Report on Dagstuhl-Seminar 02401:

Algorithms and Complexity for Continuous Problems

September 29 – October 4, 2002

The seminar was devoted to the computational solution of continuous problems. Concrete algorithms and their analysis were discussed as well as complexity results were presented. Important continuous problems arise in different areas, and different techniques for analysis of these problems are necessary. Therefore the seminar attracted researchers from computer science, mathematics and applied mathematics, and statistics. There were altogether 46 participants representing 13 countries, among them 20 from Germany and 8 from the US. Together with senior and well recognized scientists, young prospective colleagues, some of them having just finished their diploma or master thesis, were also invited and presented their results. The abstracts of 40 lectures are included in this report.

A substantial part of the talks were devoted to numerical integration, with emphasis on problems with a large number of variables, and the algorithms under investigation were mainly Monte Carlo or quasi Monte Carlo methods. In some of these talks the computer-based construction of good deterministic cubature formulas was addressed.

A number of talks dealt with non-linear or operator equations, the latter being sometimes analyzed in a statistical setting with noisy data. Probabilistic concepts also played a role as a tool for analysis, e.g., for a problem from computational geometry or for global optimization, or as a part of the problem formulation itself, e.g., for solving stochastic differential equations.

Recently a new quantum model of computation has been introduced. Since quantum computers are potentially much more powerful than the classical ones, the quantum model is attracting great attention. At the Dagstuhl seminar 00931 in 2000 the first result concerning quantum algorithms for continuous problems was presented. Thereafter the quantum model has been included into several research projects related to the topics of the seminar. The current question is for what continuous problems the quantum model of computation offers an essential speed-up in solving them. There were 6 talks in which results on quantum complexity of summation, function approximation and integration were presented.

A selection of results presented at this conference will be published as invited papers in the Journal of Complexity.

The IBFI invited participants to a ceremony during our seminar, since Joe Traub, one of the seminar's organizers, had his 70th birthday in 2002. We are grateful for this exceptional event, as well as for excellent working environment, the support, and the hospitality at Schloss Dagstuhl.

L. Plaskota, K. Ritter, I.H. Sloan, and J.F. Traub

Abstracts in Alphabetical Order by Speaker's Surname:

Abstracts of Talks

(in alphabetical order by speaker's surname)

Efficient Generation of Low Discrepancy Sequences

Emanouil Atanassov
joint work with Michael Mascagni

The generation of the low-discrepancy sequence frequently takes an important fraction of the total CPU time. For the purpose of reducing the generation time we used extended instruction sets, available on most modern CPU. These instruction sets are already used in scientific computing, for example in optimized BLAS, LAPACK, FFT subroutines. They can be utilized in a C program through assembly language statements or compiler intrinsics. We studied the possibility to use such instructions in order to speed-up the generation of some of the most popular low-discrepancy sequences - the Sobol sequence, the Halton sequence and some modifications of them. We developed and tested vectorized versions of the previously described algorithms. The vectorization could be performed in a straight-forward manner, but more sophisticated algorithms showed better performance. On various computer architectures we were able to generate the coordinates of these sequences with speed, comparable to the MHz rate of the processor, usually around 50% for single precision and 25% for double precision in the case of the Sobol sequence, and around 15% for single precision and 10% for double precision in the case of the modified Halton sequence. The vectorized versions outperform the sequential version by a factor of around 4. An easy to use efficient interface to these subroutines was also presented.

On the Power of Various Real-Valued Quantum Queries

Arvid Bessen

It has been shown that the approximate computation of numerical problems on quantum computers is often much faster than on a classical computer in numbers of queries, where a query is the only way the quantum computer can gain information about the specific problem. Examples include the computation of Boolean functions and the computation of the mean and the median. These algorithms for "discrete" problems could even be extended to solve "continuous" problems like integration.

Different query definitions were given and our aim is to review them and to show that these definitions are not equivalent. To achieve this result we prove lower bounds for the simple evaluation problem, which show an exponential gap in the numbers of queries needed. The main tool in this lower bound proof is a relationship between quantum algorithms and trigonometric polynomials that we will establish.

Comparison of Three Algorithms for Finding All Vertices of a Polyhedron - Based on a Probabilistic Analysis

Karl Heinz Borgwardt

Three different algorithms can be applied to solve the following problem:

Given m points $a_1, \ldots, a_m \in \mathbf{R}^n$. Find all vertices of the polyhedron

$$X := \{x \in \mathbf{R}^n \mid a_1^T x \leq 1, \ldots, a_m^T x \leq 1\}.$$

We compare their average-case behaviour under the following Stochastic Model:

$a_1, \ldots, a_m$ are distributed identically, independently and symmetrically under rotations on the unit sphere.

