Two-round n-out-of-n and multi-signatures and trapdoor commitment from lattices

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Intro

Background & Motivation

- Two approaches to lattice-based signatures among the NIST PQC standardization finalists:
  - Hash-and-sign [GPV08]: Falcon
  - **Fiat–Shamir with aborts** [Lyu09]: Dilithium

- Renewed interest in multi-party signing: upcoming NIST standardization, Blockchain, etc.
  - Many existing works on round-efficient n-party signatures in the **discrete log** setting (cf. Drijvers et al. [DEF+19]).

- FSwA-style signature has a structure similar to **Schnorr**.

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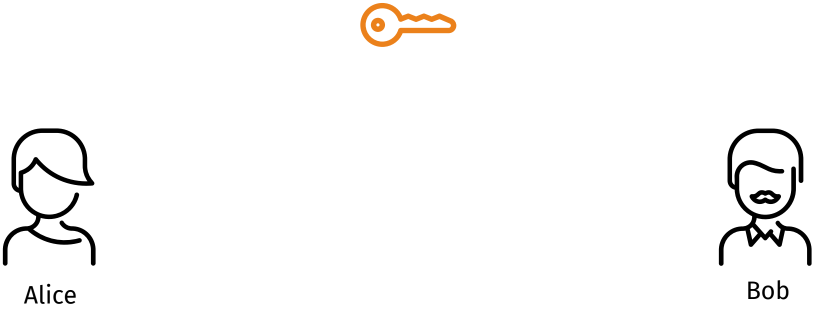
2-out-of-2 Signing



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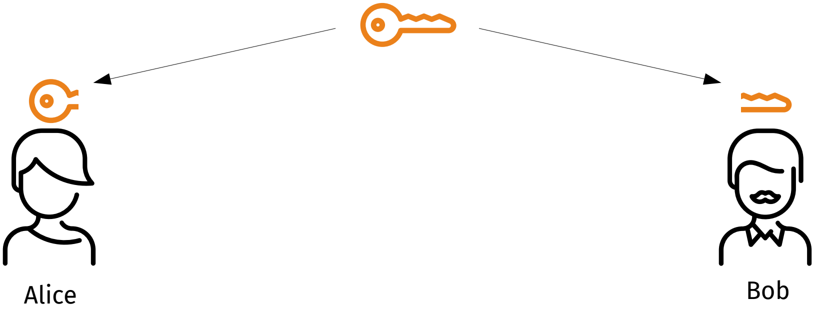

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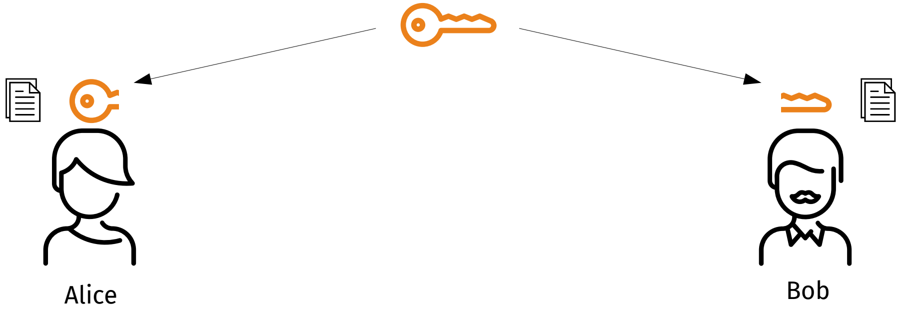

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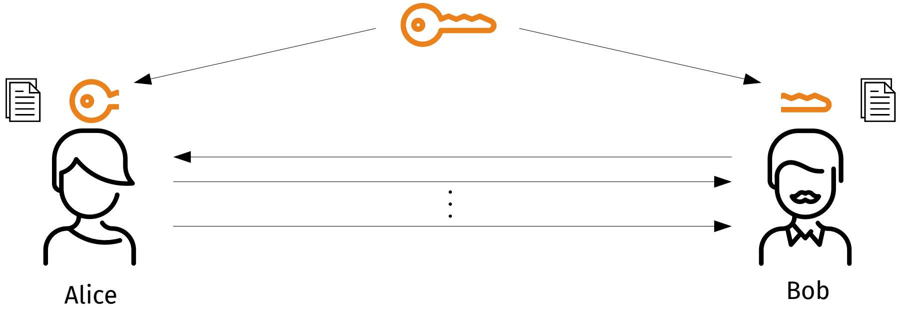

