Zero Knowledge Proofs:

Challenges, Applications, and Real-world Deployment

NIST Workshop on Privacy Enhancing Cryptography

September 26th, 2024

Tjerand Silde & Akira Takahashi

 \Box NTNU



AlgoCRYPT CoE

AI Research



1) Introduction to Zero Knowledge Proof (Akira)

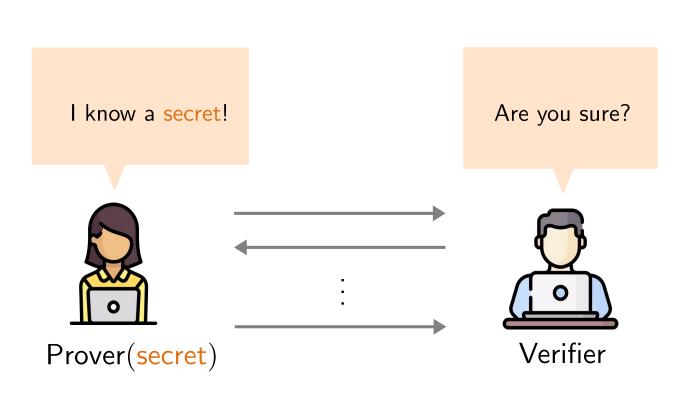
2) Technical Challenges (Akira)

3) Real-World Applications (Tjerand)

4) Insights from ZKP Workshop (Tjerand)

5) Resources and Standards (Tjerand)

What is Zero Knowledge Proof?

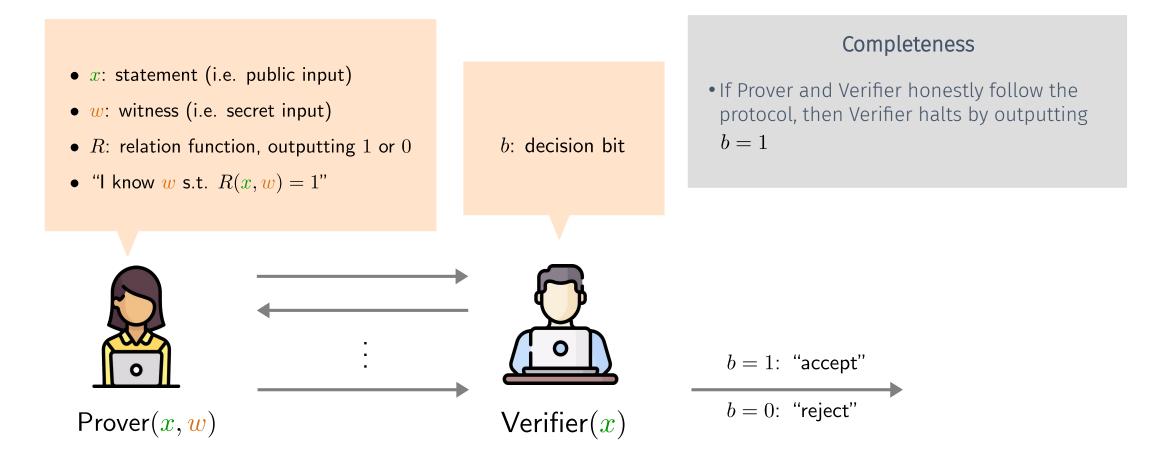


Basics

- ZKP is a two-party protocol, consisting of **Prover** and **Verifier**
- With ZKP, Prover can convince Verifier that she has some secret information without disclosing the secret
- Example: "I know **sk** corresponding to **pk**"
- Long history of research starting from the '80s [GMR85]. Lots of efficiency improvements during the last decade

• cf. **ZK-SNARK** (Succinct Noninteractive Argument of Knowledge)

Syntax of ZKP



Security Goals of Zero Knowledge Proof

- *x*: statement (i.e. public input)
- *w*: witness (i.e. secret input)
- R: relation function, outputting 1 or 0
- "I know w s.t. R(x, w) = 1"

• Tries to steal w

Verifier(x)