The incremental Beneath-Beyond-Algorithm determines in sequence the polyhedra $X_k := \{x \in \mathbf{R}^n \mid a_1^T x \leq 1, \ldots, a_k^T x \leq 1\}$ for $k = n, n+1, \ldots, m$. Each update from X_{k-1} to X_k destroys several vertices and edges and creates new ones. On the average n vertices have to be produced for one to survive until $X = X_m$ is constructed. This causes an expected effort of

$$C(n)[m^2 * n^3 + m^2 * \ln(m) * n^2].$$

The sequential Gift-Wrapping-Algorithm works on the surface of X and moves according to the Simplex-Algorithm from vertex to an adjacent unknown vertex. If all adjacent vertices are known, the same path is used for backtracking until a vertex with unknown neighbours is reached. The need to find the way back requires storing the data of the path and causes an expected effort of

$$C(n)[m^2 * n + m * \ln(m) * n^2].$$

The Fukuda-Avis-Algorithm avoids storing data. It is based on the fact that for a fixed Simplex-Rule from every vertex a Simplex-Path leads to the optimal vertex. The collection of all these paths forms a tree. The essence of this algorithm is the construction of this tree by a depth-first search with an expected effort of

$$C(n)[m^2 * n].$$

Convergence Rate for an Optimization Algorithm Under the 2-Fold Integrated Wiener Measure

James M. Calvin

We describe an algorithm that approximates the (global) minimum of a twice continuously differentiable function defined on the unit interval. The error converges to zero at rate $\exp(-n\log(n)^{-1/4})$ for the subclass of functions with a unique minimizer in the interior of the interval and that have strictly positive second derivative at the minimizer.

For a probabilistic analysis we consider a conditional 2-fold integrated Wiener measure with boundary conditions to ensure that, with probability one, the minimizer is unique and lies in the interior of the interval. Under this measure the probability that the error is less than $\exp(-n\log(n)^{-1/4})$ converges to one.

The Continuing Search for Lattice Rules: Exploiting Structured Matrices

Ronald Cools and Hilde Govaert

Lattice rules are a type of integration rules designed for periodic multivariate functions on a unit cube. The construction of efficient lattice rules for higher dimensions is an area of ongoing research.

In [1] the results of a large scale computer search for optimal lattices in 3 and 4 dimensions are reported. A well defined set of lattices which probably, but not certainly, contains the optimal lattices was searched. The results are called K-optimal lattice rules. The fundamental problem with this search is the fact that the amount of time required grows as δ^{s^2} where δ is the degree and s is the dimension.

In [2] a subset of the above class of lattices generated by a particular type of structured matrices was investigated and it turned out that many 3 and 4-dimensional K-optimal lattices belong to that class. This is a big achievement because searching in this class requires an amount of time 'only' proportional to δ^s. In this talk we described ongoing research that extends this approach to 5 and 6 dimensions. The rules we obtained so far are more efficient, i.e. they use less lattice points, than previously known results of the same degree. Furthermore, searching in 6 dimensions takes only a fraction of the time needed previously in the 4 dimensional search for K-optimal rules, for the same δ.

References

[1] R. Cools and J.N. Lyness. Three- and four-dimensional K-optimal lattice rules of moderate trigonometric degree. *Math. Comp.*, 70(236):1549–1567, 2001.

[2] J. Lyness and T. Sørevik. Four-Dimensional Lattice Rules Generated by Skew-circulant Matrices. *Math. Comp.*, To appear.

Approximation of Gaussian Random Vectors

Jakob Creutzig

Let E be a separable Banach space with topological dual E^*, let μ be a Gaussian measure on a Banach space E, and denote $\tau_\mu : E^* \to L_2(E,\mu)$ the canonical embedding. We establish close correspondences between the average Kolmogorov widths of μ in E and the classical Gel'fand widths of the compact embedding τ_μ. Secondly, we find that the average linear widths of μ are estimable by the average Kolmogorov widths of μ times a logarithmic factor, in contrast to the case of classical Kolmogorov and linear widths of operators.

The results enable us to give a necessary and a sufficient condition for the tractability of the approximation problem for a sequence μ_d of Gaussian measures on Banach spaces E_d, in terms of tractability of the approximation problem related to the embeddings τ_{μ_d}. There is a small gap between the necessary and the sufficient condition, and only the case Λ^{all} is considered.