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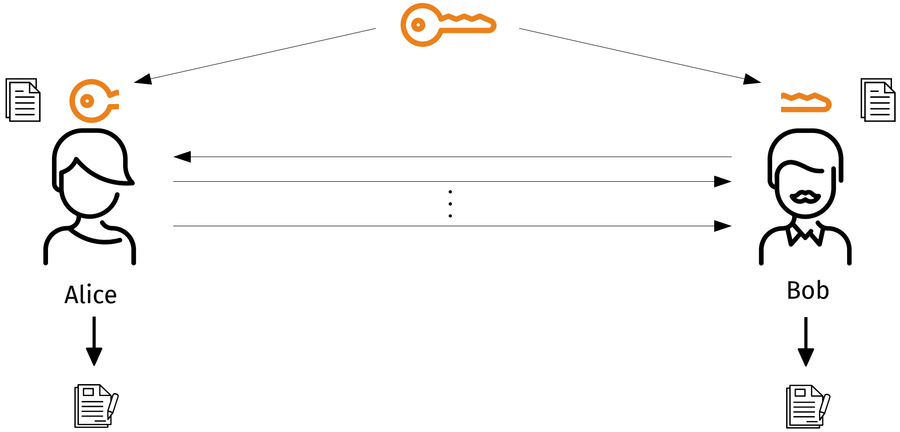

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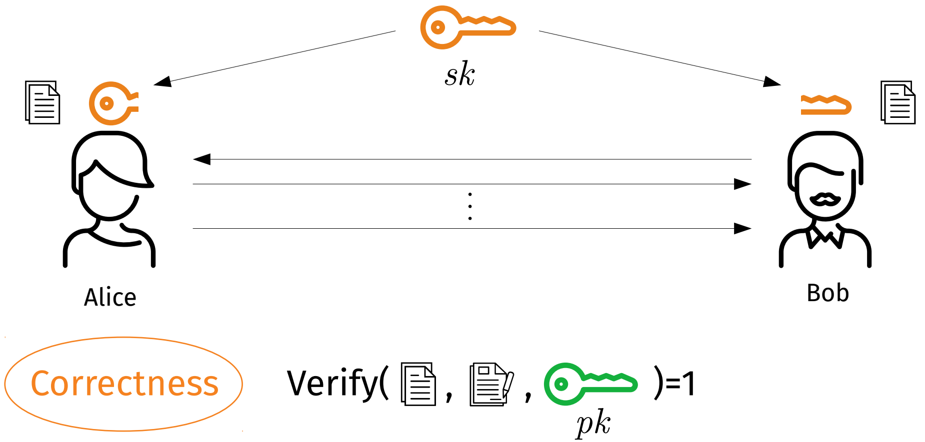
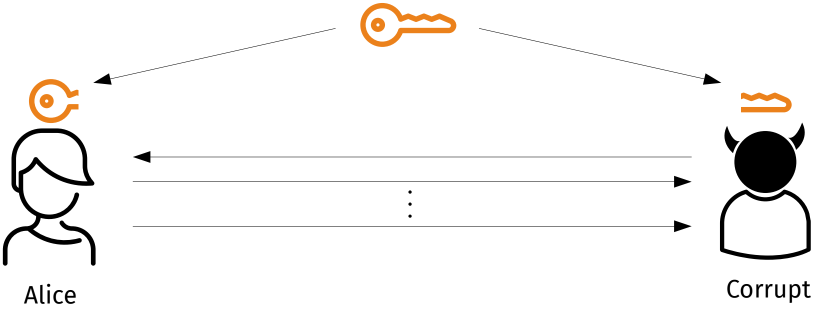


2-out-of-2 Signing : Game-based Security



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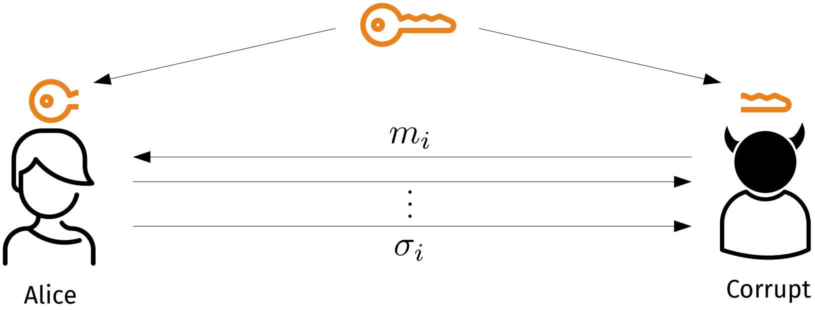

