \{\neg exp,\ \neg inv\})$ results in the program $(\mathcal{P}_3 \setminus \{exp \leftarrow \top,\ inv \leftarrow \top\}) \cup \{exp \leftarrow \bot,\ inv \leftarrow \bot\}$. Its weak completion now admits a least model where inv and exp are false, and (now) so is rem. Thus the variability in the truth of rem indicates that inv and exp are noise variables.

Let us now consider statements (1) to (6), and in particular statements (5) and (6). While practising in London during 1848 may be deemed necessary for the level of experience Snow had with Cholera patients, his apprenticeship during 1831 may be deemed non-necessary by some individuals. That is while one may not readily imagine a possibility where *Snow did not practise in London, but had experience with Cholera patients*, one may imagine *Snow not doing the apprenticeship yet having experience.* The reader is pointed out that the opposite may also hold for some individuals where they may consider apprenticeship to be necessary but the London practice to be non-necessary. Or some individuals may even consider both antecedents to be non-necessary for the consequent. For the sake of modelling and demonstration purposes, we assume the first case. So, we consider the conditional *if* app *then* exp, where app is deemed non-necessary for exp. And consider *if* $prac$ *then* exp, where $prac$ is deemed necessary for exp, owing to which we characterize $\neg prac$ as an abnormality with respect to the former conditional.[4] Thus, the program $\mathcal{P}_4$ has the clauses:

$$\{rem \leftarrow int \wedge \neg ab_{int},\ ab_{int} \leftarrow \bot,\ int \leftarrow conv \wedge \neg ab_{conv},\ ab_{conv} \leftarrow \bot,$$
$$conv \leftarrow exp \wedge \neg ab_{exp},\ ab_{exp} \leftarrow \bot,\ ab_{exp} \leftarrow \neg inv,$$
$$conv \leftarrow inv \wedge \neg ab_{inv},\ ab_{inv} \leftarrow \bot,\ ab_{inv} \leftarrow \neg exp,$$
$$exp \leftarrow prac \wedge \neg ab_{prac},\ ab_{prac} \leftarrow \bot,$$
$$exp \leftarrow app \wedge \neg ab_{app},\ ab_{app} \leftarrow \bot,\ ab_{app} \leftarrow \neg prac,$$
$$inv \leftarrow \top,\ prac \leftarrow \top,\ app \leftarrow \top\},$$

where $prac$ denotes *Snow was practising in London during the 1848 outbreak* and app denotes *Snow was an apprentice for a physician during the 1831 outbreak.* ab_{prac} and ab_{app} are abnormality predicates. The last three clauses represent facts. The reader may note that we now have the additional clause $ab_{app} \leftarrow \neg prac$, and that the assumption $ab_{app} \leftarrow \bot$ is overridden by $ab_{app} \leftarrow \neg prac$ in $wc(\mathcal{P}_4)$. The least model of $wc(\mathcal{P}_4)$ is:
$\mathcal{M}_{wc(\mathcal{P}_4)} = \langle\{inv,\ prac,\ app,\ exp,\ conv,\ int,\ rem\}, \{ab_{prac},\ ab_{app},\ ab_{inv},\ ab_{exp},\ ab_{conv},\ ab_{int}\}\rangle$.
Here, app and $prac$ are both true and so is rem. Because $prac$ has been deemed necessary for exp, irrespective of whether he worked as an apprentice or not, as long as Snow did not practice in London in 1848 he would not have the level of experience. Reasoning counterfactually about the situation we may thus imagine, *if Snow would not have been practising in London then he would not have the experience* viz. *if* $\neg prac$ *then* $\neg exp$. Upon evaluating $\neg prac$ with respect to $\mathcal{M}_{wc(\mathcal{P}_4)}$, we find that it is false. Hence, this is a subtractive counterfactual conditional. Therefore revision using $rev(\mathcal{P}_4, \{\neg prac\})$ leads us to the program $(\mathcal{P}_4 \setminus \{prac \leftarrow \top\}) \cup \{prac \leftarrow \bot\}$. Its weak completion admits a least model where $prac$ is false, and so is rem. The variability in the truth of rem indicates that $prac$ is a noise variable.

Now, reconsidering statements (5) and (6) from a different angle, we might ask ourselves the following - *"if Snow was a physician's apprentice in 1831 does it necessarily mean that he would have a high level of experience with Cholera patients?"* or that *"if Snow practised in London in 1848, does it necessarily that he would have a high level of experience with Cholera patients?"*. In other words, we may imagine a possibility where *Snow did practise in London in 1848 but due to some (additional) reasons he could not have a high level of experience with Cholera patients.* Or that, *he was an apprentice but certain reasons hindered his opportunity to have the needed experience.* As discussed in Section 3, imagining such (alternative) possibilities count for statements (5) and (6) being comprehended as *factual.* Meaning in such a case, given *if* A *then* C and *affirming* A, both C and $\neg C$ are deemed possible. For the current moment, we are particularly interested in this latter possibility. Within the WCS it can be modelled by considering A as an observation and applying abduction in order to explain it using the set of abducibles for factual conditionals, $\mathcal{A}^f_{\mathcal{P}}$, mentioned in Section 3. We attempt to clarify the process in the current context in what follows. We consider the program $\mathcal{P}_5 = \mathcal{P}_4 \setminus \{prac \leftarrow \top,\ app \leftarrow \top\}$, and consider $prac$ and app to be observations instead, meaning $\mathcal{O} = \{prac,\ app\}$. Now we apply abduction in order to explain $\mathcal{O}$. Following the definition of abducibles as described in Section 2, since both $prac$ and