Recent Results on Adaptive Wavelet Schemes for Operator Equations

Stephan Dahlke

The aim of this talk is to give an overview on recent developments concerning the numerical treatment of (elliptic) operator equations by means of adaptive wavelet methods. The first part of the presentation is devoted to the theoretical analysis of adaptive schemes. We intend to give a rigorous answer to the fundamental question: when does adaptivity pay? It turns out that the efficiency of an adaptive scheme is determined by the regularity of the exact solution of the operator equation in the specific scale $B^\alpha_\tau(L_\tau(\Omega))$, $\tau := (\alpha/d + 1/p)^{-1}$ of Besov spaces. Thus, at least for some simple model problems, the regularity of the solutions in this scale is investigated with the aim of determining the largest α for which the solution is in $B^\alpha_\tau(L_\tau(\Omega))$. It turns out that the Besov regularity is high enough to justify the use of adaptive schemes.

In the second part of the talk, we shall discuss some recent developments concerning the practical realization of adaptive wavelet schemes. Using the equivalence of Sobolev norms with weighted sequence norms of wavelet expansions, it is possible to derive reliable and efficient a posteriori error estimators. Moreover, based on these error estimators it is possible to derive an adaptive refinement strategy which is guaranteed to converge for a large set of problems including operators of negative order. We finish this talk with some numerical experiments for the Poisson equation on a L–shaped domain.

On Adaptive Inverse Estimation of Linear Functionals in Hilbert Scales

Alexander Goldenshluger
joint work with Sergei V. Pereverzev

We address the problem of estimating the value of a linear functional $\langle f, x\rangle$ from random noisy observations of $y = Ax$ in Hilbert scales. Both the white noise and density observation models are considered. We propose an estimation procedure that adapts to unknown smoothness of x, of f, and of the noise covariance operator. It is shown that accuracy of this adaptive estimator is only by a logarithmic factor worse than one could achieve in the case of known smoothness. As an illustrative example, the problem of deconvolving a bivariate density with singular support is considered.

Quantum Approximation of Sobolev Embeddings

Stefan Heinrich

We study function approximation in the quantum model of computation. We consider embeddings between finite dimensional L_p-spaces and Sobolev embeddings. Matching upper and lower bounds for the quantum query complexity are derived and an optimal quantum algorithm is indicated.

Is the Mandelbrot Set Computable?

Peter Hertling

We discuss the question whether the Mandelbrot set is computable. The computability notions which we consider are studied in Computable Analysis and will be introduced and discussed. We show that the exterior of the Mandelbrot set, the boundary of the Mandelbrot set, and the union of its hyperbolic components satisfy certain natural computability conditions. We conclude that the two–sided distance function of the Mandelbrot set is computable if the Hyperbolicity Conjecture is true.

On the Global Error of Methods Based on Multiple Itô-Integrals

Norbert Hofmann
joint work with Thomas Müller–Gronbach

We analyze the $L_2([0,1])$-error of general numerical methods based on multiple Itô-integrals for pathwise approximation of scalar stochastic differential equations on the interval $[0,1]$. We show that the minimal error that can be obtained is of order $N^{-1/2}$, where N is the number of multiple Itô-integrals that are evaluated. As a consequence, there are no Itô-Taylor methods of higher order with respect to the global L_2-error on $[0,1]$, which is in sharp contrast to the well known fact that arbitrary high orders can be achieved by these methods with respect to the error at the discretization points.

Quadratic Forms of Gaussian Vectors (Quadratic Approximations in Risk Measurement)

Stefan Jaschke

An overview of approximation methods to the cumulative distribution function of quadratic functions of Gaussian vectors is given. This is a classical problem in statistics, since quadratic forms and their ratios are ubiquitous in linear regression, time series analysis, and spatial statistics.

The problem of computing value-at-risk, i.e., the quantile, of a portfolio is a more recent application that leads to the above problem, in much higher dimensions than before, however. An overview of approximation methods and their computational costs are given, covering (a) series expansions, (b) numerical Fourier inversion, (c) inverse Cornish-Fisher expansions, (d) saddle-point approximations, (e) Monte-Carlo methods, and (f) methods of limited accuracy like moment matching and tail approximations.

The conclusion is that the value-at-risk problem has exposed weaknesses in the traditionally used approximations. A study of Monte-Carlo and QMC-methods, whose computational costs increase only linearly in the number of non-zeros of the matrix that defines the quadratic form, needs to be performed.

Complexity of Nonlinear Two-Point Boundary-Value Problems

Bolesław Kacewicz

We study upper and lower bounds on the worst-case ε-complexity of nonlinear two-point boundary-value problems. We deal with general systems of equations with general nonlinear boundary conditions, as well as with second order scalar problems. The complexity depends significantly on the problem being solved and on the type of information allowed. For general problems the ε-complexity for the right-hand side functions having r $(r \geq 2)$ continuous bounded partial derivatives turns out to be of order $(1/\varepsilon)^{1/r}$ for standard information, and $(1/\varepsilon)^{1/(r+1)}$ for linear information. For second order scalar problems, linear information is even more powerful. The ε-complexity in this case is shown to be of order $(1/\varepsilon)^{1/(r+2)}$, while for standard information it remains at the same level as in the general case.

