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Technical Note 155

THE ENERGY PARAMETER B FOR STRONG BLAST WAVES

D. L. JONES

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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NBS Boulder Laboratories
Boulder, Colorado

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ABSTRACT

The energy parameter B used in the strong blast wave equations is calculated for monatomic and diatomic gases. Three geometries, spherical, cylindrical, and plane are considered. Comparisons are made with previously published values of B. Tables and curves of the distribution functions are given for each case. The equations of the blast waves, in the similarity solution, are compiled for the six cases. An application of the analysis of a cylindrical blast wave from an exploding wire is given.

THE ENERGY PARAMETER B FOR STRONG BLAST WAVES

DONALD L. JONES

1. Introduction

The theoretical treatment of strong spherical blast waves, assuming similarity, has been made by Taylor[1]. This work was extended by Sakurai[9] to the case of cylindrical and plane blast waves. Although Taylor developed some approximate solutions for monatomic and other gases, the main emphasis in these analyses has been on solutions for air, a diatomic gas.

In the present work, three ideal situations have been computed which differ in the geometry of their initial conditions. Energy is instantaneously released: (1) at a point to produce a spherical shock wave, (2) along a line to generate a cylindrical shock, and (3) over a plane to yield a plane shock. The shock disturbance is assumed to be similar at all times, changing only its linear dimensions with increasing time. It is also assumed that the gases are perfect, with constant specific heat ratios. Energy losses from ionization and radiation are neglected.

Under these assumptions the distance R of the shock front from its initial position is related to the time by the expression

$$t = \frac{1}{c}\left(\frac{E}{B\rho_0}\right)^{-1/2} R^c \qquad (1)$$

where E is the energy released, ρ_0 is the ambient density ahead of the shock, c is a numerical constant equal to 5/2, 4/2, or 3/2 for spherical, cylindrical, or plane shocks respectively. B is a numerical constant depending upon the geometry of the shock wave and the specific heat ratio γ.

In spite of the idealizations, equation (1) describes real explosions of exploding wires[2,3], cylindrical charges of high explosives[4], and even atomic bombs[5]. In order to compare experimental results with theory using equation (1), the value of B is needed with a precision at least as great as that of the experimental data. For the present work three significant digits are adequate. The precision claimed for the previously published values for B is three digits also. However, it will be demonstrated that few were correct to more than two digits and one was not correct even in the first digit.

A table of values of B, calculated by the author, with a precision of three digits, is given for each geometry and for both diatomic and monatomic gases. The blast wave expressions for each of the six cases are compiled in Appendix A. This report constitutes the more complete discussion referred to in an earlier publication[6]*.

2. Procedure

Taylor[1], in his original work, gives a thorough discussion of the similarity method. It is sufficient here to

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* The author is indebted to N. Gerber, BRL, Aberdeen Proving Ground, Maryland, for pointing out an error in the previous publication.

indicate that the assumption of similarity is consistent with the equations of motion and continuity and with the equation of state of a perfect gas. The conditions at the shock front are given by the familiar Rankine-Hugoniot relations. Distribution functions are developed as a convenient method for representing the pressure, density and flow velocity at all points in the blast wave.

In the computation of the parameter B, it is first necessary to integrate the differential equations of the distribution functions given in Appendix C. Graphs and tables of the solutions of these equations are shown in Appendix B for each of the six cases. The abscissa η is the ratio r/R, where R is the distance from the origin to the shock front and r is an intermediate point. The distribution functions f, φ, and ψ are all dimensionless functions of η. The function f is related to the pressure ratio across the front; ψ is the density ratio; and φ is related to the radial velocity of the front. It should be noted that the given boundary conditions in Appendix C are correct only for strong shocks, i.e., for which the pressure ratio across the front exceeds 10. As shown in Appendix C, B is the integral of a geometry dependent function of the distribution functions. Following Taylor, B is found by evaluating first the differential equations step by step and then numerically integrating from the table obtained.

A Runge-Kutta integration technique given by Gill[7] was used to evaluate the differential equations of the distribution functions. Gill developed the method for automatic computation, the main advantage being that only one set of values at the boundary is required to initiate the computation. The boundary conditions at the shock front provide the necessary initial values. One change in the method of Gill was required since the present computations were performed with a floating point machine, whereas his technique contained a means of reducing rounding errors on a fixed point computer. Obitts[8] has determined that the use of floating point arithmetic reduces the accumulation of rounding error within a step, so that the application of a bridging term as developed by Gill would be unnecessary.

In computing the tables of the distribution functions the procedure followed was to select an arbitrary interval for $\Delta\eta$ and compute the table from $\eta = 1$ to $\eta = 0$. Then the interval was halved and a new table was computed. If the new table agreed with the previous table to six significant digits the last table computed was accepted as correct. If such agreement did not exist another table was computed. This procedure was repeated until the desired agreement was obtained.

The tables of the distribution functions in Appendix B are not nearly as complete as the original tables calculated for this work; many intermediate values are not listed. It was necessary to use an interval of 0.0005 in η to obtain an adequate precision in the integration of the differential equations for the distribution function graphs.

The differential equations of the distribution functions are quite well behaved, as evidenced by the graphs in Appendix B. Also, the computer word length is in excess of ten digits, while the distribution function tables are only required to be accurate to six digits at most. These two facts, when coupled with the relatively short length of the tables ($\sim$ 2000 entries), allow a straight-forward evaluation of the distribution functions without the serious loss of accuracy from truncation and round-off that often plagues numerical evaluation of differential equations.

Preliminary computations of B were made on an IBM 650 computer, but the large number of calculations required in the step by step computations indicated the need for a faster machine. Subsequently the program was placed on a CDC 1906 computer and all computations were performed with that equipment.

