LadderLeak

Breaking ECDSA with Less than One Bit of Nonce Leakage

Real World Crypto 2021 (ePrint 2020/615, appeared at CCS '20)

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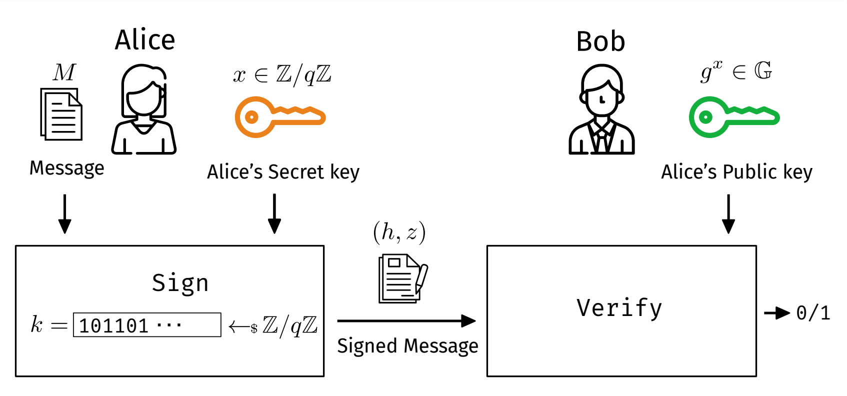
Nonce = Number used only once

“Nonce” in ECDSA/Schnorr-type Schemes

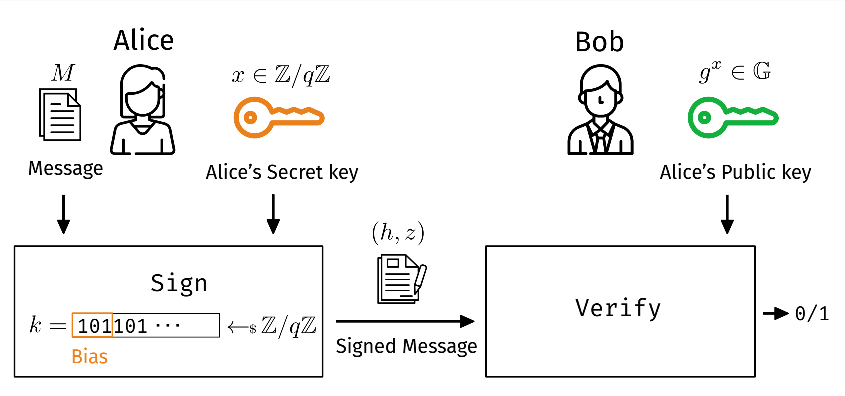


- k is a uniformly random value satisfying

$$k \equiv \underbrace{z}_{\text{public}} + \underbrace{h}_{\text{public}} \cdot x \mod q.$$

- k should **NEVER** be reused/exposed as $x = (z - z')/(h' - h) \mod q$

Risk of Biased/Leaky Nonces



- What if k is slightly biased ?
- Secret key x is recovered by solving the **hidden number problem (HNP)**

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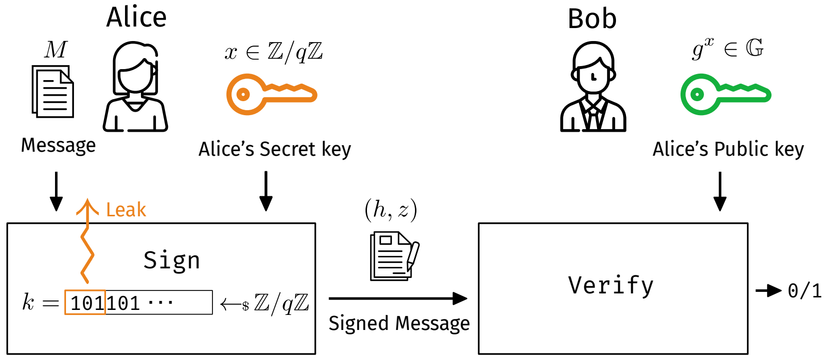