[4]This is in lines with the prior discussion regarding statement (3) and (4).

app are undefined in $\mathcal{P}_5$, $\mathcal{A}_{\mathcal{P}_5} = \{prac \leftarrow \top,\ prac \leftarrow \bot,\ app \leftarrow \top,\ app \leftarrow \bot\}$. Moreover, comprehending statements (5) and (6) as factual entails extending the set of abducibles that can be derived from $\mathcal{P}_5$ to $\mathcal{A}^e_{\mathcal{P}_5} = \mathcal{A}_{\mathcal{P}_5} \cup \mathcal{A}^f_{\mathcal{P}_5}$, where $\mathcal{A}^f_{\mathcal{P}_5}$ includes $\{ab_{app} \leftarrow \top,\ ab_{prac} \leftarrow \top\}$. While there is a minimal explanation for $\mathcal{O}$, viz. $\{prac \leftarrow \top,\ app \leftarrow \top\}$, there is also a non-minimal explanation viz. $\{prac \leftarrow \top,\ ab_{prac} \leftarrow \top,\ app \leftarrow \top,\ ab_{app} \leftarrow \top\}$. Adding the former to $\mathcal{P}_5$ would again result in the program $\mathcal{P}_4$. However, the latter results in a program $\mathcal{P}_5{}'$:

$$\{rem \leftarrow int \wedge \neg ab_{int},\ ab_{int} \leftarrow \bot,\ int \leftarrow conv \wedge \neg ab_{conv},\ ab_{conv} \leftarrow \bot,$$
$$conv \leftarrow exp \wedge \neg ab_{exp},\ ab_{exp} \leftarrow \bot,\ ab_{exp} \leftarrow \neg inv,$$
$$conv \leftarrow inv \wedge \neg ab_{inv},\ ab_{inv} \leftarrow \bot,\ ab_{inv} \leftarrow \neg exp,$$
$$exp \leftarrow prac \wedge \neg ab_{prac},\ ab_{prac} \leftarrow \bot,\ ab_{prac} \leftarrow \top,$$
$$exp \leftarrow app \wedge \neg ab_{app},\ ab_{app} \leftarrow \bot,\ ab_{app} \leftarrow \neg prac,\ ab_{app} \leftarrow \top,$$
$$inv \leftarrow \top,\ prac \leftarrow \top,\ app \leftarrow \top\}.$$

Its weak completion, $wc(\mathcal{P}_5{}')$, admits the least model:
$\mathcal{M}_{wc(\mathcal{P}_5{}')} = \langle\{app,\ prac,\ inv,\ ab_{app},\ ab_{prac},\ ab_{inv}\}, \{exp,\ ab_{exp},\ conv,\ ab_{conv},\ int,\ ab_{int},\ rem\}\rangle$, where app, $prac$, ab_{app} and ab_{prac} are true, but exp is false. The reader is pointed out that the additional clauses $\{ab_{prac} \leftarrow \top\}$ and $\{ab_{app} \leftarrow \top\}$ signify that there could be (other additional) reasons which could have hindered Snow's experience with Cholera patients. This is in line with additive counterfactuals as discussed in [12]. In other words, reasoning counterfactually one may thus use statements such as *if something abnormal had happened with respect to his apprenticeship, then John would not have the experience*, i.e. *if* ab_{app} *then* $\neg exp$ or, *if something abnormal had happened with respect to his practice, then John would not have the experience*, i.e. *if* ab_{prac} *then* $\neg exp$. Information here is not taken away like in case of (the previous) subtractive counterfactual statements, but rather added to the simulation of reality. As rem is false in $\mathcal{M}_{wc(\mathcal{P}_5{}')}$, it signifies variability in the outcome. Thus ab_{app} and ab_{prac} are noise variables which could be explored further.

The case of non-necessary antecedents begs a lengthier discussion than the current spatial constraints of the paper would allow. Given *if* A *then* C, comprehending A to be non-necessary for C signifies that in case of $\neg A$, both $\neg C$ and particularly C are deemed possible. This implies two least models, one in which C is false, and the other where C is true. Both can be computed by employing the abductive framework using the extended set of abducibles $\mathcal{A}^{nn}_{\mathcal{P}}$ mentioned in Section 3, as discussed in detail in [10]. However, the implications of dealing with these multiple least models when identifying noise variables need further analysis. For now, as a small example let us consider statement (6) and explore the atom deg. Thus, we consider the program $\mathcal{P}_6 = (\mathcal{P}_4 \setminus \{prac \leftarrow \top\}) \cup \{prac \leftarrow deg \wedge \neg ab_{deg},\ ab_{deg} \leftarrow \bot\}$. In statement (6), the antecedent may be deemed non-necessary for the consequent. Meaning, considering the question, *"if John had not received his medical degree in London, could he (yet) be practising there during the 1848 outbreak?"*, one may deem it possible for *John to not have received his medical degree in London* but *to have been practising there*. This can be modelled within the WCS using the aforementioned abductive framework as follows. Supposing $\mathcal{O} = \{\neg deg\}$ to be an observation for which we look for an explanation, leads us to apply abduction. Since deg is undefined in $\mathcal{P}_6$, $\mathcal{A}_{\mathcal{P}_6}$ includes $\{deg \leftarrow \top,\ deg \leftarrow \bot\}$. Furthermore as deg here is a non-necessary antecedent,