The computed values of B for the six cases considered are shown in Table 1.

TABLE 1 - ENERGY PARAMETER B

	Spherical	Cylindrical	Plane
$\gamma = 7/5$	5.33	3.94	1.22
$\gamma = 5/3$	3.08	2.26	0.678

Once B is known accurately it is a simple matter to apply equation (1), its derivative, and the Rankine-Hugoniot relations to compute the theoretical time, velocity, pressure, and temperature for a shock front propagating in a known gas. The theoretical values can then be used to compare with experimental data or to predict experimental parameters. Equations for these computations are given in Appendix A. These equations are grouped according to the geometry of the shock and further subdivided, as necessary, into monatomic and diatomic gases.

If the distance of a shock front from the origin is measured as a function of time, application of the equations in Appendix A allow determination of the energy in the shock, the front velocity, particle velocity immediately behind the front, and the pressure and temperature in the shock front.

3. Discussion

Since the initial work of Taylor on spherical blasts, several others have made further calculations on the spherical as well as cylindrical and plane blast waves. A list of authors and the B values they have obtained is shown in Table 2. The entries listed for Sakurai[9] were calculated from his published J values. Harris[10] developed an approximate method for calculating B for any γ, but even for $\gamma = 5/3$ his values are in error by more than 20 o/o. The values listed for Sedov[11] are obtained from graphs* and the

* The author is indebted to Dr. H.T. Yang for calling his attention to this work and also to reference 12.

spherical case appears closely related to the work of Taylor.

When the computed values of B in Table 1 are compared with the previously published values some discrepancies appear. For the plane shock wave with $\gamma = 7/5$, the value of 2.04 given by Lewis et al[13] disagrees with our value by 67 o/o. This deviation probably resulted from a mistake in their integrand of B. Their equation (24) contained a term $\left(\frac{\gamma}{\gamma-1}\right)$ which should have been $\left[\frac{1}{\gamma(\gamma-1)}\right]$. In the cylindrical case Lin's[14] value of 3.85 for $\gamma = 7/5$ gives a disagreement of 2.3 o/o. Presumably this was caused by inadequate evaluation of the distribution function differential equations. The agreement with the B values from Taylor for the spherical shocks is remarkable in view of the fact that his were made without the aid of an electronic computer.

TABLE 2. B VALUES

	Spherical				Cylindrical			Plane		
	$\gamma=5/3$	$\gamma=7/5$	$\gamma=1.3$	$\gamma=6/5$	$\gamma=5/3$	$\gamma=7/5$	$\gamma=6/5$	$\gamma=5/3$	$\gamma=7/5$	$\gamma=6/5$
Taylor[1]	3.04	5.36	7.28	10.79						
Sakurai[9]	3.04	5.35			2.22	3.94		0.678	1.21	
Lin[14]						3.85				
Rogers[12]		5.36		10.8		4.03	8.10		1.22	2.52
Lewis[13]									2.04	
Rouse[15]						3.965				
Sedov[11]	3.11	5.32	6.94	10.9	2.20	4.00	8.16	0.675	1.22	2.45
Jones[6]	3.08	5.33			2.26	3.94		0.678	1.22	
Gerber[16]					2.26	3.94				

4. Application

We can now apply the equation for cylindrical shocks to the case of an exploding wire in air. Radius-time observations of the shock wave were made simultaneously on three frequencies with the microwave Doppler technique[6] as shown on the left in Fig. 1. The data for each frequency, scaled from these traces, are given in Table 3.

TABLE 3 - Shock Wave Data

λ = 3.0 cm		λ = 1.2 cm		λ = 0.84 cm	
R cm	t μsec	R cm	t μsec	R cm	t μ sec
		.329	1.38	1.26	2.55
		.629	1.80	1.47	3.07
3.00	7.27	.929	2.24	1.68	3.42
3.75	9.68	1.23	2.78	1.89	3.86
4.50	12.82	1.53	3.21	2.10	4.42
5.25	21.14	1.83	3.43	2.31	4.87
		2.13	4.49	2.52	5.50
		2.43	5.22	2.73	6.10
		2.73	6.05	2.94	6.83
		3.03	6.99	3.15	7.45
		3.33	7.94	3.36	8.33
		3.63	9.97	3.57	8.99
		3.93	10.16	3.78	9.96
		4.23	11.27	3.99	10.80
		4.53	12.42	4.20	11.83
		4.83	13.96	4.41	12.72
		5.13	15.60	4.62	13.82
		5.43	17.88	4.83	14.99
		5.73	20.15	5.04	16.09
		6.03	22.89	5.25	17.27
		6.32	25.67	5.45	18.60
		6.62	28.17	5.69	19.96
		6.92	31.13	5.88	21.38
		7.22	34.91	6.09	22.90
		7.52	39.32	6.30	24.56
		7.82	44.65	6.51	25.86
				6.72	27.51

The air density ρ_0, as determined from the ambient pressure of 30 cm.Hg. is 4.63×10^{4} gm cm^{-3}. We now go to Appendix A, to the column for cylindrical shocks in a diatomic gas. The value of the parameter B is 3.94. The time-radius equation is

$$t = \frac{1}{2}\left(\frac{E}{B\rho_0}\right)^{-\frac{1}{2}} R^2 \qquad \text{(A1)}$$

Since the parameter B, the energy E, and the ambient density ρ_0 are all constants, a graph of R^2 as a function of time will yield a straight line with slope

$$m = 2\left(\frac{E}{B\rho_0}\right)^{\frac{1}{2}} \quad . \qquad \text{(2)}$$

The measured slope of the straight line portion of the curve in Fig. 1 is 1.99×10^{8} cm^{2} sec^{-1}. Upon solving equation

(2) for E, the energy in the shock is found to be 182.5 joules cm^{-1} of wire length. Examination of Fig. 1 shows that the data follow the straight line for only part of the shock trajectory. The positive curvature at the beginning results from the finite time of the delivery of energy to the shock during the explosion of the wire. The negative curvature later represents departure of the shock trajectory from the strong blast relation.