QMC Methods for the Solution of Differential Equations with Multiple Delayed Arguments

Reinhold F. Kainhofer
joint work with Robert F. Tichy

We apply the Runge-Kutta (Quasi-) Monte Carlo methods of Stengle [6, 7], Lécot, Coulibaly and Koudiraty [4, 1, 3] to delay differential equation with multiple retarded arguments

$$y'(t) = f\left(t, y(t), y(t-\tau_1(t)), \ldots, y(t-\tau_k(t))\right), \quad t \geq t_0, k \geq 1,$$

thus extending our previous result of one retarded argument [2]. The RKQMC methods are combined with the Hermite interpolation method (e.g. [5]) to interpolate the retarded values from the already calculated solution. For heavily oscillating delay differential equations, this method can reduce the error considerably, and even delay instabilities in the numerical solution by some orders of magnitude.

References

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[7] G. Stengle. Error analysis of a randomized numerical method. *Numer. Math.*, 70:119–128, 1995.

Quasi-Monte Carlo Methods for Some Linear Algebra Problems. Convergence and Complexity

Aneta Karaivanova
joint work with Michael Mascagni

We present quasi-Monte Carlo analogs of Monte Carlo methods for some linear algebra problems such as solving systems of linear equations, computing extreme eigenvalues and matrix inversion. We consider crude and importance sampling type Monte Carlo methods. Reformulating the problems as solving integrals or integral equations with special kernels and domains permits us to analyze the quasi-Monte Carlo methods with bounds from numerical integration. Standard Monte Carlo methods for integration provide a convergence rate of $O(N^{-1/2})$ using N samples. Quasi-Monte Carlo methods use quasi-random sequences with the resulting convergence rate for numerical integration as good as $O((logN)^k)N^{-1})$ where k is the dimension of the sequence. In the studied here quasi-Monte Carlo methods the dimension k of the used sequences does not depend on the matrix size n but on its eigenvalue spectrum.

We have shown theoretically and through numerical tests, that the use of quasi-random sequences improves both the magnitude of the error and the convergence rate of the Monte Carlo methods for considered linear algebra problems. We also analyze the complexity of corresponding quasi-Monte Carlo algorithms and compare them to the complexity of the analogous Monte Carlo and deterministic algorithms.

Stratification by Rank-1 Lattices

Alexander Keller

Many rendering problems can only be solved using Monte Carlo integration. The noise and variance inherent with the statistical method efficiently can be reduced by stratification. So far only uncorrelated stratification methods were used that in addition depend on the dimension of the integration domain. Based on rank-1 lattices we present a new stratification technique that removes this dependency on dimension, is much more efficient by correlation, is trivial to implement, and robust to use. The superiority of the new scheme is demonstrated for standard rendering algorithms.

Spatial Discretization of Dynamical Systems

Peter E. Kloeden

The effects of spatial discretization on the dynamics of a discrete time dynamical system, essentially an autonomous difference equation, was discussed. The possible collapse of complicated behaviour onto simple cyclic behaviour was discussed. In view the existence of spurious cyclic solutions it was argued that it was more appropriate to investigate and compare invariant measures of the spatially discretized solution. It was then shown that

semi-invariant measures were in fact robuster to spatial discretization and could be approximated by the invariant probability vectors of suitably constructed Markov chains, so constructed as to avoid collapsing effects. These results are taken from the paper

P. Diamond, P.E. Kloeden and A. Pokrovskii, Interval stochastic matrices, a combinatorial lemma, and the computation of invariant measures, J. Dynamics & Diff. Eqns.7 (1995), 341–364.

Sharp Error Bounds on Quantum Boolean Summation in Various Settings

Marek Kwas
joint work with Henryk Woźniakowski

We study the quantum summation (**QS**) or the amplitude estimation algorithm of Brassard, Høyer, Mosca and Tapp, see [1], that approximates the arithmetic mean of a Boolean function defined on N elements. We improved error bounds of this algorithm presented in [1] in the worst-probabilistic setting, and present new error bounds in the average-probabilistic setting.

In particular, in the worst-probabilistic setting, we prove that the error of the **QS** algorithm using less than M quantum queries is $3\pi/(4M)$ with probability $8/\pi^2$ which improves the error bound $\pi/M+\pi^2/M^2$ of [1]. We also present error bounds with probability $p \in (1/2, 8/\pi^2]$, and show that they are sharp for large M and N/M.