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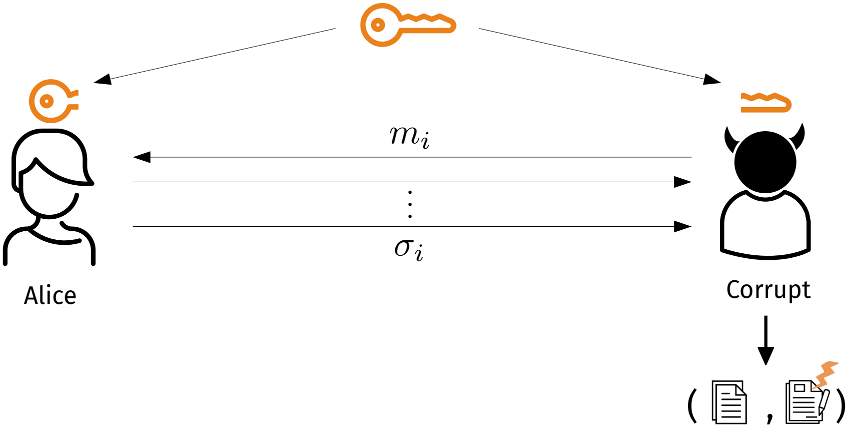

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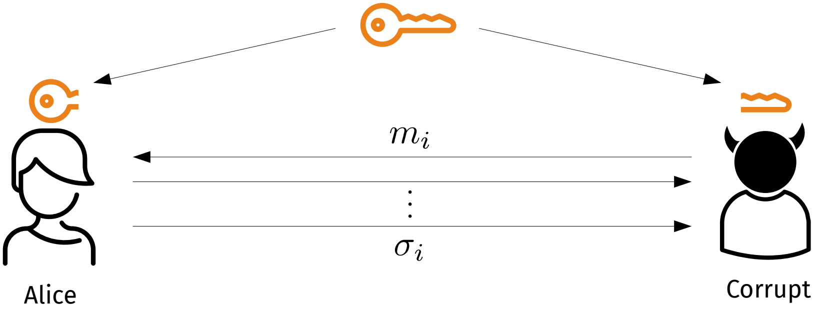

Unforgeability Pr[Verify(, ,)=1 ∧ $\neq m_i$] is negl.

Fiat–Shamir with Aborts: Dilithium ID

$\boxed{P(\mathbf{A}, \mathbf{s})}$		$\boxed{V(\mathbf{A}, \mathbf{t} = \mathbf{A}\mathbf{s})}$
$\mathbf{y} \leftarrow\!\!\$\ D^{\ell+k} // D$ is Gaussian or uni.		
$\mathbf{w} := \mathbf{A}\mathbf{y}$	$\xrightarrow{\mathbf{w}}$	$c \leftarrow\!\!\$\ C \subset R$
$\mathbf{z} := c\mathbf{s} + \mathbf{y}$	$\xleftarrow{c}$	
If $\mathrm{RejSamp}(c\mathbf{s}, \mathbf{z}) = 0:$ restart	$\xrightarrow{\mathbf{z}}$	$\mathbf{A}\mathbf{z} - c\mathbf{t} = \mathbf{w} \wedge \|\mathbf{z}\|$ is small

- Operate on a vector of polynomials in a quotient ring $R_q = \mathbb{Z}_q[X]/(f(X))$.
- Secret key is a small $\mathbf{s} \in R_q^{\ell+k}$; public key consists of $\mathbf{A} = [\mathbf{A}'|\mathbf{I}]$ with random $\mathbf{A}' \in R_q^{k\times\ell}$ and $\mathbf{t} = \mathbf{A}\mathbf{s}$.
- $\mathbf{z} \in R_q^{\ell+k}$ has to be small $\leadsto$ c and $\mathbf{y}$ have to be small as well.
- RejSamp = rejection sampling: force $\mathbf{z}$ to be independent of $\mathbf{s}$ (**non-linear operation**)

