Summary of the Dagstuhl Seminar 04401 (26.09.-01.10.2004) "Algorithms and Complexity for Continuous Problems" organized by Th. Müller-Gronbach, E. Novak, K. Petras and J. F. Traub

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1 Motivation

The goal of this workshop was to bring together researchers from different communities working on computational aspects of continuous problems.

Continuous computational problems arise in many areas of science and engineering. Examples include path and multivariate integration, function approximation, optimization, as well as differential, integral and operator equations. Understanding the complexity of such problems and constructing efficient algorithms is both, important and challenging.

The workshop was of a very interdisciplinary nature with invitees from, e.g., computer science, numerical analysis, discrete, applied and pure mathematics, physics, statistics and scientific computation. Many of the lectures were presented by Ph. D. students.

Compared to earlier meetings, several very active research areas received more emphasis. These include Quantum Computing, Complexity and Tractability of high-dimensional problems, Stochastic Computation, and Quantization, which was an entirely new field for this workshop.

Due to strong connections between the topics treated at this workshop many of the participants initiated new cooperations and research projects.

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The meeting was the eighth in a series of Dagstuhl-workshops on "Algorithms and Complexity for Continuous Problems". This topic belongs to the focus group research areas of the Foundation of Computational Mathematics Society and is also the topic of the Working Group 1.1 of the International Federation for Information Processing. The work of the attendants was supported by a variety of funding agencies including the Deutsche Forschungsgemeinschaft, the National Science Foundation and the Defense Advanced Research Projects Agency (USA), the Australian Research Council, and the State Committee for Research (Poland).

Selected papers from the workshop will be published in a special issue of the Journal of Complexity.

2 Complexity and Regularization of Ill-Posed Problems

Ill-posed inverse problems arise in a number of applications. Usually they have a form of parameter identification problems connected with the situation that the physical law (e.g., the type of differential equation) governing the process is known, but quantitative information about physical parameters (equation coefficients) is not available. Then the problem is to recover the values of these parameters from noisy observations of the underlying process. As a rule, such a problem is ill-posed in the sense that the parameters depend in a discontinuous way on the measurements, and its numerical treatment requires the application of special regularization methods.

A new approach to regularization in the case of numerical differentiation with unknown smoothness of the function was presented in the talk by Pereverzev.

A challenging question for future research is the optimal regularization with the use of a Quantum Computer. It seems that a new concept of regularization will be necessary for answering this question.

3 Non-Linear Approximation

Approximation theory and widths are fundamental tools for many problems in information based complexity. For instance, they are heavily used in the worst case analysis of operator equations. Non-linear approximation and widths are particularly related to adaptive algorithms. On the Dagstuhl conference, we learned about different facets of non-linear approximation.

One direction was the progress in approximation by manifolds and the related non-linear n-widths. A rather different topic was the selection of good approximations from infinite series by thresholding.

The talks in this area gave the impression that there will be further impact of non-linear methods in approximation to the investigation of more general analytic problems.

4 Tractability of High-Dimensional Numerical Problems

A number of talks dealt with the tractability and strong tractability of multivariate problems. Tractability means that it is possible to reduce the initial error of a d-variate problem by ε in cost bounded from above by a polynomial in d and in ε^{-1}. If the cost can be bounded independently of d, the corresponding problem is said to be strongly tractable. Moreover the corresponding algorithms that reduce the error in polynomial cost are called polynomial-time and strongly polynomial-time algorithms, respectively.

In the talk by J.O. Wojatszczyk, a Ph.D. student, it was shown that the problem of approximating $\int_{[0,1]^d} f(\mathbf{x})\,\mathrm{d}\mathbf{x}$ is not strongly tractable even for the class of smooth functions with all partial derivatives uniformly bounded.