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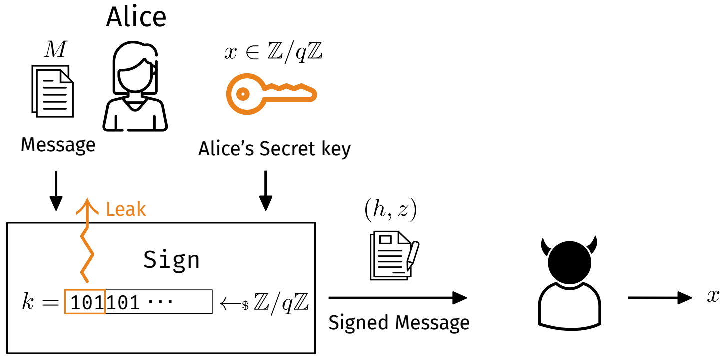

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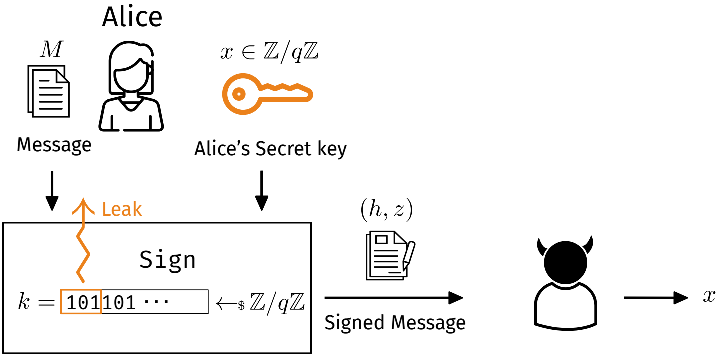

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Randomness Failure in the Real World

- Poorly designed/implemented RNGs
- Predictable seed (`srand(time(0))`)
- VM resets ⤳ same snapshot will end up with the same seed
- Side-channel leakage
- and many more…

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Technology

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Technology reporter, BBC News

6 January 2011

Share

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    - Allows us to exploit tiny amount of nonce leakage per signature
2. Novel class of cache timing attacks against the Montgomery ladder scalar multiplication in OpenSSL `1.0.2u` and `1.1.0l`, and RELIC 0.4.0.
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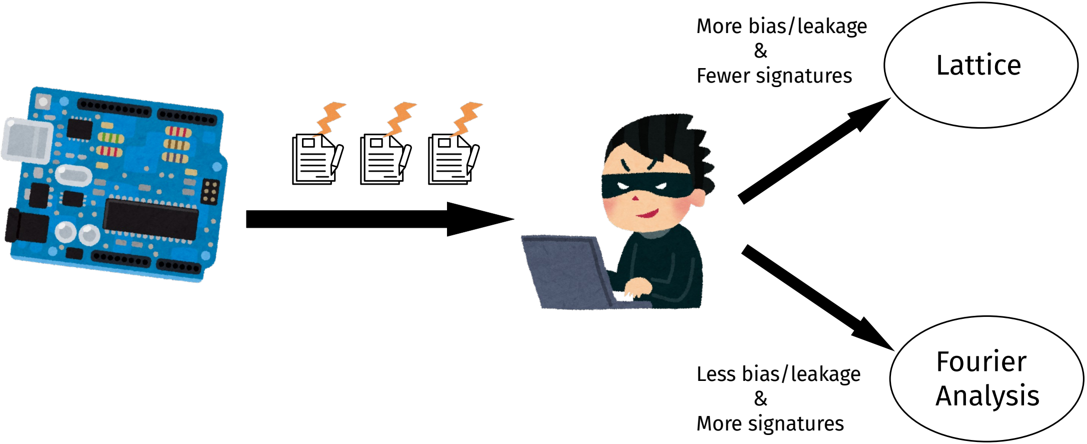
How to Exploit Nonce Leakage