$\mathcal{A}^e_{\mathcal{P}_6} = \mathcal{A}_{\mathcal{P}_6} \cup \mathcal{A}^{nn}_{\mathcal{P}_6}$ where $\mathcal{A}^{nn}_{\mathcal{P}_6}$ includes $\{\, prac \leftarrow \top\}$. While there is a minimal explanation to $\mathcal{O}$ viz. $\{deg \leftarrow \bot\}$, there is also a non-minimal explanation, viz. $\{deg \leftarrow \bot,\ prac \leftarrow \top\}$. The latter results in the revised program $\mathcal{P}_6 \cup \{deg \leftarrow \bot,\ prac \leftarrow \top\}$. Its weak completion admits a least model where we find that $prac$ is true, although deg is false. Summing up in simpler words, there could be some reason due to which John practised in London (i.e. $prac$ could be true) even if he had not received his medical degree there (i.e. deg was false). Such an exercise in turn motivates further questions along the lines - *"what if this particular reason had not occurred?"*. This so-called reason hints at being a noise variable and could be explored further.

In all that has been demonstrated so far, we have attempted to illustrate the identification of some of the noise variables in our system such as, int, exp, $conv$, inv, $prac$, ab_{app}, ab_{prac} etc. Thus we have identified the system noise. At this point it must also be acknowledged that while there are variables which may contribute to noise, there may also be those which do not. One possible means to identify the latter could be to guage the *relevance* of the antecedent of a conditional with respect to the consequent, in line with [28]. For example, consider $\mathcal{P}_7 = \mathcal{P}_6 \cup \{deg \leftarrow \top,\ wint \leftarrow \top\}$, where $wint \leftarrow \top$ represents the fact that *the winter of 1848 was particularly harsh*, and a conditional, *if Snow received his medical degree from London then the winter of 1848 was particularly harsh* viz. *if* deg *then* $wint$. In the common knowledge of an individual, deg is very likely not relevant to $wint$. This may be modelled following [29], using the notion of the so-called *strong relevance*, the core idea of which is to check whether $wint$ loses support as soon as the support of deg is withdrawn.[5] The reader may observe that both deg and $wint$ are true in $\mathcal{M}_{\mathcal{P}_7}$. Now, removing the support of deg from $\mathcal{P}_7$ which leads to $\mathcal{P}_7{}' = \mathcal{P} \setminus \{deg \leftarrow \top\}$, we still find that $wint$ is true in $\mathcal{M}_{\mathcal{P}_7{}'}$. Hence, in this case we may conclude that deg is not strongly relevant to $wint$. The said individual may thus ask - *"if Snow had not received his medical degree from London, would the winter of 1848 still be harsh?"*. And the answer may well be *yes*. With this remark we cease the discussion about variables that do not add to noise, as it requires further research and is best reserved for another occasion.

6. Conclusion

Noise just like bias, may be undesirable in much of our decision-making and judgment. As the human race journeys further into the age of digitalization and artificial intelligence becomes more and more involved in our daily lives, creating systems which minimize bias and noise, both of which contribute to errors in judgement seems important. In order to create any kind of AI system with decision-making capabilities with minimal noise and bias, it is essential to discuss how they can be identified in our own, humane judgments, whether it be in economy, judiciary, education, healthcare, or even personal. Regardless of whether the decisions are recurrent or singular the aim is to identify and minimize the noise in both. In this paper, we have particularly looked into singular decisions because in comparison with recurrent decisions the noise in these systems may be less apparent. Considering the singular decision to remove the hand pump in Broad Street which not only helped save the lives of many during the 1854 Cholera outbreak

[5] Unfortunately the current spatial constraints of the paper disallows us from being more detailed about strong relevance, but an interested reader is encouraged to read [29].

in London, but also influenced healthcare for the better around the world, we have attempted to demonstrate how the historical outcome was not entirely free from noise. Had some of the influencing causal factors been tweaked, the outcome may have been different. The outcome's variability thus becomes apparent. To that end, we have used the WCS framework to model the identification of some of these so-called noise variables using counterfactual reasoning. The protoypical modelling is not limited to the discussion in this paper however and there is scope for future development and general formalization. Some avenues that present themselves for furture exploration through the current exercise are modelling non-necessary antecedents as noise variables, handling obligational conditionals when reasoning counterfactually and eventually minimizing noise in a system.

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