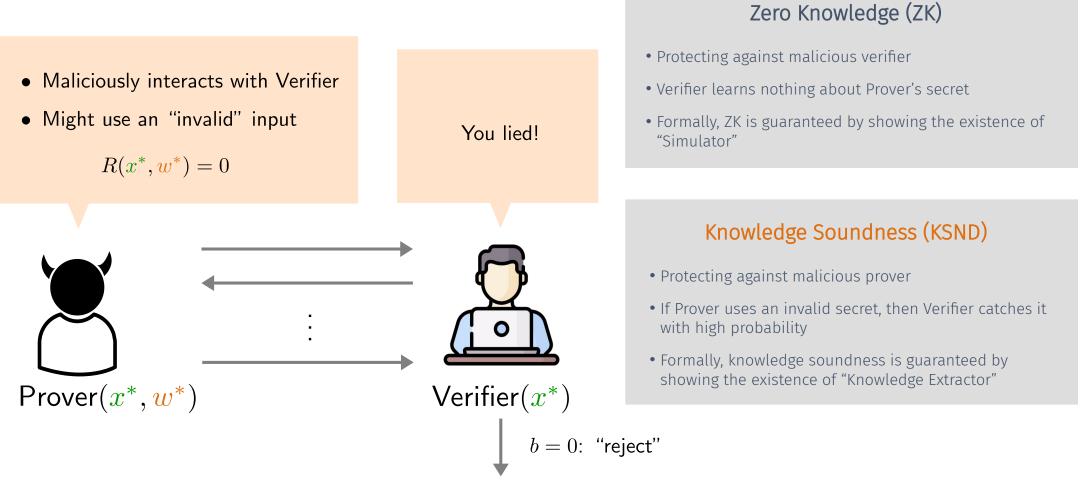
Zero Knowledge (ZK)

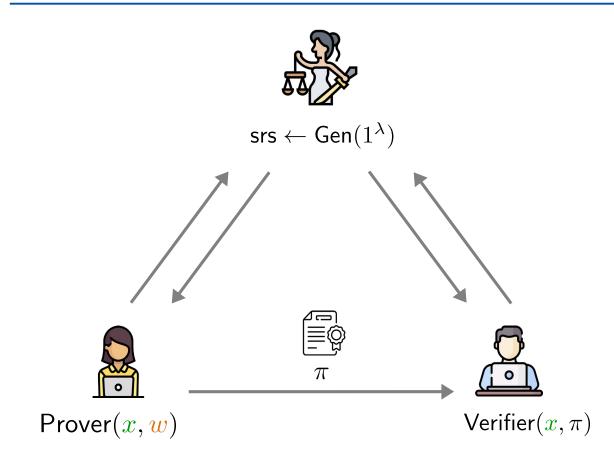
- Protecting against malicious verifier
- Verifier learns nothing about Prover's secret
- Formally, ZK is guaranteed by showing the existence of "Simulator"



 $\mathsf{Prover}(x, w)$

Security Goals of Zero Knowledge Proof



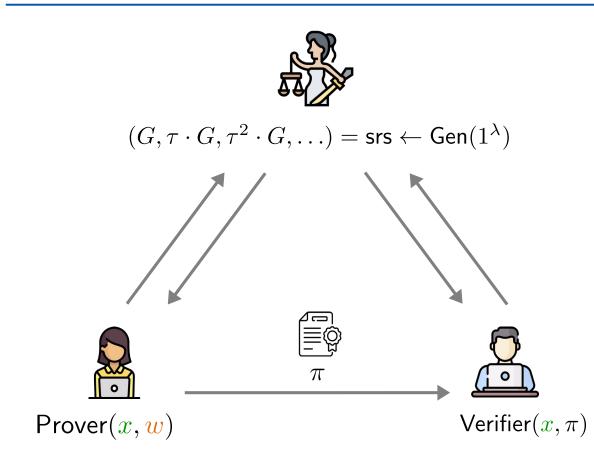


Removing Interactions

- \bullet Ideally, Prover should create a one-shot proof string π
- Verifier checks π asynchronously
- \bullet Such π is reusable and can be checked by potentially many verifiers

Types of Trusted Setup

- Structured Reference String (SRS)
- Hash function modeled as Random Oracle

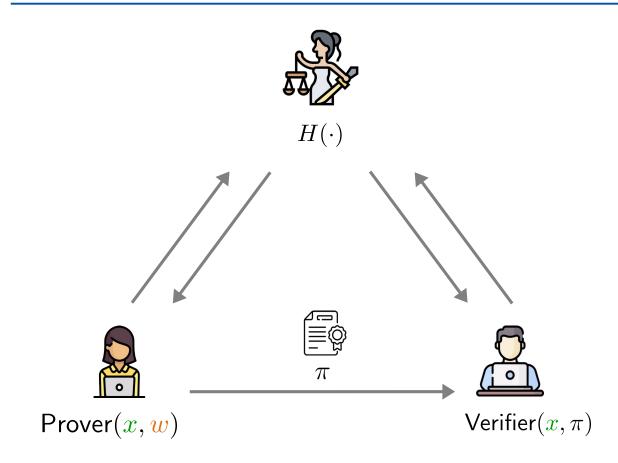


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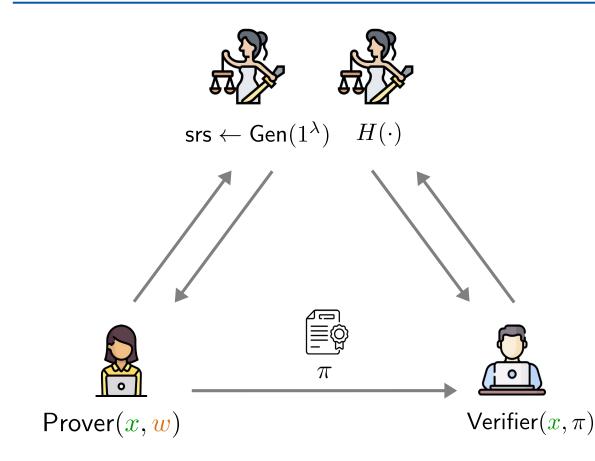