After finding the energy in the shock the velocity U at any point can be calculated from relation (A3)

$$U = \left(\frac{E}{B\rho_0}\right)^{1/2} R^{-1}. \qquad \text{(A3)}$$

For instance, when the radius is 3.93 cm the velocity is 25.5×10^4 cm sec^{-1} and the particle velocity immediately behind the shock front is 5/6 U, or 21.2×10^4 cm sec^{-1}.

The expression for pressure at the shock front is:

$$P_1 = 7/6\ R^{-2} \frac{E}{\gamma B} \qquad \text{(A4)}$$

giving a value of 2.28×10^7 dynes cm^{-2} or 22.6 atmospheres at the radius 3.93 cm. At the radius 5.5 cm where the shock trajectory diverges from the strong blast relation the pressure is 12.6 atmospheres.

Calculation of the temperature with expression (A5) gives a temperature of 1130^oK at the radius 3.93 cm.

In assuming the specific heat ratio γ to be constant, the effects of excitation, dissociation, and ionization have been neglected. These effects are such that they can decrease γ to below 4/3. If for a given geometry of shock the value of B is available for several values of γ, the correct value of B could be estimated. Careful experiments could, at least in principle, determine the value of B experimentally by making use of equation (1) and the fact that quite precise values of the energy E can be known.

ACKNOWLEDGEMENT

It is a pleasure to acknowledge the help of Mrs. J. Herman, who wrote the computer program for the numerical solution of the differential equations of the distribution functions; M. Addison, who performed the numerical integrations from the distribution function tables; and particularly R. Gallet, whose suggestions and criticisms were invaluable in the preparation of this report.

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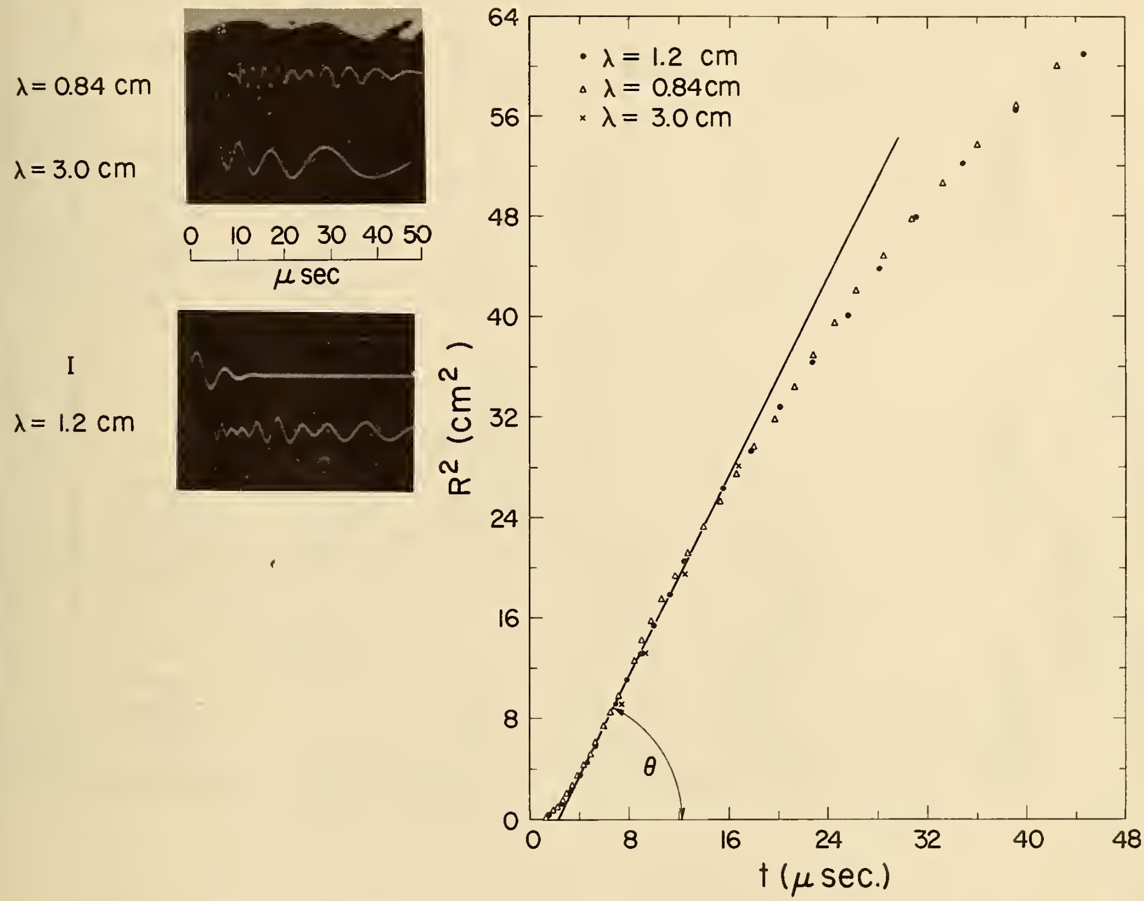


Figure 1. Oscilloscope traces of microwave Doppler measurements of the expanding cylindrical shock front from a wire explosion, are shown on the left. A plot of the square of the radius of the ionization front with time is on the right. The straight line portion of the plotted points represents agreement with the theory. This explosion was made in air at a pressure of 30 cm Hg. The energy released into the 4 cm long, 18 mil copper wire was 500 joules per centimeter of wire length.