In the average-probabilistic setting, we prove that the **QS** algorithm has error of order $\min\{M^{-1}, N^{-1/2}\}$ iff M is divisible by 4. This bound is optimal as recently shown in [2]. For M not divisible by 4, the **QS** algorithm is far from being optimal if $M \ll N^{1/2}$ since its error is proportional to M^{-1}.

References

[1] G. Brassard, P. Høyer, M. Mosca, A. Tapp, Quantum Amplitude Estimation and Amplification, http://arXiv.org/abs/quant-ph/0005055

[2] A. Papageorgiou, in progress.

Geometry of Linear Ill-Posed Problems in Variable Hilbert Scales

Peter Mathé
joint work with Sergei V. Pereverzev

We discuss the best possible accuracy of recovering the unknown solution from linear ill-posed problems in variable Hilbert scales. Here, a priori smoothness of the unknown solution is expressed in terms of general source conditions, given through index functions. Emphasis is on geometric concepts.

To this end, the notion of regularization is appropriately generalized, and the interplay between qualification of regularization and index function becomes visible.

Finally we propose a general adaptation strategy, which turns out to be optimal for a variety of source conditions.

A Step Towards a Complexity Theory for Analog Systems

Klaus Meer
joint work with Marco Gori

We consider analog models of computation. A problem is given as a relation in $\mathbb{R}^* \times \mathbb{R}^*$. Problem solving is related to finding a global minimizer of an element in a family of Lyapunov-functions corresponding to the problem instance. Depending on the structure of local minima of the Lyapunov-functions we define several complexity classes (analogues of P and NP) and prove some completeness results for these classes.

Approximation by Gaussian Networks

Hrushikesh N. Mhaskar

A Gaussian network is a function of the form

$$\mathbf{x} \mapsto \sum_{k=1}^{N} a_k \exp(-\|\mathbf{x}-\mathbf{x}_k\|^2/2).$$

The minimal separation parameter for this network is defined to be

$$\min_{1\le j<k\le N} \|\mathbf{x}_k - \mathbf{x}_j\|.$$

Let the set of all Gaussian networks with the minimal separation parameter at least $1/m$ be denoted by G_m. The notation $W\Pi_{n,s}$ will denote the class of all expressions of the form $\exp(-\|\mathbf{x}\|^2/2)P(\mathbf{x})$, where P is a polynomial of s variables with coordinatewise degree at most n. We proved that for a function $f \in L^2(\mathbf{R}^s)$, the following statements (a) and (b) are equivalent:
(a)

$$\|f\|_{\mathbf{R}^s\setminus[-t,t]^s} = O(t^{-\beta})$$

and

$$\text{dist}\,(f, G_m) = O(m^{-\beta}).$$

(b)

$$\text{dist}\,(f, W\Pi_{n,s}) = O(n^{-\beta/2}),$$

In particular, a polynomial rate of approximation of the nonlinear Gaussian approximation process is equivalent to a similar rate for the linear process of weighted approximation. Similar results are given also for the other L^p norms, but with a mild additional condition on the number of evaluations of the Gaussian function.

Strong Approximation of SDE's at a Single Point

Thomas Müller-Gronbach

We study strong approximation of the solution X of a stochastic differential equation

$$dX(t) = a(t,X(t))\,dt + \sigma(t,X(t))\,dW(t), \qquad t \in [0,1],$$

at the point $t = 1$. Arbitrary numerical methods $\widehat{X(1)}$ are compared by their mean squared error and the mean number of observations of the one-dimensional driving Brownian motion W. For a large class of equations we present the order of convergence of the minimal errors that can be achieved together with the corresponding asymptotic constants. Moreover, we provide an asymptotically optimal method which is based on an adaptive step-size control.

Tractability of Approximation for Weighted Korobov Spaces on Classical and Quantum Computers

Erich Novak
joint work with Ian H. Sloan and Henryk Woźniakowski

We study the approximation problem (or problem of optimal recovery in the L_2-norm) for weighted Korobov spaces with smoothness parameter α. The weights γ_j of the Korobov spaces moderate the behavior of periodic functions with respect to successive variables. The non-negative smoothness parameter α measures the decay of Fourier coefficients.

The periodic functions are defined on $[0,1]^d$ and our main interest is when the dimension d varies and may be large. We consider algorithms using two different classes of information. The first class $\Lambda^{\rm all}$ consists of arbitrary linear functionals. The second class $\Lambda^{\rm std}$ consists of only function values and this class is more realistic in practical computations.