Fiat–Shamir with Aborts: Dilithium ID vs. Schnorr ID

$$
\begin{array}{lcl}
\boxed{P(G, \mathbf{s})} & & \boxed{V(G, \mathbf{t} = [\mathbf{s}]G)} \\
\mathbf{y} \leftarrow_{\$} \mathbb{Z}_q & & \\
\mathbf{w} = [\mathbf{y}]G & \xrightarrow{\quad \mathbf{w} \quad} & c \leftarrow_{\$} C \subset R \\
\mathbf{z} := c\mathbf{s} + \mathbf{y} & \xleftarrow{\quad c \quad} & \\
 & \xrightarrow{\quad \mathbf{z} \quad} & [\mathbf{z}]G - c\mathbf{t} = \mathbf{w}
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Security of FSwA

- Soundness from **Module-SIS** and **Module-LWE**
  - Suppose $P^*(\mathbf{A}, \mathbf{t})$ can correctly answer c and c' for the same $\mathbf{w}$

$$\rightsquigarrow \mathbf{Az} - c\mathbf{t} = \mathbf{w} = \mathbf{Az}' - c'\mathbf{t}$$

  - $(\mathbf{A}, \mathbf{t} = \mathbf{As}) \approx^c (\mathbf{A}, \mathbf{t} \leftarrow_{\$} R_q^k)$ due to LWE.
  - Then using P^* find a non-zero solution to the SIS problem wrt $[\mathbf{A}|\mathbf{t}]$:

$$[\mathbf{A}|\mathbf{t}] \begin{bmatrix} \mathbf{z} - \mathbf{z}' \\ c' - c \end{bmatrix} = \mathbf{0}.$$

- Non-aborting statistical HVZK
  - If protocol doesn't abort: simulator outputs $(\mathbf{w} = \mathbf{Az} - c\mathbf{t}, c, \mathbf{z} \leftarrow_{\$} D^{\ell+k})$.

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Two-party Signing from FSwA

Our results

- Two-round multi-party FSwA signing with full security proof in CROM
- Two instantiations: n**-out-of-**n signatures and **multi-signatures**.
- This talk: focused on $n = 2$, but the approach can be generalized to $n > 2$.

Comparison with previous lattice-based multi-party signing

	Functionality	# Rounds	Type	Security	Building blocks
[BGG+18]	t-out-of-n	1	FSwA	Lyubashevsky ’12	Threshold FHE
[BKP13]	t-out-of-n	1	H&S	GPV ’08	Honest-majority MPC
Our DS_3	n-out-of-n	3	FSwA	MLWE	Homomorphic COM
Our DS_2	n-out-of-n	2	FSwA	MLWE & MSIS	Homomorphic TDCOM
[BS16]	Multisig	3	FSwA	DCK	—
[FH20]	Multisig	3	FSwA	Heuristic assumption / QROM	—
Our MS_2	Multisig	2	FSwA	MLWE & MSIS	Homomorphic TDCOM

Bare-bone 2-party signing: Schnorr vs Dilithium

$P_1(\mathbf{s}_1, pk = [\mathbf{s}_1 + \mathbf{s}_2]G)$		$P_2(\mathbf{s}_2, pk)$
$\mathbf{y}_1 \leftarrow\$ \mathbb{Z}_q; \mathbf{w}_1 = [\mathbf{y}_1]G$	$\xrightarrow{\quad \mathbf{w}_1 \quad}$	
$c \leftarrow \mathsf{H}(\mathbf{w}_1 + \mathbf{w}_2, m, pk)$	$\xleftarrow{\quad \mathbf{w}_2 \quad}$	
$\mathbf{z}_1 = c\mathbf{s}_1 + \mathbf{y}_1$		
	$\xrightarrow{\quad \mathbf{z}_1 \quad}$	
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Output $((\mathbf{w}_1 + \mathbf{w}_2, \mathbf{z}_1 + \mathbf{z}_2), m)$		

- Round 1: Exchange “commitments” $\mathbf{w}_i$ and locally derive a joint challenge c
- Round 2: Compute signature shares $\mathbf{z}_i$ and exchange them

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$\mathbf{z}_1 = c\mathbf{s}_1 + \mathbf{y}_1$