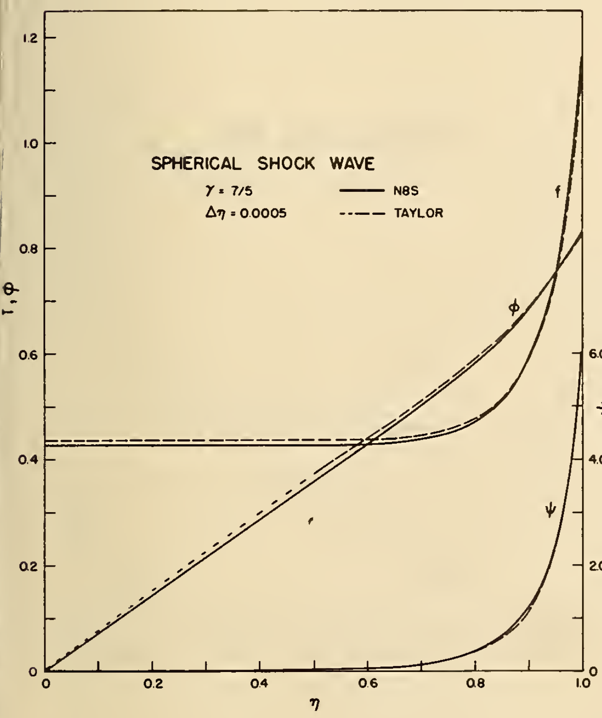
APPENDIX A. SHOCK WAVE EQUATIONS BASED ON SIMILARITY

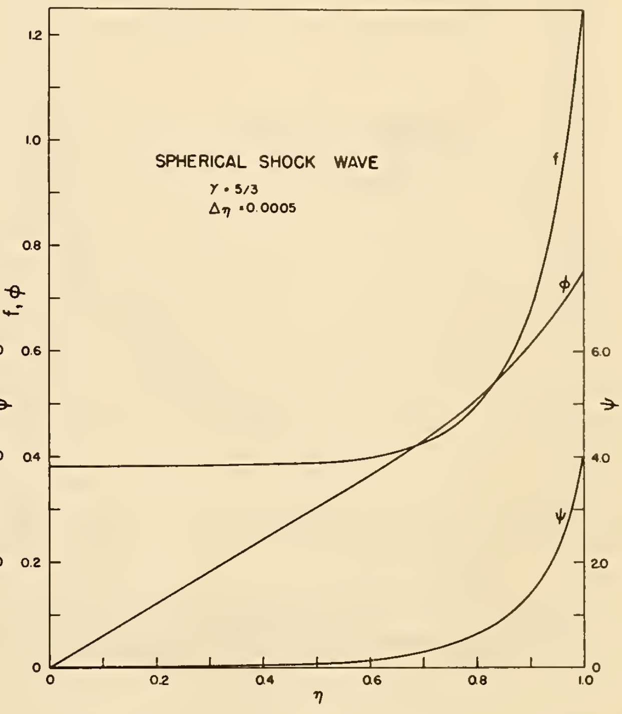
		SPHERICAL		CYLINDRICAL		PLANE	
		MONATOMIC	DIATOMIC	MONATOMIC	DIATOMIC	MONATOMIC	DIATOMIC
	γ	5/3	7/5	5/3	7/5	5/3	7/5
	B	3.08	5.33	2.26	3.94	.678	1.22
(A1)	t	$\left[\frac{2}{5}\left(\frac{E}{B\rho_0}\right)^{-1/2} R^{5/2}\right]$		$\left[\frac{1}{2}\left(\frac{E}{B\rho_0}\right)^{-1/2} R^{2}\right]$		$\left[\frac{2}{3}\left(\frac{E}{B\rho_0}\right)^{-1/2} R^{3/2}\right]$	
(A2)	U	$\left[\left(\frac{E}{B\rho_0}\right)^{1/2} R^{-3/2}\right]$		$\left[\left(\frac{E}{B\rho_0}\right)^{1/2} R^{-1}\right]$		$\left[\left(\frac{E}{B\rho_0}\right)^{1/2} R^{-1/2}\right]$	
(A3)	u	$\left[3/4\ U\right]$	$\left[5/6\ U\right]$	$\left[3/4\ U\right]$	$\left[5/6\ U\right]$	$\left[3/4\ U\right]$	$\left[5/6\ U\right]$
(A4)	P_1	$\left[5/4\ R^{-3}\ \frac{E}{\gamma B}\right]$	$\left[7/6\ R^{-3}\ \frac{E}{\gamma B}\right]$	$\left[5/4\ R^{-2}\ \frac{E}{\gamma B}\right]$	$\left[7/6\ R^{-2}\ \frac{E}{\gamma B}\right]$	$\left[5/4\ R^{-1}\ \frac{E}{\gamma B}\right]$	$\left[7/6\ R^{-1}\ \frac{E}{\gamma B}\right]$
(A5)	T_1	$\left\lvert\frac{1}{4}\ \frac{P_1T_0}{P_0}\right\rvert$	$\left\lvert\frac{1}{6}\ \frac{P_1T_0}{P_0}\right\rvert$	$\left\lvert\frac{1}{4}\ \frac{P_1T_0}{P_0}\right\rvert$	$\left\lvert\frac{1}{6}\ \frac{P_1T_0}{P_0}\right\rvert$	$\left\lvert\frac{1}{4}\ \frac{P_1T_0}{P_0}\right\rvert$	$\left\lvert\frac{1}{6}\ \frac{P_1T_0}{P_0}\right\rvert$

Where γ is the ratio of specific heats, B is the energy constant, t is the time at radius R, U is the shock front velocity, u is the individual particle velocity, P_1 is the pressure in the shock front, and T_1 is the temperature in the shock front.