How to solve the HNP: Lattice vs Fourier analysis



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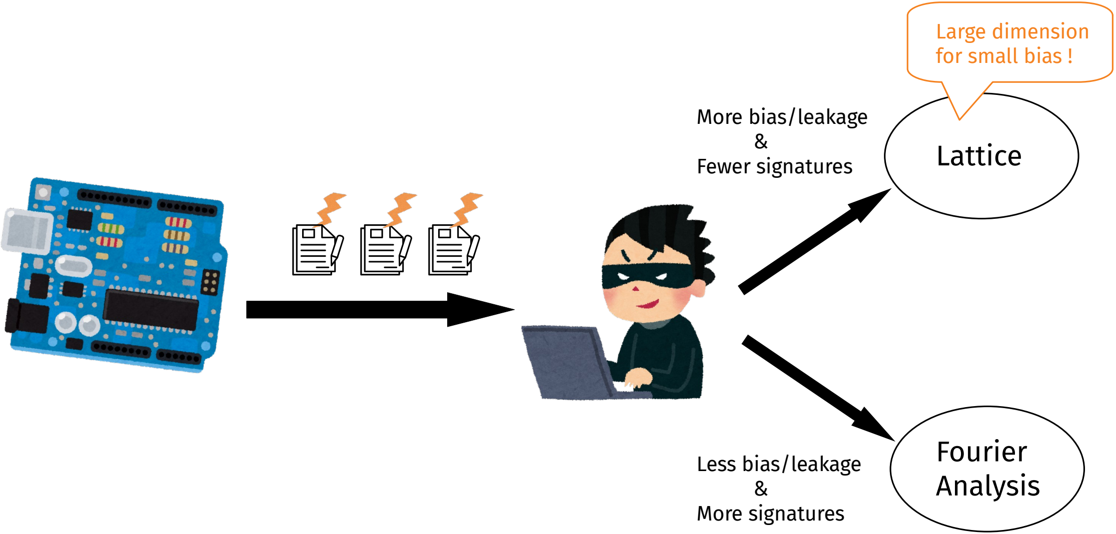

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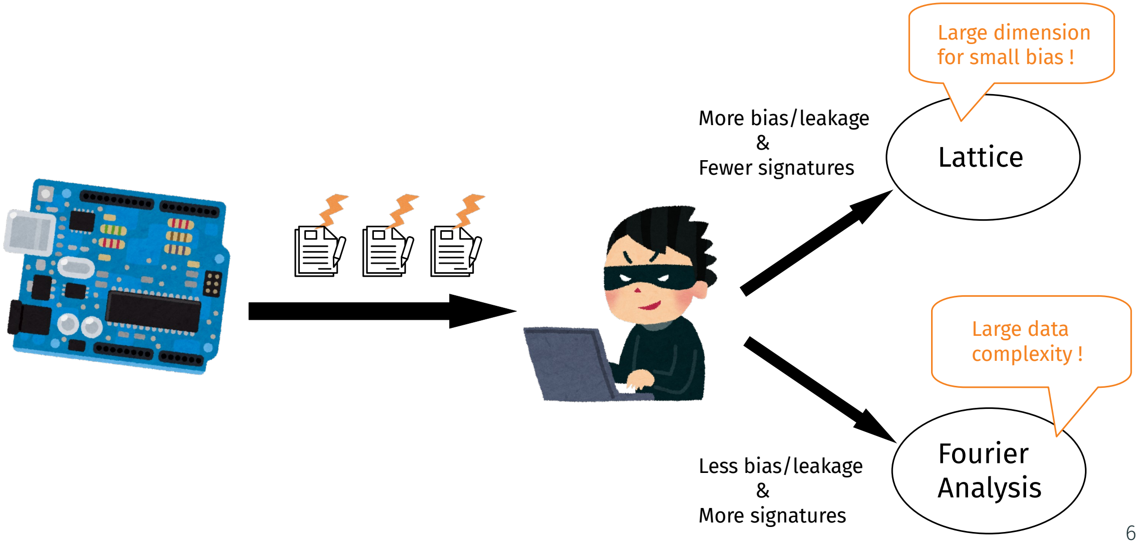

Questions

- Can we reduce #signatures for Fourier analysis-based attack?
- Can we attack even **less than 1-bit of nonce leakage** (= MSB is only leaked with prob. < 1)?