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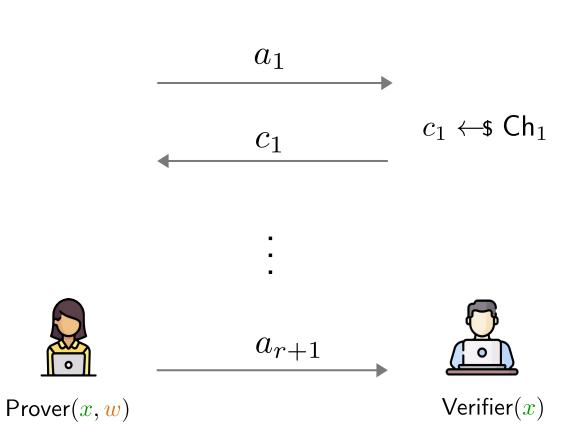


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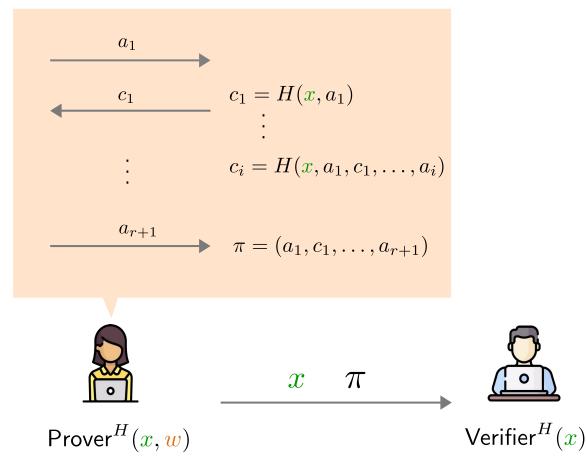
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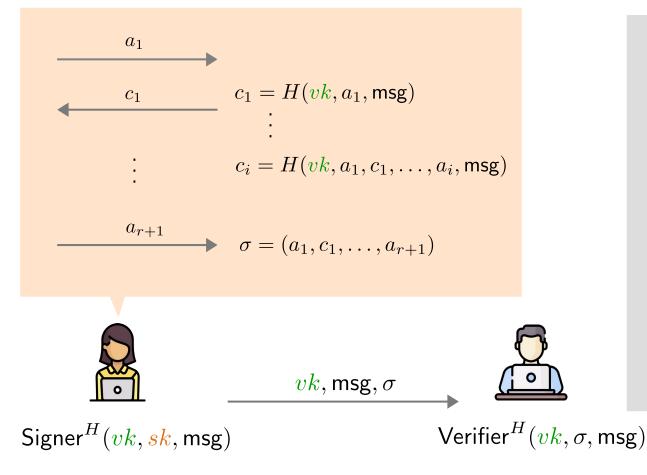
Modular Design of NIZK

- Step 1. Construct a "public-coin" interactive protocol
 - Verifier does not require a secret state
 - ZK against semi-honest Verifier (Honest-Verifier ZK)
- **Step 2.** NI Prover and Verifier obtain challenge by locally hashing a partial transcript so far
- Bonus: By hashing the message, FS-NIZK gives rise to a **digital signature**
- Example: Schnorr/EdDSA, CRYSTALS-Dilithium, PLONK family, Bulletproofs, etc.
- Many modern SNARKs are constructed from (Polynomial) Interactive Oracle Proofs converted to NIZK via Fiat-Shamir [BCS16, CHMMVW19, BFS19, GWC19, CFFQR20,...]