APPENDIX B

Distribution Function Curves and Tables



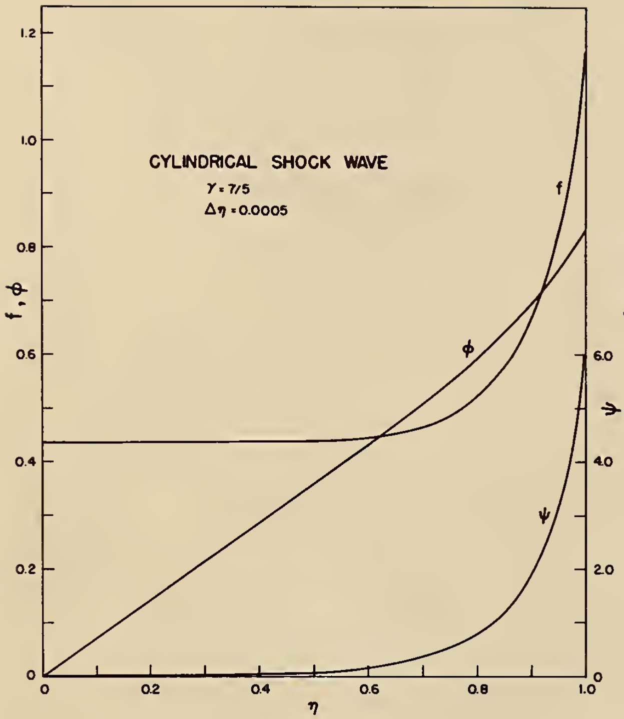


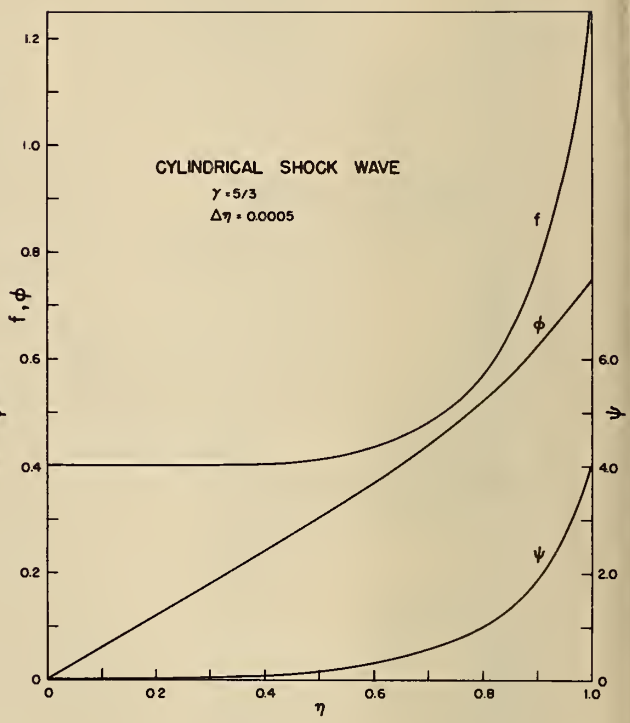
SPHERICAL SHOCK WAVE $\gamma = 7/5$

η	f	ϕ	ψ
1.00	1.6666667	0.8333333	6.0000000
0.98	0.9549096	0.7985383	4.0659380
0.96	0.8131019	0.7664041	2.8848390
0.94	0.7145236	0.7368696	2.1197298
0.92	0.6439796	0.7097627	1.5997640
0.90	0.5923293	0.6848413	1.2321970
0.88	0.5538214	0.6618317	0.9637827
0.88	0.5247000	0.6404581	0.7624754
0.84	0.5024335	0.6204610	0.6081833
0.82	0.4852673	0.6016069	0.4878446
0.80	0.4719555	0.5836921	0.3926865
0.78	0.4615936	0.5665425	0.3166384
0.76	0.4535121	0.5500115	0.2553826
0.74	0.4472068	0.5339773	0.2057690
0.72	0.4422924	0.5183392	0.1654446
0.70	0.4384709	0.5030149	0.1326138
0.68	0.4355095	0.4879377	0.1058782
0.66	0.4332247	0.4730534	0.0841315
0.64	0.4314716	0.4583190	0.0664839
0.62	0.4301348	0.4438998	0.0522113
0.60	0.4291229	0.4291686	0.0407188
0.58	0.4283631	0.4147040	0.0315129
0.56	0.4277978	0.4002893	0.0241843
0.54	0.4273808	0.3859118	0.0183904
0.52	0.4270769	0.3715615	0.0138456
0.50	0.4268580	0.3572309	0.0103111
0.46	0.4265928	0.3288081	0.0055132
0.42	0.4284600	0.3000141	0.0027858
0.38	0.4264093	0.2714335	0.0013149
0.34	0.4263880	0.2428587	0.0005709
0.30	0.4263773	0.2142882	0.0002233
0.26	0.4263748	0.1857144	0.0000783
0.22	0.4263738	0.1571429	0.0000218
0.18	0.4263736	0.1285715	0.0000048
0.14	0.4263736	0.1000001	0.0000007
0.10	0.4263736	0.0714288	0.0000000
0.06	0.4263736	0.0428577	0.0000000
0.02	0.4263736	0.0142911	0.0000000