If $\mathsf{RejSamp}(c\mathbf{s}_1, \mathbf{z}_1) = 0 : \mathbf{z}_1 := \perp$ $\xrightarrow{\mathbf{z}_1}$

If $\mathbf{z}_i = \perp$: restart $\xleftarrow{\mathbf{z}_2}$

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- Round 1: Exchange “commitments” $\mathbf{w}_i$ and locally derive a joint challenge c
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Two issues of bare-bone protocol

1. Simulation of rejected $(\mathbf{w}_i, c, \perp)$
    - Not a problem for single-user signing or NIZK
    - Problematic in **interactive** FSwA protocols
    - Just sending Commit($\mathbf{w}_i$) is not enough: need $\mathbf{w}_1 + \mathbf{w}_2$ **before** computing challenge
2. Malicious P_2 can choose the first message depending on P_1's output!
    - Naive: **extra round for "committing to commitment"** to construct an honest party simulator
    - Potential **concurrent attack** (variant of Drijvers et al. [DEF+19] against Schnorr multisigs)

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Our solutions

1. Simulation of rejected $(\mathbf{w}_i, c, \perp)$
   - Send homomorphic $\mathsf{Commit}(\mathbf{w}_i)$
   - Hide $\mathbf{w}_i$ until the rejection sampling succeeds while computing $\mathbf{w}_1 + \mathbf{w}_2$ earlier.
2. Malicious P_2 could choose $\mathbf{w}_2$ depending on $\mathbf{w}_1$!
   - Use trapdoor homomorphic commitment to avoid an extra round

First step: Three-round protocol from "double" commitments

$P_1(\mathbf{s}_1, \mathbf{t} = \mathbf{A}(\mathbf{s}_1 + \mathbf{s}_2), ck)$		$P_2(\mathbf{s}_2, \mathbf{t}, ck)$
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$com_1 \leftarrow \mathsf{Commit}_{ck}(\mathbf{w}_1; r_1)$	$\xleftarrow{h_2 = \mathsf{H}(com_2)}$	
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Signature verification

- Vf$(com, \mathbf{z}, r, m, ck, (\mathbf{A}, \mathbf{t}))$:
  1. Get a challenge $c \leftarrow \mathsf{H}(com, m, \mathbf{t})$
  2. Reconstruct committed $\mathbf{w} = \mathbf{A}\mathbf{z} - c\mathbf{t}$
  3. Verify

$$\|\mathbf{z}\| \text{ is small} \wedge \mathsf{Open}_{ck}(com, r, \mathbf{w}) = 1$$

- Correctness holds since
  - Linearity of $f_{\mathbf{A}}(\mathbf{x}) = \mathbf{A}\mathbf{x}$:

$$\mathbf{A}\mathbf{z} - c\mathbf{t} = \mathbf{A}(\mathbf{z}_1 + \mathbf{z}_2) - c(\mathbf{A}\mathbf{s}_1 + \mathbf{A}\mathbf{s}_2) = \mathbf{w}_1 + \mathbf{w}_2$$

  - Homomorphism of the commitment: $\mathsf{Open}_{ck}(com, r, \mathbf{w}) = \mathsf{Open}_{ck}(com_1 + com_2, r_1 + r_2, \mathbf{w}_1 + \mathbf{w}_2) = 1$
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Security

☺ Provably Secure!

- If protocol doesn't abort: Honest party oracle can be simulated with the NA-HVZK simulator
- If protocol aborts: Hiding commitment reveals nothing about $\mathbf{w}_i$
- Security reduction to (Module) LWE without the forking lemma, thanks to the **lossy ID technique** (Abdalla et al. [AFLT16])

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Efficiency

☺ No expensive machinery like FHE, MPC, Gaussian sampling over lattices, etc.

- L^2-norm of $\mathbf{z}$ grows by a factor of $\sqrt{n}$: given n discrete Gaussian samples $\mathbf{z}_i \sim D_\sigma$, their sum $\mathbf{z} = \mathbf{z}_1 + \ldots + \mathbf{z}_n$ is statistically close to $D_{\sqrt{n}\sigma}$.
- Need to wait for all n parties to pass the rejection sampling: if each party succeeds with prob. $1/M$ then the entire protocol restarts M^n times
  - To keep M^n constant, σ grows by a factor of n.
  - Or parallel repetition is required.

