JOURNAL OF RESEARCH of the National Bureau of Standards
Vol. 86, No. 6, November–December 1981

Automatic Computing Methods for Special Functions. Part IV. Complex Error Function, Fresnel Integrals, and Other Related Functions

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July 15, 1981

Accurate, efficient, automatic methods for computing the complex error function to any precision are detailed and implemented in an American Standard FORTRAN subroutine. A six significant figure table of erfc z, e^{z^2} erfc z, and e^{z^2} erfc$(-z)$ is included for z in polar coordinate form with the modulus of z ranging from 0 to 9. The argand diagram is given for erf z.

Key words: Argand diagram; complex error function; continued fraction; Dawson's function; FORTRAN subroutine; Fresnel integrals; key values; line broadening function; plasma dispersion function; Voigt function.

1. Introduction

In computing many of the functions of mathematical physics, for example, Fresnel integrals, Dawson's integral, Voigt function, plasma dispersion function, etc., difficulties are frequently encountered. Since these functions may be expressed in terms of the error function of complex argument, we have chosen this function for Part IV.[1] The major part of the coding of the power series, continued fraction and asymptotic expansion computations for complex arguments will carry over equally well for other functions.

As Part I was devoted to the error function of a real variable, the probability function and other related functions, Part IV will only emphasize those functions and pitfalls due to complex arguments.

While accuracy over the entire domain of definition remains our main concern, the methods employed ensure efficiency, portability and ease of programming and modification.

If one supplies approximate values for the maximum machine value, minimum machine value, the upper bound of the sine, cosine routine, and the upper bound to the acceptable relative error and gives the square root of π to the required number of significant figures, the detailed methods will work for computations ranging from very low precision to multi-precision.

The argand diagram of erf z is included as well as the implementing ANS FORTRAN program and a six significant figure table of erfc z, e^{z^2} erfc z and e^{z^2} erfc$(-z)$ for z in polar coordinate form with the modulus of z ranging from 0 to 9.

2. Mathematical properties

Relevant formulas are collected here for completeness and ease of reference. In keeping with the convention of the Handbook [1],[2] $z = x + iy$ is a complex variable.

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[1] Part I. Error, Probability, and Related Functions. J. Res. Nat. Bur. Stand. (U.S.). 74(3): 211-224; 1970. Part II. The Exponential Integral $E_n(x)$. J. Res. Nat. Bur. Stand. (U.S.). 78(4): 199-216; 1974. Part III. The Sine, Cosine, Exponential Integrals, and Related Functions. J. Res. Nat. Bur. Stand. (U.S.). 80(2): 291-311; 1976.

[2] Figures in brackets indicate literature references at the end of this paper.

A. Definitions

$$\operatorname{erf} z = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2}\, dt$$

$$\operatorname{erfc} z = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-t^2}\, dt = 1 - \operatorname{erf} z$$

(The path of integration is subject to the restriction $\arg t \to \alpha$ with $|\alpha| < \frac{\pi}{4}$ as $t \to \infty$ along the path. If Rt^2 remains bounded to the left, $\alpha = \frac{\pi}{4}$ is permissible.)

$$w(z) = e^{-z^2}\left(1 + \frac{2i}{\sqrt{\pi}} \int_0^z e^{t^2}\, dt\right) = e^{-z^2} \operatorname{erfc}(-iz) = e^{\xi^2} \operatorname{erfc} \xi \quad (\xi = -iz)$$

$$= \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{e^{-t^2}\, dt}{z - t} = \frac{2iz}{\pi} \int_0^\infty \frac{e^{-t^2}\, dt}{z^2 - t^2} \quad (I\, z > 0)$$

with $F(z) = e^{-z^2} \int_0^z e^{t^2}\, dt$ (Dawson's Function)

and $Z(z) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-t^2}\, dt}{t - z}$ (Plasma Dispersion Function)

$$E(z) = C(z) + i\, S(z) = \int_0^z e^{i\pi t^2/2}\, dt = \int_0^z \cos\left(\frac{\pi t^2}{2}\right) dt + i \int_0^z \sin\left(\frac{\pi t^2}{2}\right) dt$$

$$= \frac{1+i}{2} \operatorname{erf}\left[\frac{\sqrt{\pi}}{2}(1-i)\, z\right] \qquad \text{(Fresnel Functions)}$$

$$W(x,t) = \frac{\omega}{(4\pi t)^{1/2}} \int_{-\infty}^{\infty} \frac{e^{-u^2}\, du}{u^2 + \omega^2} = \left(\frac{\pi}{4t}\right)^{1/2} e^{\omega^2} \operatorname{erfc} \omega$$

where

$$\omega = (1 - ix)/2\, t^{1/2}$$

$$= U(x,t) + i\, V(x,t) \qquad \text{(Voigt Function)}$$

$$= \frac{1}{(4\pi t)^{1/2}} \int_{-\infty}^{\infty} \frac{e^{-(x-y)^2/4t}}{1+y^2}\, dy + \frac{i}{(4\pi t)^{1/2}} \int_{-\infty}^{\infty} \frac{y e^{-(x-y)^2/4t}}{1+y^2}\, dy$$

$$\mathrm{H}(a,u) = \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{e^{-t^2}\, dt}{(u-t)^2 + a^2} = \frac{1}{a\sqrt{\pi}}\, U(u/a,\ 1/4a^2) \qquad \text{(Line Broadening Function)}$$