SPHERICAL SHOCK WAVE $\gamma = 5/3$

η	f	ϕ	ψ
1.00	1.2500000	0.7500000	4.0000000
0.98	1.0742631	0.7205468	3.1711570
0.96	0.9382844	0.6922655	2.5506850
0.94	0.8318917	0.6652500	2.0781430
0.92	0.7478278	0.6395580	1.7124410
0.90	0.8808284	0.6152091	1.4251380
0.88	0.6270086	0.5921885	1.1962170
0.86	0.5834808	0.5704420	1.0114007
0.84	0.5480582	0.5499034	0.8803535
0.82	0.5190766	0.5304815	0.7355092
0.80	0.4952548	0.5120780	0.6312624
0.78	0.4755976	0.4945907	0.5434138
0.78	0.4593271	0.4779185	0.4687820
0.74	0.4458281	0.4819843	0.4049320
0.72	0.4348111	0.4466376	0.3499789
0.70	0.4252828	0.4318549	0.3024478
0.86	0.4175248	0.4175408	0.2811725
0.68	0.4110770	0.4036278	0.2252208
0.64	0.4057283	0.3900559	0.1938380
0.62	0.4012954	0.3787727	0.1884083
0.60	0.3978388	0.3837320	0.1424218
0.58	0.3946287	0.3508939	0.1214517
0.56	0.3921608	0.3382241	0.1031378
0.54	0.3901509	0.3258931	0.0871705
0.52	0.3885222	0.3132758	0.0732835
0.50	0.3872111	0.3009504	0.0812432
0.48	0.3853329	0.2785085	0.0419021
0.42	0.3841741	0.2522570	0.0277518
0.38	0.3834892	0.2281213	0.0178808
0.34	0.3831054	0.2040527	0.0108988
0.30	0.3829042	0.1800208	0.0060878
0.28	0.3828072	0.1580070	0.0031985
0.22	0.3827853	0.1320021	0.0015072
0.18	0.3827498	0.1080005	0.0006109
0.14	0.3827452	0.0840002	0.0001972
0.10	0.3627442	0.0600002	0.0000434
0.06	0.3627440	0.0360005	0.0000044
0.02	0.3827440	0.0120048	0.0000000