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Two-round protocol

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If $\mathsf{RejSamp}(c\mathbf{s}_1, \mathbf{z}_1) = 0 : (\mathbf{z}_1, r_1) := (\perp, \perp)$	$\xrightarrow{\mathbf{z}_1, r_1}$	
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Output $((com_1 + com_2, \mathbf{z}_1 + \mathbf{z}_2, r_1 + r_2), m)$		

How to drop the extra round?

$P_1(\mathbf{s}_1, \mathbf{t} = \mathbf{A}(\mathbf{s}_1 + \mathbf{s}_2), ck)$ $\qquad$ $P_2(\mathbf{s}_2, \mathbf{t}, ck)$

$\mathbf{y}_1 \leftarrow_\$ D^{\ell+k}; \mathbf{w}_1 = \mathbf{A}\mathbf{y}_1$ $\quad \xrightarrow{com_1 = \mathsf{Commit}_{ck}(\mathbf{w}_1; r_1)}$

$c \leftarrow \mathsf{H}(com_1 + com_2, m, \mathbf{t})$ $\quad \xleftarrow{com_2 = \mathsf{Commit}_{ck}(\mathbf{w}_2; r_2)}$

$\mathbf{z}_1 = c\mathbf{s}_1 + \mathbf{y}_1$

If $\mathsf{RejSamp}(c\mathbf{s}_1, \mathbf{z}_1) = 0 : (\mathbf{z}_1, r_1) := (\perp, \perp)$ $\quad \xrightarrow{\mathbf{z}_1, r_1}$

If $\mathbf{z}_i = \perp$: restart $\quad \xleftarrow{\mathbf{z}_2, r_2}$

Output $((com_1 + com_2, \mathbf{z}_1 + \mathbf{z}_2, r_1 + r_2), m)$

Simulation fails!

$\boxed{\mathsf{Sim}(\mathbf{t}_1, \mathbf{t} = \mathbf{t}_1 + \mathbf{A}\mathbf{s}_2, ck)}$ $\boxed{\mathcal{A}(\mathbf{s}_2, \mathbf{t}, ck)}$

$\mathbf{z}_1 \leftarrow_\$ D^{\ell+k}; c \leftarrow_\$ C; \mathbf{w}_1 = \mathbf{A}\mathbf{z}_1 - c\mathbf{t}_1$ $\xrightarrow{com_1 = \mathsf{Commit}_{ck}(\mathbf{w}_1; r_1)}$

com_2 is not known! $\rightsquigarrow$ can't program RO such that

$\mathsf{H}(com_1 + com_2, m, \mathbf{t}) := c$ $\xleftarrow{com_2 = \mathsf{Commit}_{ck}(\mathbf{w}_2; r_2)}$

With prob. $1 - 1/M : (\mathbf{z}_1, r_1) := (\perp, \perp)$ $\xrightarrow{\mathbf{z}_1, r_1}$

If $\mathbf{z}_i = \perp :$ restart $\xleftarrow{\mathbf{z}_2, r_2}$

Output $((com_1 + com_2, \mathbf{z}_1 + \mathbf{z}_2, r_1 + r_2), m)$

Simulation fails!

$\mathsf{Sim}(\mathbf{t}_1, \mathbf{t} = \mathbf{t}_1 + \mathbf{A}\mathbf{s}_2, ck)$		$\mathcal{A}(\mathbf{s}_2, \mathbf{t}, ck)$
$\mathbf{z}_1 \leftarrow_\$ D^{\ell+k}; c \leftarrow_\$ C; \mathbf{w}_1 = \mathbf{A}\mathbf{z}_1 - c\mathbf{t}_1$	$\xrightarrow{com_1 = \mathsf{Commit}_{ck}(\mathbf{w}_1; r_1)}$	
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If $\mathbf{z}_i = \perp$: restart	$\xleftarrow{\mathbf{z}_2, r_2}$	
Output $((com_1 + com_2, \mathbf{z}_1 + \mathbf{z}_2, r_1 + r_2), m)$		

Also: If ck is fixed then the same concurrent attack applies!