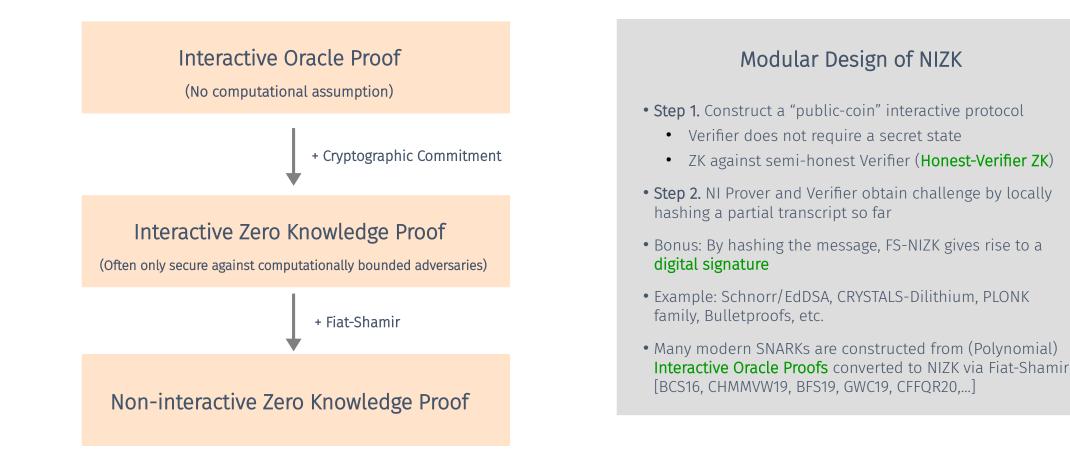
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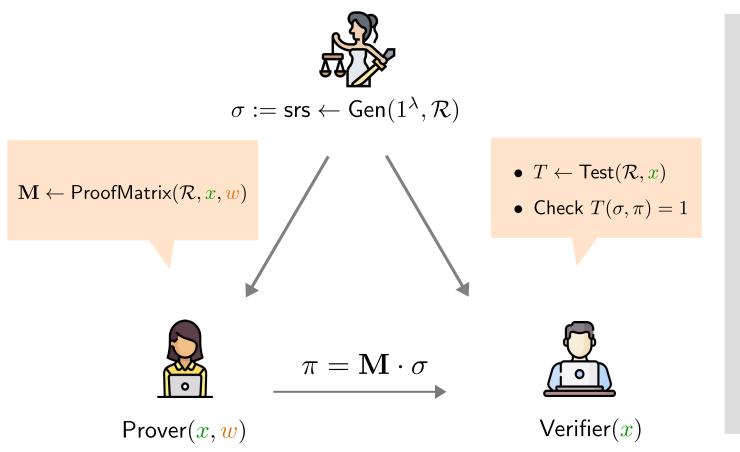


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Paradigm of NIZK II: Linear Interactive Proofs [GGPR13,BCI+13]



NIZK without Fiat-Shamir

- **Step 1.** srs generator outputs a relation-dependent vector
- **Step 2.** NI Prover applies linear transformation to srs
- **Step 3.** NI Verifier derives a testing function, allowing to check whether correct linear transformation has been applied
- Example: [Groth16]
- Important: Prover and Verifier should never learn internal randomness of Gen; otherwise, malicious prover can easily prove a false statement

1) <u>Balancing Generality, Efficiency and Assumptions</u>

2)<u>Advanced Security</u>

3)<u>Interoperability</u>

Types of ZKP

General-Purpose ZKP

- Supports arbitrary NP relation R
- Relation is often described using an arithmetic circuit

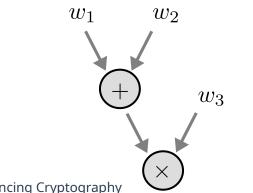
 $\mathcal{R}_C = \{(x, \boldsymbol{w}) : C(x, \boldsymbol{w}) = 1\}$

• Pros:

- Can prove correct execution of *any* program
- Suitable for verifiable and outsourced computation

• Cons:

- circuit gets complex for certain non-linear computations
- E.g., elliptic curve arithmetic, comparison, table lookup, etc.



Specialized ZKP

• Designed for particular type of NP relation R

 $\mathcal{R}_{\text{DL}} = \{ (X, \boldsymbol{w}) : X = \boldsymbol{w} \cdot G \}$ $\mathcal{R}_{\text{SIS}} = \{ (\mathbf{x}, \mathbf{w}) : \mathbf{x} = \mathbf{A}\mathbf{w} \mod q, \|\mathbf{w}\| \le \beta \}$ $\mathcal{R}_{\text{Lookup}} = \{ (\mathbf{x}, \mathbf{w}) : \mathbf{w} \text{ is a subvector of } \mathbf{x} \}$

- Pros:
 - Can prove and verify designated relations efficiently
 - Sufficient for some useful applications, e.g., proof of correct encryption, distributed key generation, signatures, etc.
- Cons:
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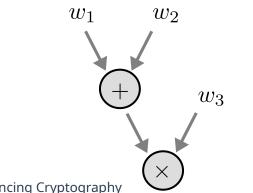
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Proof Size

• To minimize a trust assumption, SRS should be avoided Smaller proof saves storage and communication bandwidth • Better alternative: only trust the security of hash function modeled as RO (aka **transparent** setup), e.g., • Groth16 requires only 3 group elements from pairingfriendly curves Bulletproofs, Brakedown, STARK, LaBRADOR, MPC/VOLEin-the-Head, etc. • State-of-the-art Polymath [Lip24] and PARI [DMS24] achieve even smaller proof sizes! • Middle-ground solution: allows different parties to update SRS (aka **updatable SRS**) [GKMMM18] Setup, Prover and Verifier Cost **Scalability** • Universal Setup: Setup outputs SRS once and for all • How can we prove a large statement efficiently? for arbitrary circuits [GKMMM18] • **Proof Aggregation**: aggregate many, $srs \leftarrow Setup; srs_C \leftarrow Derive(srs, C)$ asynchronously generated proofs, e.g., SnarkPack • Verifier sub-linear in |C|Incrementally Verifiable Computation [Valiant08]: succinct proof of incremental computations via • Prover time linear in #non-linear gates recursion or folding, e.g., Halo2, Nova, etc.