We want to know when the approximation problem is tractable. Tractability means that there exists an algorithm whose error is at most ε and whose information cost is bounded by a polynomial in the dimension d and in ε^{-1}. Strong tractability means that the bound does not depend on d and is polynomial in ε^{-1}. In this paper we consider the worst case, randomized and quantum settings.

Average Case Quantum Lower Bounds for Computing the Boolean Mean

Anargyros Papageorgiou

We consider the average case approximation of the boolean mean by quantum algorithms. For two distributions on the set of inputs we show sharp error and query lower bounds. We show conditions, for a class of distributions, under which similar lower bounds hold. Our results extend the optimality of the quantum summation algorithm of Brassard et al. We also consider the worst expected error of quantum algorithms and show a query lower bound.

Some Results on Learning Theory

Ye Peixin

In this report we present some new results of several problems which are close related to learning theory. These problems have numerous applications and their recent importance in the learning theory sparked our interest in these subjects. In section 2 we present some estimates of I-functional which is close related to approximation error of Cucker-Smale theory of learning, In section 3 we obtain the exact order of Monte Carlo Integration error for anisotropic classes, this subject is part of Cucker-Smale theory of sampling error. In section 4 we investigate some nonlinear-approximation problems which is close related to sampling error of Cucker-Smale theory of learning and computational learning theory. We also give a greedy algorithm which realize best n-term approximation for various function classes.

Discretization Strategy for Ill-Posed Problems in Hilbert Space

Sergei V. Pereverzev
joint work with Peter Mathé

We consider inverse problems represented in the form of ill-posed linear operator equation. Smoothness of its solution is measured with respect to variable Hilbert scale generated by equation operator. Usually this smoothness is unknown, and in our first talk it has been shown how to choose the parameter of regularization method in such a way that it provides the optimal order of accuracy without knowledge of smoothness. By the way, this new principle for choosing regularization parameter gives the answer for some old question coming from the Theory of ill-posed problems. Namely, for the first time one has a posteriori principle that allows to reach the best order of accuracy for all linear ill-posed problems which in principle can be treated in optimal way by the regularization method with fixed qualification. In general this new principle can be numerically realized only in combination with appropriate discretization strategy. In our second talk we show how such a strategy can be developed on the base of the Theorem about structure of the error arising after discretization and regularization. Namely, discretization parameter should be chosen in dependence of regularization one in such a way that discretization error could not dominate. Comparison of this adaptive strategy with non-adaptive one shows that it allows to reduce essentially the amount of discrete information required for effective regularization.

Use Derivatives in Numerical Integration!

Knut Petras

In some practical problems, integrals have to be calculated with very high precision. If we therefore increase the working precision of the computer, the cost for applying arithmetic operations grows slower than that for applying elementary functions and, in particular, special functions. If once a more complicated function is evaluated, the first m derivatives can often be obtained by applying $O(m^2)$ multiplications and additions. Hence, up to a certain order of the derivative, derivative calculation is cheaper than function evaluation. This leads to results on the optimal number of derivatives a Hermitian quadrature rule should use if applied to an analytic integrand that involves either only arithmetic operations and elementary functions or additionally certain special functions.

Monte Carlo Integration with Few Random Bits

Harald Pfeiffer
joint work with Stefan Heinrich and Erich Novak

We study the approximation of means of p-summable sequences and of integrals of functions from Sobolev classes $W_p^r([0,1]^d)$ in the restricted randomized setting. A restricted randomized algorithm is a randomized algorithm where as randomness only random bits are allowed. Especially we are interested in a low number of random bits. For the two problems considered we show that restricted Monte Carlo is almost as good as general Monte Carlo.

As to the integration of functions from $W_p^r([0,1]^d)$, with the use of about $O(\log n)$ random bits we get for the n-th minimal error in the restricted randomized setting the same rate of convergence as for that in the general randomized setting. We use a discretization technique that consists of splitting the problem into a sequence of discrete summation problems of p-summable sequences. That allows us to apply the results on the complexity of computing the mean of such sequences in the restricted randomized setting.

Weighted Discrepancy of Scrambled Nets

Friedrich Pillichshammer

Until now the most powerful constructions of low discrepancy point sets are based on the concept of (t,m,s)-nets in base b. Several years ago A. Owen introduced a randomization of such point sets such that the net property is preserved. This randomization is based on scrambling the base b digits of the components of the points. Owens aim was to combine the benefits of both, MC and QMC methods. In our talk we consider the expectation of weighted L^2 discrepancy of scrambled $(0,m,s)$-nets in base b. The concept of weighted discrepancy was introduced by Sloan and Woźniakowski to give a general form of a Koksma-Hlawka inequality which takes imbalances in the "importance" of the projections of the integrand into account. Finally we discuss the obtained bound for several choices of weights as for example for the unweighted case or for the case where the sum of the weights is finite.