$\rightsquigarrow$ Need per-message keys $ck = \mathsf{H}(m, \mathbf{t})$

Solution: Straight-line simulation with trapdoor commitment (Damgård ’00)

- Commitment key generation outputs an extra trapdoor td
- Given td a commitment can be opened to any message!
- **Simulation sketch**
  1. Honest party simulator sends out a “fake” commitment $com_1 = \mathsf{TCommit}_{ck}(td)$ in the first round
  2. com_1 can be later equivocated to anything depending on the derived joint challenge c.

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Simulation with TDCOM

$\boxed{\mathsf{Sim}(\mathbf{t}_1, \mathbf{t} = \mathbf{t}_1 + \mathbf{A}\mathbf{s}_2, ck, td)}$ $\boxed{\mathcal{A}(\mathbf{s}_2, \mathbf{t}, ck)}$

$\xrightarrow{com_1 \ = \ \mathsf{TCommit}_{ck}(td)}$

$c \leftarrow \mathsf{H}(com_1 + com_2, m, \mathbf{t})$ $\xleftarrow{com_2 \ = \ \mathsf{Commit}_{ck}(\mathbf{w}_2; r_2)}$

$\mathbf{z}_1 \leftarrow_\$ D^{\ell+k}; \mathbf{w}_1 = \mathbf{A}\mathbf{z}_1 - c\mathbf{t}_1$

$r_1 \leftarrow \mathsf{Eqv}_{ck}(td, com_1, \mathbf{w}_1)$

With prob. $1 - 1/M : (\mathbf{z}_1, r_1) := (\bot, \bot)$ $\xrightarrow{\mathbf{z}_1, r_1}$

If $\mathbf{z}_i = \bot :$ restart $\xleftarrow{\mathbf{z}_2, r_2}$

Output $((com_1 + com_2, \mathbf{z}_1 + \mathbf{z}_2, r_1 + r_2), m)$

Simulation with TDCOM

$\mathsf{Sim}(\mathbf{t}_1, \mathbf{t} = \mathbf{t}_1 + \mathbf{A}\mathbf{s}_2)$		$\mathcal{A}(\mathbf{s}_2, \mathbf{t})$
$ck \leftarrow \mathsf{H}(m, \mathbf{t})$		
//Invoke $(ck, td) \leftarrow \mathsf{TCGen}$ and program $\mathsf{H}(m, \mathbf{t}) := ck$	$\xrightarrow{com_1 = \mathsf{TCommit}_{ck}(td)}$	
$c \leftarrow \mathsf{H}(com_1 + com_2, m, \mathbf{t})$	$\xleftarrow{com_2 = \mathsf{Commit}_{ck}(\mathbf{w}_2; r_2)}$	
$\mathbf{z}_1 \leftarrow_\$ D^{\ell+k}; \mathbf{w}_1 = \mathbf{A}\mathbf{z}_1 - c\mathbf{t}_1$		
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Our two-round protocol: the final form

$P_1(\mathbf{s}_1, \mathbf{t} = \mathbf{A}(\mathbf{s}_1 + \mathbf{s}_2))$		$P_2(\mathbf{s}_2, \mathbf{t})$
$ck \leftarrow \mathsf{H}(m, \mathbf{t})$		$ck \leftarrow \mathsf{H}(m, \mathbf{t})$
$\mathbf{y}_1 \leftarrow_\$ D^{\ell+k}; \mathbf{w}_1 = \mathbf{A}\mathbf{y}_1$	$\xrightarrow{com_1 = \mathsf{Commit}_{ck}(\mathbf{w}_1; r_1)}$	
$c \leftarrow \mathsf{H}(com_1 + com_2, m, \mathbf{t})$	$\xleftarrow{com_2 = \mathsf{Commit}_{ck}(\mathbf{w}_2; r_2)}$	
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If $\mathbf{z}_i = \perp :$ restart	$\xleftarrow{\mathbf{z}_2, r_2}$	
Output $((com_1 + com_2, \mathbf{z}_1 + \mathbf{z}_2, r_1 + r_2), m)$		

Summary of the two-round protocol

- Per-message ck prevents the concurrent k-list sum attack.
- TDCOM requires **computationally binding** $\rightsquigarrow$ security proof relies on the forking lemma (leading to a larger security loss)
- Paper describes how to instantiate a lattice-based TDCOM from Baum et al's commitment [BDL+18] + Micciancio–Peikert lattice trapdoor [MP12].