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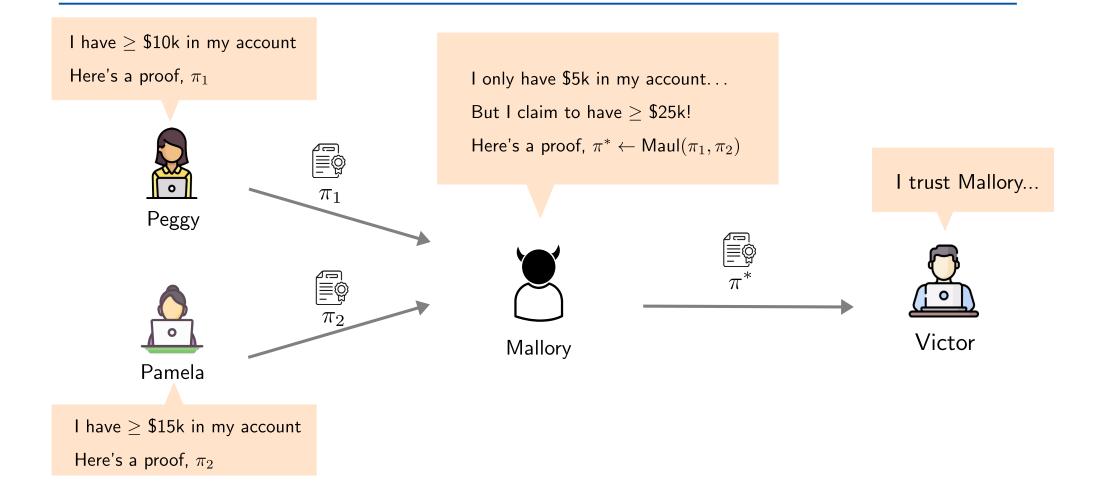
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1) <u>Balancing Generality, Efficiency and Assumptions</u>

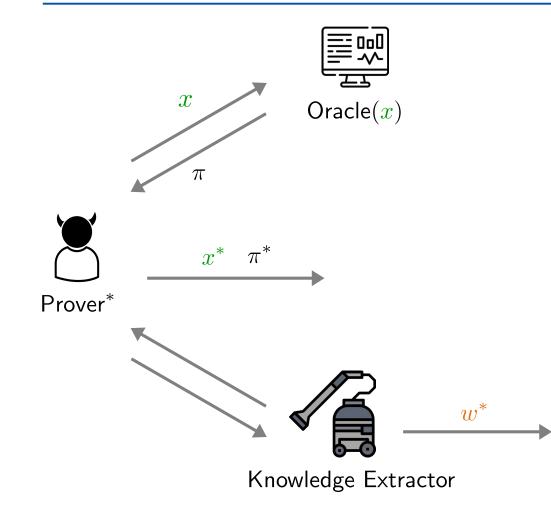
2) Advanced Security

3)<u>Interoperability</u>

ZK and Knowledge Soundness are not Enough: Malleability Attacks



Combined Notion: Simulation-Extractability [Sah99]



SIM-EXT Security

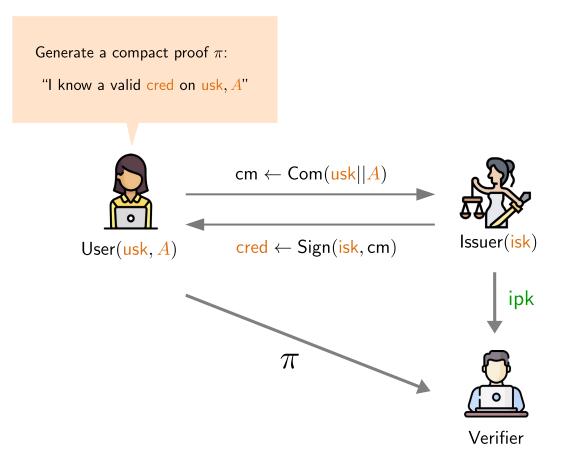
- 1. Prover^* obtains fresh proof from Oracle
- 2. Prover* outputs "forgery" (x^*,π^*)
- 3. If (x^*, π^*) is accepting and not recorded by Oracle, then Prover^{*} must know the corresponding witness w^*
- Intuitively, SIM-EXT guarantees **non-malleability:** a cheating prover cannot maul existing proofs to create a new one, without knowing a valid witness
- Cf. (S)EUF-CMA for signature and IND-CCA for PKE
- Crucial property NIZK should satisfy if used as a subroutine of another protocol
- Many practical NIZK schemes turn out to be SIM-EXT [GKKNZ22] [GOPTT22] [DG23] [FFKR23] [KPT23] [Lib24] [FFR24]
- Some schemes satisfy UC security [Canetti01] accepting some idealized setup [CF24] [BFKT24]

