15-th Canadian Mathematical Olympiad 1983

May 4, 1983

1. Find all posiitive integers x,y,z,w such that $w! = x! + y! + z!$.

2. For each real number r let T_r be the transformation of the plane that takes the point (x,y) into the point $(2^r x, r2^r x + 2^r y)$. Let $\mathscr{F} = \{T_r \mid r \in \mathbb{R}\}$. Find all curves $y = f(x)$ whose graphs remain unchanged by every transformation in $\mathscr{F}$.

3. The area of a triangle is determined by the lengths of its sides. Is the volume of a tetrahedron determined by the areas of its faces?

4. Prove that for every prime number p, there are infinitely many positive integers n such that p divides $2^n - n$.

5. Show that the gepmetric mean of a set S of n positive numbers is equal to the geomteric mean of the geometric means of all nonempty subsets of S.