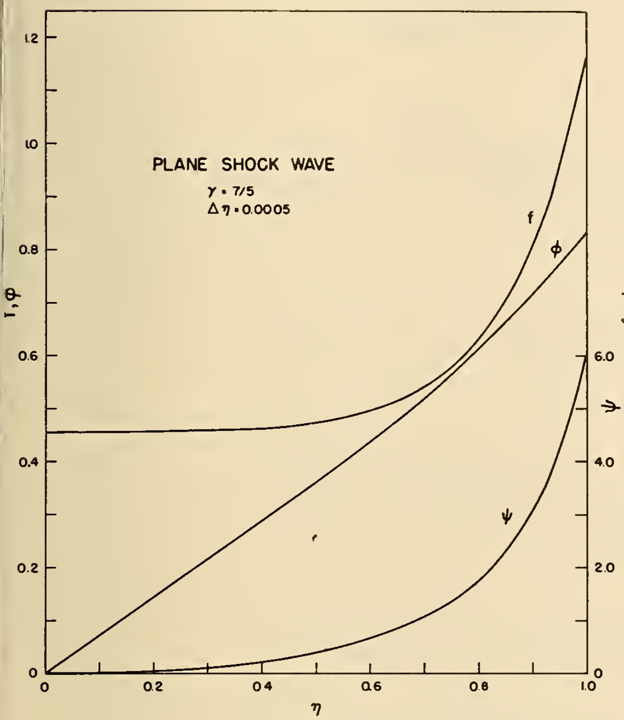


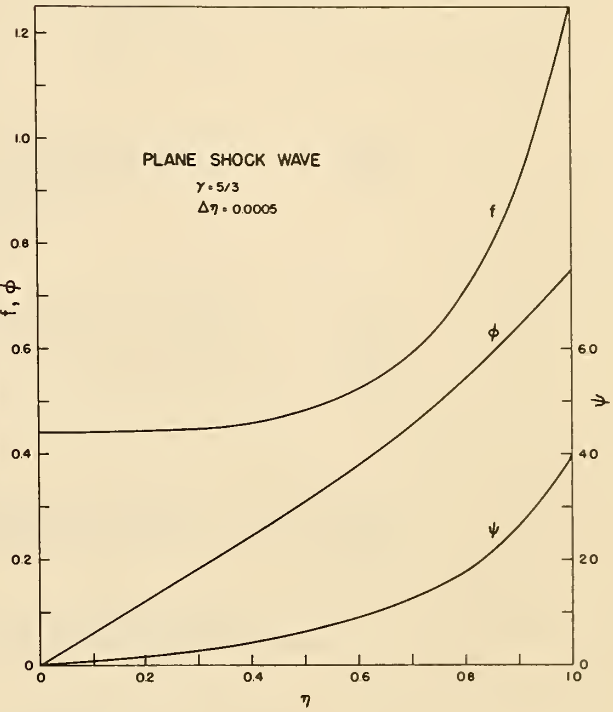
CYLINDRICAL SHOCK WAVE $\gamma = 7/5$

η	f	ϕ	ψ
1.00	1.1666667	0.8333333	6.0000000
0.98	1.0087050	0.8034607	4.5786812
0.96	0.8899788	0.7749885	3.5750860
0.94	0.7990885	0.7479352	2.8450400
0.92	0.7284405	0.7222810	2.3000562
0.90	0.6728230	0.6979718	1.8838684
0.88	0.6285670	0.6749282	1.5596000
0.86	0.5930338	0.6530528	1.3024355
0.84	0.5642897	0.6322389	1.0952829
0.82	0.5408940	0.6123767	0.9261213
0.80	0.5217575	0.5933577	0.7863344
0.78	0.5060450	0.5750787	0.6696371
0.76	0.4931084	0.5574437	0.5713691
0.74	0.4824382	0.5403648	0.4880211
0.72	0.4730630	0.5237630	0.4169113
0.70	0.4663589	0.5075679	0.3559606
0.68	0.4603620	0.4917175	0.3035349
0.66	0.4554241	0.4761573	0.2583322
0.64	0.4513680	0.4608401	0.2193013
0.62	0.4480468	0.4457254	0.1855819
0.60	0.4453378	0.4307780	0.1564612
0.58	0.4431384	0.4159682	0.1313413
0.56	0.4413622	0.4012707	0.1097143
0.54	0.4399365	0.3866642	0.0911445
0.52	0.4387998	0.3721309	0.0752541
0.50	0.4379006	0.3576561	0.0617126
0.46	0.4366468	0.3288348	0.0405442
0.42	0.4359044	0.3001272	0.0256782
0.38	0.4354853	0.2714857	0.0155515
0.34	0.4352622	0.2428806	0.0089126
0.30	0.4351517	0.2142943	0.0047653
0.26	0.4351019	0.1857170	0.0023297
0.22	0.4350820	0.1571436	0.0010105
0.18	0.4350752	0.1285716	0.0003705
0.14	0.4350733	0.1000000	0.0001054
0.10	0.4350730	0.0714286	0.0000196
0.06	0.4350730	0.0428572	0.0000015
0.02	0.4350730	0.0142858	0.0000000

CYLINDRICAL SHOCK WAVE $\gamma = 5/3$

η	f	ϕ	ψ
1.00	1.25000000	0.75000000	4.00000000
0.98	1.11889770	0.72406996	3.40401587
0.96	1.00979580	0.69881980	2.91710821
0.94	0.91856441	0.67430297	2.51606138
0.92	0.84194494	0.65056232	2.18313064
0.90	0.77734414	0.62762827	1.90463512
0.88	0.72268220	0.60551771	1.66994251
0.86	0.67627944	0.58423390	1.47073103
0.84	0.63677099	0.56376711	1.30044470
0.82	0.60304183	0.54409599	1.15388876
0.80	0.57417669	0.52518945	1.02692422
0.78	0.54942108	0.50700868	0.91623762
0.76	0.52815056	0.48950933	0.81916317
0.74	0.50984636	0.47264345	0.73354669
0.72	0.49407587	0.45636127	0.65763965
0.70	0.48047696	0.44061262	0.59001648
0.68	0.46874527	0.42534810	0.52950987
0.66	0.46101581	0.41418863	0.48821598
0.64	0.44989525	0.39608280	0.42617275
0.62	0.44237382	0.38199378	0.38189071
0.60	0.43590108	0.36821307	0.34176467
0.58	0.43034077	0.35470391	0.30533470
0.56	0.42557523	0.34143260	0.27221346
0.54	0.42150242	0.32836847	0.24207297
0.52	0.41803344	0.31548376	0.21463385
0.50	0.41509053	0.30275355	0.18965663
0.46	0.41051802	0.27766983	0.14628772
0.42	0.40733001	0.25296749	0.11060634
0.38	0.40517079	0.22853072	0.08154633
0.34	0.40376130	0.20427231	0.05823621
0.30	0.40288348	0.18012851	0.03993098
0.26	0.40236896	0.15605447	0.02596513
0.22	0.40209061	0.13202000	0.01572103
0.18	0.40195567	0.10800602	0.00860806
0.14	0.40189980	0.08400136	0.00404967
0.10	0.40188285	0.06300027	0.00170839
0.06	0.40187790	0.03600007	0.00031876
0.02	0.40187758	0.01200017	0.00001181





PLANE SHOCK WAVE $\gamma = 7/5$

η	f	ϕ	ψ
1.00	1.1686667	0.8333333	6.0000000
0.98	1.0698629	0.8085881	5.1830443
0.98	0.9874815	0.7843701	4.5079704
0.94	0.9170344	0.7607013	3.9452009
0.92	0.8565355	0.7375980	3.4721819
0.90	0.8043842	0.7150897	3.0714380
0.88	0.7592791	0.6931198	2.7294514
0.86	0.7201535	0.8717444	2.4355374
0.84	0.8881270	0.8509348	2.1812357
0.82	0.8564682	0.8306782	1.9597920
0.80	0.8305883	0.8109492	1.7857772
0.78	0.8079080	0.5917308	1.5947958
0.76	0.5880802	0.5729950	1.4432772
0.74	0.5708580	0.5547137	1.3082979
0.72	0.5553829	0.5388573	1.1874583
0.70	0.5419740	0.5193957	1.0787897
0.68	0.5302000	0.5022985	0.9805945
0.86	0.5198638	0.4855358	0.8915828
0.84	0.5107942	0.4890784	0.8105313
0.62	0.5028438	0.4528982	0.7385425
0.60	0.4958825	0.4389882	0.6687903
0.58	0.4897978	0.4212632	0.6085950
0.58	0.4844903	0.4057591	0.5493814
0.54	0.4798722	0.3904338	0.4988814
0.52	0.4758659	0.3752865	0.4480195
0.50	0.4724023	0.3802381	0.4031013
0.46	0.4888835	0.3305302	0.3232846
0.42	0.4828357	0.3011885	0.2551968
0.38	0.4599789	0.2721143	0.1974318
0.34	0.4580183	0.2432292	0.1488419
0.30	0.4587218	0.2144727	0.1085310
0.26	0.4559119	0.1857994	0.0757501
0.22	0.4554394	0.1571788	0.0498355
0.18	0.4551889	0.1285827	0.0301589
0.14	0.4550734	0.1000028	0.0180857
0.10	0.4550305	0.0714290	0.0089355
0.08	0.4550198	0.0428572	0.0019339
0.02	0.4550184	0.0142857	0.0001241