A Monte Carlo Algorithm for Weighted Integration Over $\mathbf{R}^d$

Leszek Plaskota
joint work with P. Gajda, Y. Li, and Greg W. Wasilkowski

We present and analyze a new randomized algorithm for numerical computation of weighted multiply integrals over the *unbounded* domain $\mathbb{R}^d$. The algorithm and its desirable theoretical properties are derived based on certain stochastic assumptions about the integrands. It is easy to implement, enjoys $O(n^{-1/2})$ convergence rate, and uses only standard random number generators. Numerical results are also included.

On the Complexity of a Stochastic Heat Equation

Klaus Ritter
joint work with Katrin Mittmann and Thomas Müller-Gronbach

We study the stochastic partial differential equation

$$U_t(t,x) = U_{xx}(t,x) + \sigma(t,x,U(t,x)) \cdot \xi(t,x),$$

where $t,x \in [0,1]$ and ξ is a space-time white noise. We impose the boundary and initial conditions $U(\cdot,0) = U(\cdot,1) = 0$ and $U(0,\cdot) = 0$, for simplicity. A solution U can be expressed in terms of a sequence of independent Brownian motions $(B_k(t))_{t\in[0,1]}$ with $k \in \mathbb{N}$.

We are interested in a global approximation $\widehat{U}$ of U on $[0,1]\times[0,1]$, and we define the error of $\widehat{U}$ by

$$e(\widehat{U}) = \left(E\left(\int_{[0,1]\times[0,1]} (U(t,x) - \widehat{U}(t,x))^2 \, d(t,x) \right) \right)^{1/2}.$$

The available information about $(B_k)_{k\in\mathbb{N}}$ consists of a finite number of values of finitely many Brownian motions. As a rough measure of the cost $c(\widehat{U})$, we consider the total number of evaluations of the Brownian motions.

In the case $\sigma = 1$, we show that

$$\inf\{e(\widehat{U}) : c(\widehat{U}) \le n\} \asymp n^{-1/6}.$$

This optimal order of minimal errors is achieved by a simple algorithm.

We add that the results extend to the case of a general additive noise, i.e., $\sigma(t,x,U(t,x)) = \widetilde{\sigma}(t,x)$.

A Recursive Algorithm for the Infinity-Norm Fixed Point Problem

Spencer Shellman

We present an algorithm that solves the fixed point problem $||f(x)-x|| \leq \varepsilon$, where $||.||$ is the infinity norm, for any function $f : [0,1]^d \to [0,1]^d$ that is Lipschitz continuous with constant 1 w.r.t. the infinity norm. The algorithm, called PFix, has better than exponential worst-case complexity. We prove an upper bound that is binomial in $d+r$ and r, where r is the ceiling of $\log_2 1/\varepsilon$.

Approximation from Sparse Grids and Sobolev Spaces of Dominating Mixed Smoothness

Winfried Sickel

For a given 2π-periodic and continuous function f we denote by $I_n f$ the trigonometric polynomial of order less than or equal to n which interpolates f at the points $t_k = 2\pi k/(2n+1)$, $k \in \mathbb{Z}$. Approximation of f by $I_j f$ is of the same order as best approximation by trigonometric polynomials of degree less than or equal to n as long as f belongs to a periodic Besov space $B^s_{p,q}$ with $s > 1/p$ and $1 < p < \infty$.
Here we are discussing the quality of approximation of a given function $f = f(x_1, x_2)$ by

$$A(q,2)\, f = \sum_{j+k \leq q} \Delta_{j,k}\, f, \qquad q = 1, 2, \dots ,$$

where $\Delta_{j,k} = \left(I_{2^j} - I_{2^{j-1}}\right) \otimes \left(I_{2^k} - I_{2^{k-1}}\right)$, $j,k \geq 1$ (modification if $\min(j,k) = 1$). This is Smolyak's construction with respect to the I_{2^j} in the bivariate situation. Let $S^r_p W(\mathbb{R}^2)$ denote the Sobolev space of dominating mixed smoothness of fractional order r. Our main result consists in the exact determination of the asymptotic behaviour of $||I - A(q,2) : S^r_p W(\mathbb{R}^2) \mapsto L_p(\mathbb{R}^2)||$. Indeed, it holds

$$||I - A(q,2) : S^r_p W(\mathbb{R}^2) \mapsto L_p(\mathbb{R}^2)|| \sim q^{1/2}\, 2^{-qr} \tag{1}$$

if $1 < p < \infty$ and $r > 1/\min(2,p)$. The tools we are using to prove the estimate from above are Fourier multiplier and Littlewood-Paley assertions (due to Lizorkin in this context), the tensor product structure of $S^r_p W$, and complex interpolation of Lizorkin-Triebel spaces of dominating mixed smoothness.