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Bleichenbacher's Attack: High-level Overview

- Step 1. Quantify the bias of nonce $K = \{k_i\}_{i \in \{1,\ldots,M\}}$
  - $\mathrm{Bias}_q(K) \approx 0$ if k is uniform in $\mathbb{Z}_q$
  - $\mathrm{Bias}_q(K) \approx 1$ if k is biased in $\mathbb{Z}_q$
  - Contribution 1: Analyzed the behavior $\mathrm{Bias}_q(K)$ when k's MSB is biased with probability < 1!
- Step 2. Find a candidate secret key which leads to the peak of $\mathrm{Bias}_q(K)$ (by computing FFT)
- Critical intermediate step: find **many small linear combinations** of integers h
  - Detect the bias peak correctly and efficiently
  - Contribution 2: Established time-data tradeoffs by applying algorithms for the **generalized birthday problem (GBP)**!

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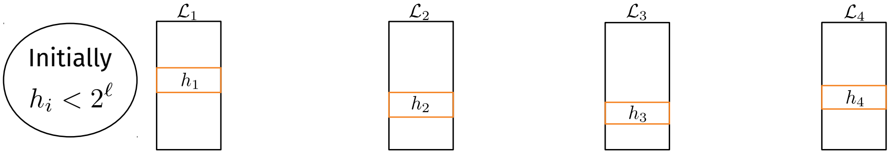
$\mathcal{K}$-list Sum for GBP (e.g., $\mathcal{K} = 4$)



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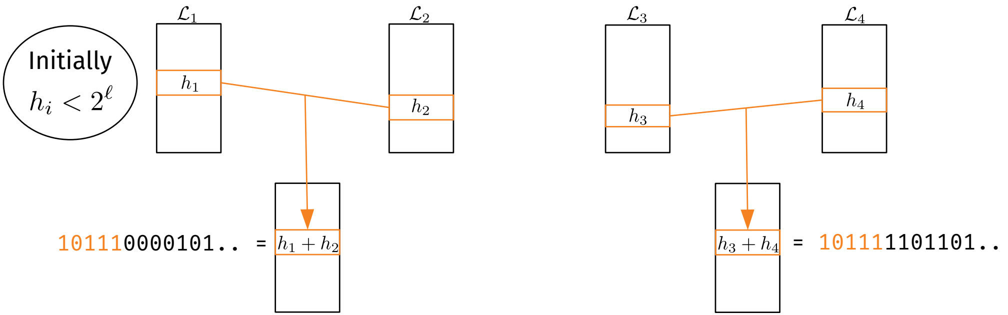

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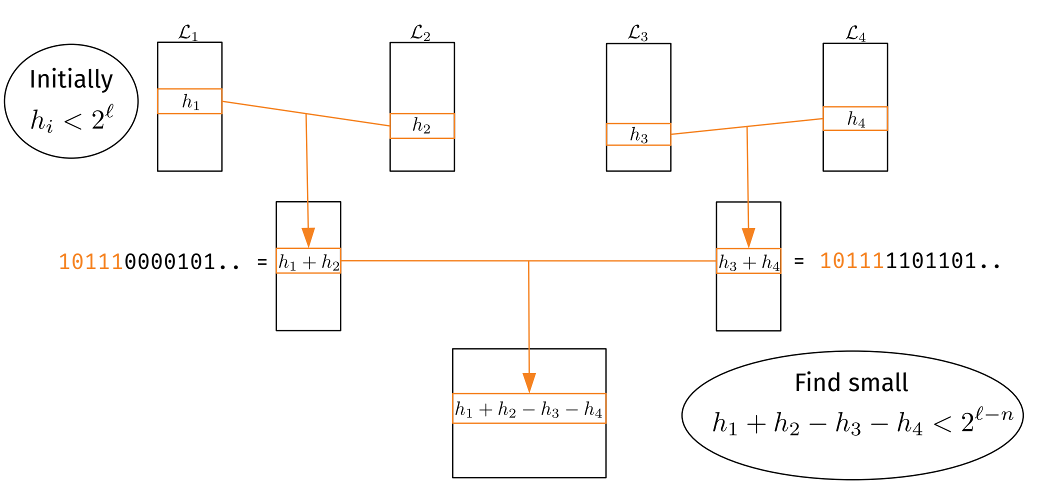