PLANE SHOCK WAVE $\gamma = 5/3$

η	f	ϕ	ψ
1.00	1.2500000	0.75000000	40000000
0.98	1.1670709	072765398	36595321
0.96	1.0926130	070563159	33548977
0.94	1.0257123	0.68395560	30817418
0.92	09655648	066284734	28362860
0.90	0.9114602	084172823	26152428
0.88	0.8627706	062120928	24157452
0.86	0.8169384	060111062	22352888
0.84	0.7794676	058144127	20716718
0.82	0.7439163	056220877	19229735
0.80	0711 8898	054341706	17874989
0.78	06830353	052506650	16837577
0.76	06570371	050715391	15504372
0.74	06336124	048967280	14483804
0.72	0812 5077	047281360	13505687
070	05934959	045596403	12820958
0.68	0.5783732	043970947	1.1801729
0.68	05609569	042383334	11040979
0.64	05470833	040831753	10332489
0.62	05348053	039314280	09870829
0.60	05233909	037828910	09051172
0.58	05133218	036373592	08469289
0.56	05042910	034948280	07921377
0.54	04982031	033544859	07404197
0.52	04889717	032187380	06914827
0.50	04825167	030611768	08450718
0.46	04718728	028158621	05589801
0.42	04831755	025571571	04808151
0.38	04566209	023037050	04089258
0.34	04518888	020543795	03430808
0.30	04480219	018081907	02825188
0.28	04454342	015843032	02266889
0.22	04436839	013220298	01760417
0.18	04425771	010808228	01300237
0.14	04419430	008402658	00890858
0.10	04418323	006000585	00537490
0.06	04415172	003800080	00249751
0.02	04414948	001200000	00048082

APPENDIX C

Blast Wave Distribution Functions and the B Integral

SPHERICAL

$$B = 4\pi \int_0^1 \left(\frac{1}{\gamma(\gamma-1)} f + \frac{\psi\,\varphi^2}{2} \right) \eta^2 \, d\eta$$

$$\frac{df}{d\eta} = f' = \frac{f\left\{-3\eta + \varphi\left(3 + \frac{1}{2}\gamma\right) - \frac{2\gamma\varphi^2}{\eta}\right\}}{\left\{(\eta - \varphi)^2 - \frac{f}{\psi}\right\}}$$

$$\frac{d\varphi}{d\eta} = \varphi' = \frac{\left\{\frac{1}{\gamma}\frac{f'}{\psi} - \frac{3}{2}\varphi\right\}}{\{\eta - \varphi\}}$$

$$\frac{d\psi}{d\eta} = \psi' = \left\{ \psi \frac{(\varphi' + 2\varphi/\eta)}{(\eta - \varphi)} \right\}$$

with $\eta = \frac{r}{R}$

where R = distance from explosion to shock front

and r = distance from explosion to intermediate point

Boundary conditions at shock front

$$f_1 = \frac{2\nu}{\gamma+1}, \quad \varphi_1 = \frac{2}{\gamma+1}, \quad \psi_1 = \frac{\gamma+1}{\gamma-1}$$

CYLINDRICAL CASE

$$B = 2\pi \int_0^1 \left(\frac{1}{\gamma(\gamma-1)} f + \frac{\psi \varphi^2}{2} \right) \eta \, d\eta$$

$$\frac{df}{d\eta} = f' = \left[\frac{2\eta (\eta-\varphi) + \gamma\varphi^2}{f - (\eta-\varphi)^2 \psi} \right] \frac{\psi f}{\eta}$$

$$\frac{d\psi}{d\eta} = \varphi' = \frac{f' - \gamma \psi \varphi}{\gamma \psi (\eta - \varphi)}$$

$$\frac{d\psi}{d\eta} = \frac{(\eta \varphi' + \varphi) \psi}{(\eta - \varphi) \eta}$$

with η, R and r same as spherical case

Boundary conditions at shock front

$$f_1 = \frac{2\gamma}{\gamma+1}$$

$$\varphi_1 = \frac{2}{\gamma+1}$$

$$\psi_1 = \frac{\gamma+1}{\gamma-1}$$

PLANE CASE

$$B = \int_0^1 \left(\frac{1}{\gamma(\gamma-1)} f + \frac{\psi \varphi^2}{2} \right) d\eta$$

$$\frac{df}{d\eta} = f = \frac{f\psi}{2} \left[\frac{\gamma \varphi + 2 (\eta - \varphi)}{f - \psi (\eta - \varphi)^2} \right]$$

$$\frac{d\varphi}{d\eta} = \varphi' = \frac{f' - \frac{1}{2} \gamma \psi}{\gamma \psi (\eta - \varphi)}$$

$$\frac{d\psi}{d\eta} = \psi' = \frac{\psi \varphi'}{\eta - \varphi}$$

with η, R and r same as in spherical case

Boundary conditions at shock front

$$f_1 = \frac{2\gamma}{\gamma+1}$$

$$\varphi_1 = \frac{2}{\gamma+1}$$

$$\psi_1 = \frac{\gamma+1}{\gamma-1}$$

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U. S. DEPARTMENT OF COMMERCE
Luther H. Hodges, *Secretary*

NATIONAL BUREAU OF STANDARDS
A. V. Astin, *Director*

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