The class $C^\infty([0,1]^d)$ is isotropic and this could be a reason of the negative result. Positive results for non-isotropic classes of functions were presented in the talk by G. W. Wasilkowski who showed in a constructive way that multivariate tensor-product problems defined over spaces of functions with limited interactions between specific variables are always tractable or even strongly tractable. Such problems appear in a number of applications including some in finance and physics.

Tractability of the classical integration problem, i.e., the problem of approximating $\int_{[0,1]^d} f(\mathbf{x})\,\mathrm{d}\mathbf{x}$ for (weighted) tensor product spaces of integrands was addressed by a number of speakers in connection with the Monte Carlo and Quasi-Monte Carlo Methods.

5 Quasi-Monte Carlo Methods

In the area of quasi-Monte Carlo (QMC) methods, a significant advance reported at this meeting concerns the fast computation of high-dimensional QMC integration rules that are optimal in suitably weighted Sobolev spaces. By definition, QMC rules are deterministic equal-weight cubature rules for the d-dimensional unit cube. In earlier Dagstuhl meetings during the second half of the last decade the tractability questions for QMC rules in weighted Sobolev spaces were answered conclusively, through the establishment of precise necessary and sufficient conditions under which the minimal worst-case errors are bounded independently of the dimension d. But the original proofs were not constructive. Constructions that achieve the theoretical bounds (modulo constants) were reported at the 2000 and 2002 Dagstuhl meetings, but the construction costs, though polynomial in both d and the number of cubature points n, were still substantial. At the present meeting Nuyens and Cools, computer scientists from Belgium, reported on a rearrangement of the component-by-component (CBC) construction for 'randomly-shifted lattice rules', that reduced the cost to $O(dn \log n)$, making the construction cost-effective even for very large values of d and n. The new algorithm makes feasible the routine CBC construction of high-quality randomly shifted lattice rules for arbitrary choices of the weights.

6 Quantum Computing

This research is concerned with the study of the potential capabilities of quantum computing in the area of continuous problems, that is, the question whether a quantum computer can solve numerical problems more efficiently than a classical computer. In the past four years a number of basic problems of analysis has been studied from this point of view. This research lead to matching upper and lower complexity bounds for the quantum setting, which puts us in a position to compare them with those for the classical deterministic and randomized setting, previously obtained in information-based complexity theory, and this way to assess the potential speedup of quantum computing over deterministic and randomized classical computing.

The seminar had 6 talks on quantum computing, which shows the lively interest in this field. In 2002 we also had 6 talks, while the 2000 seminar was the one at which the very first complexity results on quantum computing for continuous problems were presented (in one talk, by E. Novak). Specific numerical problems addressed in the 2004 talks were: path integrals, elliptic PDE, parametric integration, gradient computation and eigenvalue computation for Sturm-Liouville problems.

A challenge of the quantum setting in information-based complexity theory is the following: By now we know of exponential speedup of quantum computing over the deterministic setting for a number of numerical problems. In contrast, one usually observes a quadratic speedup over the classical randomized setting, like in the Grover's search problem. This was also visible in the talks of this seminar. No natural, numerical problem has been identified so far which shows an exponential speedup over the classical randomized setting.

Two talks were devoted to a new issue – the so-called power query. This is another (though related) challenge in this area: An oracle type which leads to exponential speedups over classical (deterministic or randomized) algorithms and, in general also over quantum algorithms with the standard query type. The seemingly very difficult open problem is to identify problems for which such an oracle is efficiently implementable.

During the seminar a special discussion devoted to the challenges of quantum computation for continuous problems was arranged.

7 Stochastic Computation and Quantization

Various real world phenomena that arise, e.g., in Physics, Biology, or Finance, can be described in terms of a time continuous system that randomly evolves in a finite or infinite dimensional state space. Typically, such systems are modelled by stochastic differential equations (SDEs) in the finite dimensional case or stochastic partial differential equations (SPDEs) in the infinite dimensional case, and one is interested in the analysis of its random dynamics. Since those equations can be solved explicitly only in exceptional cases, algorithms for approximation must be used in general.