1) <u>Balancing Generality, Efficiency and Assumptions</u>

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Example: Anonymous Credentials (High Level)

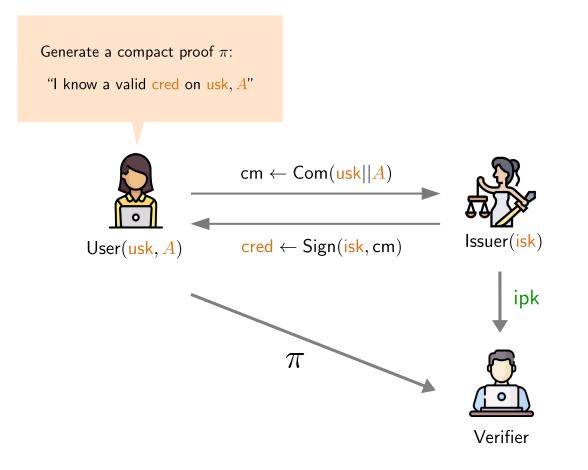


Protocol

- Issuer initially binds attributes and usk to secret credentials
- The owner of attributes produces a **proof string** in the form of ZKP
- By examining the proof string, Verifier gets convinced that User has valid attributes signed by Issuer
- Thanks to ZKP, the proof string only leaks minimum info about Prover's identity
- E.g., Verifier learns "User is => 21 years old" but nothing else

- isk: issuer secret key
- ipk: issuer public key
- usk: user secret key
- A: user attributes

Example: Anonymous Credentials (High Level)



Interoperability

- Central ZKP for AC: Proof-of-Knowledge of valid signature
- Verification algorithms of widely deployed signatures, e.g, RSA-PSS, ECDSA, EdDSA, etc. are not ZK-friendly
- Two directions:
 - Design more specialized and efficient ZKP for existing standardized schemes to retain interoperability
 - Design and standardize "ZK-friendly" primitives: Cf. BBS(+) signature

- isk: issuer secret key
- ipk: issuer public key
- usk: user secret key
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Takeaways

- ZKP allows Prover to prove the knowledge of a secret, while Verifier learns nothing about the secret
- Basic Security Properties: Knowledge Soundness and Zero Knowledge
- What kind statement do you want to prove?
 - General-purpose ZKP, Specialized ZKP, Composition of both
- Which setup assumption is suitable for deployment?
 - Trusted, Transparent, Updatable, ...
- What should you optimize?
 - Proof Size, Assumptions, Setup/Prover/Verifier Costs, Scalability.
- Advanced Security: Does the application need SIM-EXT or UC security?
- Interoperability: Standardize ZK-friendly primitives, or design standardization-friendly ZK

Zero-Knowledge Proofs: Technical Challenges, Applications, and Real-world Deployment

NIST Workshop on Privacy-Enhancing Cryptography **Tjerand Silde** & Akira Takahashi, September 26 – 2024

Content

- Introduction to ZKP
- **Technical Challenges**

Real-World Applications

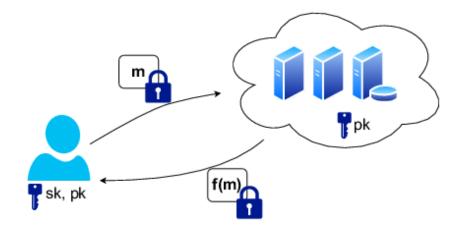
Insights from ZKP Workshop Resources and Standards



Verifiable and Outsourced Computation

Ensure that computation is conducted properly (server is the prover)

Might include secret data or algorithms, but does not have to do so



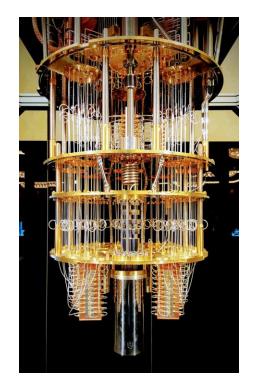
Use ZKP for compliance



Efficient (Post-Quantum) Digital Signatures

Quantum computers can break schemes based on factoring and DLOG

Can design signature schemes from zeroknowledge proofs and the Fiat-Shamir transform