Takeaways

- Multi-party FSwA signing with low round complexity & without FHE/MPC
- By deriving **per-user challenges** $c_i = \mathsf{H}(com, \mu, \mathbf{t}_i, L)$ our construction can be turned into a **two-round multi-signature** secure in the plain public-key model (= no dedicated key generation protocol is needed)
- Open questions:
  - Make the signature size less dependent on the number of parties n
  - Tighter security reduction & proof in QROM

Thank you! & Questions?

More details at `https://ia.cr/2020/1110`

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Concurrent attack against bare-bone protocol I

$\mathcal{A}$ (malicious) has $\mathbf{s}'$; P (honest) has $\mathbf{s}$; joint public key is $\mathbf{t} = \mathbf{A}(\mathbf{s}' + \mathbf{s})$

1. $\mathcal{A}$ starts k concurrent sessions on the same m; receive $\mathbf{w}_1, \ldots, \mathbf{w}_k$ from P
2. Let $\mathbf{w}^* = \mathbf{w}_1 + \ldots + \mathbf{w}_k$; Find m^*, $\mathbf{w}'_1, \ldots, \mathbf{w}'_k$ such that
$$\begin{aligned} c^* = \mathsf{H}(\mathbf{w}^*, m^*, \mathbf{t}) &= \mathsf{H}(\mathbf{w}_1 + \mathbf{w}'_1, m, \mathbf{t}) + \ldots + \mathsf{H}(\mathbf{w}_k + \mathbf{w}'_k, m, \mathbf{t}) \\ &= c_1 + \ldots + c_k \end{aligned}$$
by solving a **sparse, ternary variant of the generalized birthday problem for** $(k+1)$ **trees** [Wag02]: GBP over $\left(C = \left\{ c \in \mathbb{Z}^N : \|c\|_1 = \kappa \wedge \|c\|_\infty = 1 \right\}, +\right)$
3. $\mathcal{A}$ resumes the sessions by sending $\mathbf{w}'_1, \ldots, \mathbf{w}'_k$; P returns $\mathbf{z}_1 = c_1\mathbf{s} + \mathbf{y}_1, \ldots, \mathbf{z}_k = c_k\mathbf{s} + \mathbf{y}_k$.
4. Output a forgery $(\mathbf{w}^*, \mathbf{z}^*, m^*)$ where
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Concurrent attack against bare-bone protocol II

Why $(\mathbf{w}^*, \mathbf{z}^*, m^*)$ passes the verification:

- Thanks to the $(k+1)$-list sum solver $c^* = \mathsf{H}(\mathbf{w}^*, m^*, \mathbf{t}) = c_1 + \ldots + c_k$
- The forgery $\mathbf{z}^*$satisfies

$$\begin{aligned}\mathbf{z}^* &= c^*\mathbf{s}' + \mathbf{z}_1 + \ldots + \mathbf{z}_k \\ &= c^*\mathbf{s}' + (c_1 + \ldots + c_k)\mathbf{s} + (\mathbf{y}_1 + \ldots + \mathbf{y}_k) \\ &= c^*(\mathbf{s}' + \mathbf{s}) + (\mathbf{y}_1 + \ldots + \mathbf{y}_k)\end{aligned}$$

- Hence we have

$$\begin{aligned}\mathbf{A}\mathbf{z}^* - c^*\mathbf{t} &= \mathbf{A}(\mathbf{y}_1 + \ldots + \mathbf{y}_k) \\ &= \mathbf{w}^*\end{aligned}$$

- Verifier also checks $\|\mathbf{z}^*\|$ is small $\rightsquigarrow$ k should be sufficiently small.
  - Attack becomes easier for a general n-party setting

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TDCOM

A trapdoor commitment scheme TCOM consists of the following algorithms in addition to (CSetup, CGen, Commit, Open).

- TCGen$(cpp) \to (ck, td)$: The trapdoor key generation algorithm that outputs a key ck and the trapdoor td.
- TCommit$_{ck}(td) \to com$: The trapdoor committing algorithm that outputs a commitment com.
- Eqv$_{ck}(td, com, msg) \to r$: The equivocation algorithm that outputs randomness r.
- **Security**: for any $msg \in S_{msg}$, the distribution of (msg, ck, com, r) generated by the above algorithms is indistinguishable from the one honestly generated by CGen and Commit.