QMC for High-Dimensional Integration — Constructing Rules

Ian H. Sloan

This talk reviews recent progress in constructing quasi-Monte Carlo (QMC) rules for high-dimensional multiple integration that achieve optimal 'strong tractability' rates of convergence. The setting is that of the 'weighted' Sobolev spaces of the 1998 paper of Sloan and Woźniakowski, in which successive coordinate directions are weighted by parameters $\gamma_1, \gamma_2, \dots$, with $\gamma_1 \geq \gamma_2 \geq \cdots > 0$. The constructions, by the author with F. Kuo and S. Joe, are of (randomly) shifted lattice rules. The principles of the construction are outlined. Additionally, the talk considers recent advances in our understanding of how to choose the weights needed in the construction: a recent revelation (in joint work with J. Dick, X. Wang and H. Woźniakowski), is that in many cases the weights should not be held constant as the dimension of the integral increases, but should instead reduce to keep the sum of the weights constant, if the aim is to choose weights that give the smallest error bound.

Sparseness of Support Vector Machines

Ingo Steinwart

Classification is one of the major problems considered in learning theory. Besides the question whether a given algorithm can learn from samples it is also important to know how many samples are needed to guarantee a certain accuracy and how many samples are actually used to evaluate the decisions of the classifier. The latter is also known as the sparseness of the algorithm.

In this talk we first give a brief introduction to the standard model for classification tasks. We then discuss how many samples are needed to guarantee a certain accuracy. In particular we present some known negative results. The major part of this talk is devoted to support vector machines which are of rapidly increasing interest. After a brief description of these algorithms we first present recent results on their learning ability. We then discuss the sparseness of support vector machines. In particular we show that in typical situations they are less sparse than they are usually assumed to be.

Optimal Cubature Formulas and Nonlinear Approximation

Vladimir N. Temlyakov

We present results on a relation between construction of an optimal cubature formula with m knots for a given function class and best nonlinear m-term approximation of a special function determined by the function class. The nonlinear m-term approximation is taken with regard to a redundant dictionary also determined by the function class. The existing theory of greedy type algorithms in Banach spaces provides a constructive way of building good cubature formulas.

For details see:
V. N. Temlyakov, Cubature Formulas and Related Questions, IMI Preprint series, 2002:06, 1–38.
(website: http://www.math.sc.edu/∼imip)

Toward Derandomization of Owen's Random Scrambling

Shu Tezuka

First, I overview the worst-case, random-case, and average-case expected errors for the integration problem with respect to Owen's random scrambling scheme. Then, I introduce the i-binomial scrambling, which requires much smaller sample space compared Owen's scrambling, and show that all the three cases above still hold with the i-binomial scrambling. Finally, I discuss future research direction for derandomization of scramble nets.

On Tractability and Strong Tractability of Weighted Integration on Bounded or Unbounded Regions in $\mathbf{R}^d$

Greg W. Wasilkowski

There are number of results on tractability and strong tractability for the integration problem over bounded domains. However, the majority of them assume functions with dominating mixed partial derivatives bounded in the L_2 norm. Much less is known when the derivatives are bounded in the L_1 norm, and almost nothing when a finite L_p norm is assumed for an arbitrary p. The focus of this talk is to extend known tractability results to spaces of

functions with the derivatives bounded in L_p norms. Moreover, we consider weighted integration with the domain of integration being not necessarily bounded.

A Quantum Algorithm for Parametric Integration

Carsten Wiegand

We study parametric integration of functions from the class $C^r([0,1]^{d_1+d_2})$ to $C([0,1]^{d_1})$ in the quantum model of computation. We analyze the convergence rate of parametric integration for a particular choice of the problem parameters d_1, d_2 and r. For this special case we can show that the quantum convergence rate is faster than the rates of the optimal deterministic and randomized classical algorithms.

Tractability of Multivariate Problems (Revisited)

Henryk Woźniakowski

We consider tractability of multivariate problems in weighted Hilbert spaces. The weights that have been so far studied were independent of the dimension or were uniformly bounded. We remove these constraints and obtain necessary and sufficient conditions on the weights to get tractability and strong tractability. We also study generalized (non-exponential) tractability, and show that for the class $\Lambda^{\rm all}$ (all continuous linear functionals) we can obtain tractability for the tractability functions $x^{\ln^{\beta_i} x}$ iff $\beta_1\beta_2 \geq 1$. For the class $\Lambda^{\rm std}$ (only function values) we show that even large β_i's are not enough to get tractability without weights.