Efficient (Post-Quantum) Digital Signatures

Dilithium is a NIZK based on the quantumsafe LWE/SIS-problems

Follows a similar structure as Schnorrsignatures for DLOG Private information: $\mathbf{s}_1 \in [\beta]^m, \mathbf{s}_2 \in [\beta]^n$ Public information: $\mathbf{A} \in \mathcal{R}_{q,f}^{n \times m}, \mathbf{t} = \mathbf{A}\mathbf{s}_1 + \mathbf{s}_2 \in \mathcal{R}_{q,f}^n$

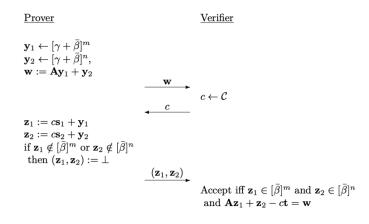


Figure 5: The basic Zero-Knowledge Proof System in which the prover knows $\mathbf{s}_1 \in [\beta]^m, \mathbf{s}_2 \in [\beta]^n$ satisfying (70) and gives a ZKPoK of knowledge of $\bar{\mathbf{s}}_1 \in [2\bar{\beta}]^m, \bar{\mathbf{s}}_2 \in [2\bar{\beta}]^n$,

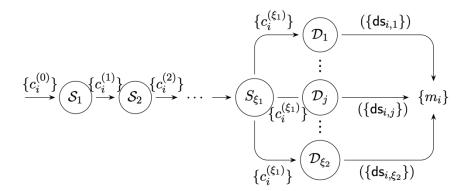
https://eprint.iacr.org/2024/1287.pdf



Proof Systems in Electronic Voting

Need to break the connection between votes and voters by shuffling

Ensure correct encryption and decryption of votes

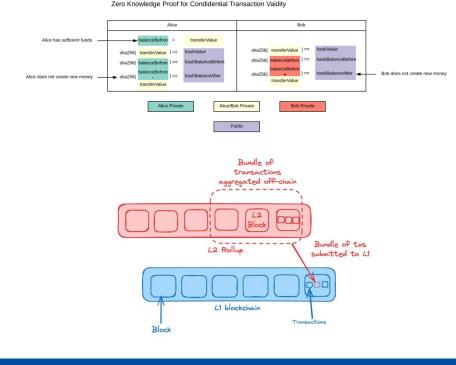




Blockchain Rollup and Private Transactions

For privacy: encrypt to make transactions private, use ZKP to ensure correctness and compliance to bank laws

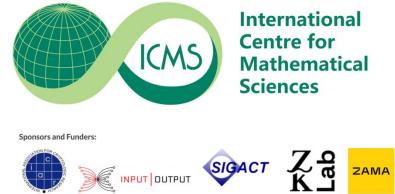
For efficiency: batch many transactions together and prove that all were correct without checking each



Content

- Introduction to ZKP
- **Technical Challenges**
- **Real-World Applications**
- Insights from ZKP Workshop

Resources and Standards





ICMS Workshop on Foundations and Applications of Zero-Knowledge Proofs

A one-week workshop about ZKPs: going from the basics to some of the most advanced applications.

All the slides and recordings are available online.

Organized w/ Elizabeth Crites and Markulf Kolweiss. icms.org.uk/ZeroKnowledgeProofs



Speakers

Jonathan Katz (UMD) Michele Ciampi (UoE) Carsten Baum (DTU) Peter Scholl (AU) Carla Rafols (UPF)

Arantxa Zapico (Ethereum) Anca Nitulescu (IOG) Lisa Kohl (CWI Amsterdam) Ngoc Khanh Nguyen (KCL) Dario Fiore (IMDEA)



Topics

- Introduction to ZKPs and their Security
- Sigma-Protocols and their Applications
- MPC-in-the-Head Techniques for ZKP and Signatures
- Group/pairing-based zkSNARK Constructions
- Polynomial Commitments for zkSNARKs
- Lattice-Based ZKPs and Polynomial Commitments
- ZKPs for Blockchain Applications
- ZKP for Machine Learning and Verifiable Computation



Lessons Learned

Recent advances in ZKP rely heavily on earlier works, and it is worthwhile to go in-depth on the foundations.

ZKP is a fast-moving field, and several invited speakers talked about new constructions published after we reached out.

ZKP has until recently been considered a theoretical field, but nowadays we see new and efficient implementations every week.

New constructions are quite complex, and it might be hard to keep up with the technical details and get a proper overview.



Content

Introduction to ZKP Technical Challenges Real-World Applications Insights from ZKP Workshop **Resources and Standards**





Zero-Knowledge Proofs MOOC

Instructors

Dan Boneh	Shafi Goldwasser	Dawn Song	Justin Thaler	Yupeng Zhang
Stanford	UC Berkeley	UC Berkeley	Georgetown University	Texas A&M University

zk-learning.org



ZKProof Standards

About **ZKProof**

ZKProof is an open-industry academic initiative that seeks to mainstream zero-knowledge proof (ZKP) cryptography through an inclusive, community-driven standardization process that focuses on interoperability and security.