Time–Data tradeoffs for 1-bit leakage

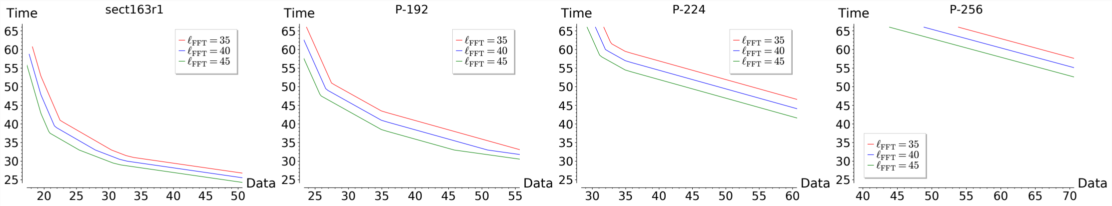

Figure 1: Time–Data tradeoff graphs (in a $\log_2$ scale) when memory is fixed to 2^{35}

* Optimized data complexity by solving the linear programming problem
* Further optimization is feasible if > 1-bit leakage is available!
    - Sample amplification via exhaustive $\mathcal{K}$-sum search

ECDSA key recovery attack: experimental records

Target	Bias	Facility	Error rate	Input	Thread (Collision)	Time (Collision)	RAM (Collision)	Recovered MSBs
NIST P-192	1-bit	AWS EC2	0	2^{29}	96×24	113h	492GB	39
NIST P-192	1-bit	AWS EC2	1%	2^{35}	96×24	52h	492GB	39
`sect163r1`	1-bit	Cluster	0	2^{23}	16×16	7h	80GB	36
`sect163r1`	1-bit	Workstation	2.7%	2^{24}	48	42h	250GB	35
`sect163r1`	2-bit	Cluster	0	1024	16	2h	96GB	32

Table 1: Computational results for the first round of Bleichenbacher

- Attack on **P-192** is made possible by our highly optimized parallel implementation.
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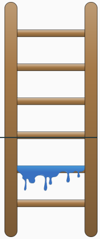
How to Acquire Nonce Leakage

LadderLeak: Tiny timing leakage from the Montgomery ladder

Algorithm 1 Montgomery ladder

Input: $P = (x, y)$, $k = (1, k_{t-2}, \ldots, k_1, k_0)$
Output: $Q = [k]P$

$k' \leftarrow$ Select $(k + q, k + 2q)$
$R_0 \leftarrow P$, $R_1 \leftarrow [2]P$
for $i \leftarrow \lg(q) - 1$ **downto** 0 **do**
    Swap (R_0, R_1) if $k'_i = 0$
    $R_0 \leftarrow R_0 \oplus R_1$; $R_1 \leftarrow 2R_1$
    Swap (R_0, R_1) if $k'_i = 0$
end for
return $Q = R_0$



Conditions for the attack to work:

- Accumulators (R_0, R_1) are in **projective coordinates**, but initialized with the base point in **affine coordinates**.
- Group order is $2^n - \delta$
- Group law is non-constant time wrt handling Z coordinates $\leadsto$ **Weierstrass model**

Experiments were carried out with Flush+Reload cache attack technique

$\leadsto$ MSB of k was detected with $> 99\ \%$ accuracy.

Software countermeasures & coordinated disclosure

- **Coordinated disclosure**: reported in December 2019 (before EOL of OpenSSL `1.0.2`)
- Fixed in April 2020 with **randomized** Z coordinates of the base point

Main takeaways

- ECDSA nonce is extremely sensitive!
  - Even <1-bit leakage/signature is exploitable, albeit with quite a few signatures as input
- HNP is still relevant nowadays
- Interesting connection between the HNP and GBP
  - Open question: Could #signatures for Bleichenbacher be as low as lattice?

Thank you! & Questions?

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