Approximation of stochastic (partial) differential equations may roughly be divided into the categories of strong approximation and weak approximation. In the first case one is approximating the trajectories of the solution. In the second case one is dealing with approximation of distributional properties of the solution.

Pathwise approximation of SDEs started with the pioneering work of

Maruyama in 1955 and intensively developed since the mid seventies. Meanwhile, the complexity of this problem in standard settings is quite well understood and (asymptotically) optimal algorithms are known. These results have been presented in former Dagstuhl conferences. However, many equations that are used in practice are not covered by these findings. A partial list includes SDEs that incorporate a memory effect, SDEs with Markovian switching, SDEs with fractional noise and jump diffusion equations.

Three talks were devoted to the complexity of SDEs in those non-standard settings. We mention in particular the results by A. Neuenkirch, a Ph.D. student, who presented sharp complexity bounds as well as new and asymptotically optimal descretization schemes for SDEs with fractional noise.

Investigation of SPDEs from an approximation point of view started only about ten years ago and is still in its infancy. We had two lectures on this topic from different perspectives. Upper bounds under rather general assumptions on the equation were given by E. Hausenblas, while K. Ritter presented sharp complexity bounds and an asymptotically optimal time discretization for the stochastic heat equation.

In the field of weak approximation we had three lectures dealing with computation of deterministic quantities. In computational practice randomized (or Monte Carlo) as well as deterministic algorithms are used in this case. Two lectures, by M. Kwas and by K. Petras, were devoted to the question, whether randomized algorithms are superior to the deterministic ones. A positive answer was given and sharp complexity bounds for both classes of algorithms were presented. The asymptotically optimal randomized methods use a new variance reduction that is based on high-dimensional approximation by means of sparse grids (Smolyak formulas).

The solution of an SDE defines a probability measure μ on a suitable function space. Weak Itô-Taylor schemes are classical randomized algorithms for weak approximation, and those methods yield measures with finite support as approximations to μ. Optimal approximation of a probability measure by measures with finite support is called the quantization problem. Motivated by applications in signal processing this problems was studied in finite-dimensional spaces already for a couple of decades. Since about 5 years the infinite-dimensional case is studied as well, and the lectures by H. Luschgy and by S. Dereich, who recently completed his Ph.D., were devoted to this problem. The results known so far focus on sharp bounds for the optimal quantization error. The problem of constructive quantization seems to be open up to now. We also had one lecture dealing with the new problem of quantization on self-similar sets in $\mathbf{R^d}$.

8 Global Optimization

We had two lectures about the difficult problem of global optimization for continuous functions $f : [0,1]^d \to \mathbf{R}$. In the lecture by J. Calvin the case $d = 1$ was studied and optimal convergence rates for the average error with respect to the Wiener measure were presented.

The second talk was given by M. Horn, by a Ph. D. student, who presented results for $d \in \mathbf{N}$ and certain Lipschitz functions with (possibly) many local extrema. For these function classes one obtains a quadratic speed up of adaptive methods over nonadaptive ones. Moreover, a universal algorithm was presented that is almost optimal for many different function classes.

9 Differential and Integral Equation

We had several lectures on differential and integral equations. Most of these lectures were also closely related to other topics of the seminar: parabolic equations (Petras), non-linear approximation and adaptive methods, quantum computing (Heinrich) and stochastic computation (including Monte Carlo methods). It was fascinating to see the great interaction between fields that are, at first glance, far away from each other.

A series of talks by S. Dahlke, E. Novak and W. Sickel was devoted to optimal linear and nonlinear approximation for elliptic problems. Here, different widths and wavelet methods played a great role.

Monte Carlo methods (randomized algorithms) were adressed in the lectures of N. Golyandina, S. Heinrich, and H. Pfeiffer, a Ph. D. student. New results about adaptive methods were presented by A. Szepessy.