Annually-held ZKProof workshops, attended by world-renowned cryptographers, practitioners and industry leaders, are the optimal forum for discussing new proposals, reviewing cutting edge projects and advancing a community reference document that will ultimately serve as a trusted specification for the implementation of ZKP schemes and protocols.

zkproof.org



Blog-posts by Matthew Green

Matthew Green in fundamentals ③ November 27, 2014 🔤 4,100 Words

Zero Knowledge Proofs: An illustrated primer

One of the best things about modern cryptography is the beautiful terminology. You could start any number of punk bands (or **Tumblrs**) named after cryptography terms like 'hard-core predicate', 'trapdoor function', 'or 'impossible differential cryptanalysis'. And of course, I haven't even mentioned the one term that surpasses all of these. That term is '*zero knowledge*'.





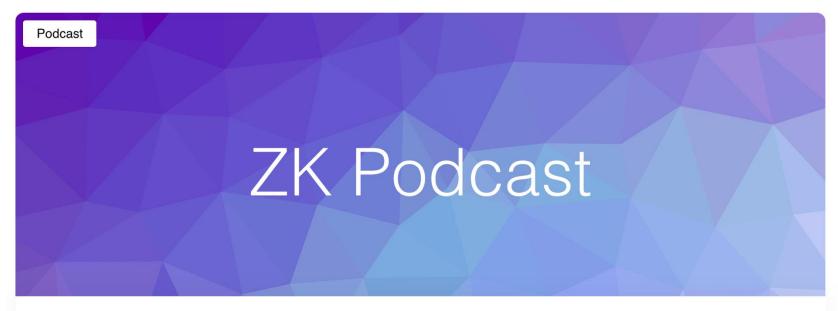
Matthew Green

I'm a cryptographer and professor at Johns Hopkins University. I've designed

blog.cryptographyengineering.com/2014/11/27/zero-knowledge-proofs-illustrated-primer

D NTNU

Zero-Knowledge Podcast



Latest Episode

Episode 340: Is Cosmos Dead? A critical look with Zaki Manian

zeroknowledge.fm



Zero-Knowledge Summit

zkSummit12

October 8th 2024 - Lisbon

zksummit.com



DARPA-Funded ZKP Research

Generating Zero-Knowledge Proofs for Defense Capabilities

Program aims to advance method for making public statements without compromising sensitive underlying information

OUTREACH@DARPA.MIL 7/18/2019



darpa.mil/news-events/2019-07-18



ZKP in EU Digital Identity Wallet

Cryptographers' Feedback on the EU Digital Identity's ARF

Carsten Baum		er Blazy	Jan Camenisch
Technical University of Denn		lytechnique	Dfinity
Jaap-Henk Hoepman Karlstad University & Radboud University	Eysa Lee Brown University	Hasso-Pl	a Lehmann attner-Institute, ity of Potsdam
Anna Lysyanskaya Brown University Johan	René Mayrhofe nnes Kepler Univer		Hart Montgomery*
Ngoc Khanh Nguyen	Bart Preneel		i shelat
King's College London	KU Leuven		ern University
Daniel Slama	Tessaro		
Universität der Bundesv	Washington		
Søren Eller T Partis		armela Troncos EPFL	50

June 2024

github.com/eu-digital-identity-wallet/eudi-doc-architecture-and-reference-framework/discussions/211



Least Authority

Building the Zero-Knowledge Community: Engagement, Events, and Advocacy

苗 September 18, 2024 🛛 🕲 Least Authority Team

leastauthority.com/blog/building-the-zero-knowledge-community-engagement-events-and-advocacy



zkSecurity

ZKSECURITY

Audits Development

We're experts

in ZKP, MPC, FHE, and advanced cryptography...

zksecurity.xyz



Trail of Bits

Serving up zero-knowledge proofs

POST FEBRUARY 19, 2021 4 COMMENTS

By Jim Miller, Senior Cryptography Analyst

Zero-knowledge (ZK) proofs are gaining popularity, and exciting new applications for this technology are emerging, particularly in the blockchain space. So we'd like to shine a spotlight on an interesting source of implementation bugs that we've seen—the Fiat Shamir transformation.

blog.trailofbits.com/2021/02/19/serving-up-zero-knowledge-proofs



Workshop at Simons Institute

Cryptography 10 Years Later: Obfuscation, Proof Systems, and Secure Computation

Monday, May 19 - Friday, Aug. 15, 2025



simons.berkeley.edu/programs/cryptography-10-years-later-obfuscation-proof-systems-secure-computation

Thank you! Questions?

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