B. Series Expansions

$$\operatorname{erf} z = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{n!\,(2n+1)}$$

$$= \frac{2}{\sqrt{\pi}}\, e^{-z^2} \sum_{n=0}^{\infty} \frac{2^n\, z^{2n+1}}{1 \cdot 3 \cdots (2n+1)}$$

C. Continued Fraction ($Rz > 0$)

$$e^{z^2} \operatorname{erfc} z = \frac{1}{\sqrt{\pi}}\left[\frac{1}{z+} \frac{1/2}{z+} \frac{1}{z+} \frac{3/2}{z+} \frac{2}{z+} \cdots\right]$$

$$= \frac{2z}{\sqrt{\pi}}\left[\frac{1}{2z^2+1} - \frac{1 \cdot 2}{2z^2+5} - \frac{3 \cdot 4}{2z^2+9} - \cdots\right] \quad \text{("Even" Form)}$$

D. Asymptotic Expansion

$$e^{z^2}\operatorname{erfc} z \sim \frac{1}{\sqrt{\pi}}\left[\frac{1}{z} + \sum_{n=1}^{\infty} \frac{(-1)^n 1 \cdot 3 \cdots (2n-1)}{z(2z^2)^n}\right] \left(z \to \infty, \left|\arg z\right| < \frac{3\pi}{4}\right)$$

E. Symmetry Relations

$$\operatorname{erf}(-z) = -\operatorname{erf} z$$

$$\operatorname{erf} \bar{z} = \overline{\operatorname{erf} z}$$

$$w(-z) = 2e^{-z^2} - w(z)$$

$$w(\bar{z}) = \overline{w(-z)}$$

$$C(-z) = -C(z),\ S(-z) = -S(z)$$

$$C(iz) = i\,C(z),\ S(iz) = -i\,S(z)$$

$$C(\bar{z}) = \overline{C(z)},\ S(\bar{z}) = \overline{S(z)}$$

3. Method

The main functions under consideration are the error function ERFZ, the complementary error function ERFCZ, and the exponential of z^2 times the complementary error function EZ2CZ. All other functions may be obtained from these three. To simplify testing, computations are performed for z in the first quadrant AZ and symmetry relations are then employed to make adjustments for other quadrants. For the special case $z = 0$, no computations are performed and the following function values are returned: ERFZ = 0, ERFCZ = 1 and EZ2CZ = 1.

Real type variables are used throughout to readily allow for double precision computation if greater accuracy is needed. The machine dependent constants are placed in a labeled section at the beginning of the subroutine. Function references are likewise grouped together when possible and attention called to the statement labels of the remaining function references. Real and imaginary parts of complex variables have R and I as final characters.

Since EZ2CZ for z in the first quadrant is machine representable even with the real and imaginary parts of z equal to the maximum machine value CMAX (provided its reciprocal is larger than the minimum machine value CMIN), checking for the range of the argument z has been omitted. However, the extensive range necessitates a fair amount of testing for overflows. Underflows are assumed to be set to zero. Overflows are set equal to the maximum machine value and an error indicator IERR set for the number of functions affected. If only EZ2CZ lies outside the machine range, IERR = 1, otherwise IERR = 3. As often as possible, computations are arranged so as to give the correct results for the three functions if they lie within the range of the machine.

In computing the modulus RHO = $\sqrt{(AZR)^2 + (AZI)^2}$ of a complex quantity AZ = AZR + i AZI in the first quadrant, RHO may lie in the machine range but $(AZR)^2$, $(AZI)^2$ or their sum may be outside the range. We select the larger ARIMX and smaller ARIMN of either AZR or AZI, and compute the ratio RMNMX = ARIMN/ARIMX. A factor of RHO called PRHO is computed as the square root of (RMNMX*RMNMX + ONE). This factor, which is greater than or equal to one and less than or equal to the square root of 2, can then be used to check for overflow. The quantity ARIMX must be less than CMAX divided by PRHO for RHO to lie in the machine range. A similar procedure is followed in computing the real and imaginary parts of $(AZ)^2$ = Z2R + i AZ2I = $(AZR)^2 - (AZI)^2 + i\,2AZR*AZI$ with first checking to ensure ARIMX is greater than or equal to 1.

Analysis has indicated and testing confirmed that the power series PS is most useful from the standpoint of accuracy and efficiency for RHO less than RHOLS(=1.5) and when AZR is less than or equal to 1 provid-

ed RHO is less than AELL ($=\sqrt{-1n(TOLER)}$) where TOLER is the upper limit for the relative error. The continued fraction expansion CF is most useful for AZR greater than 1 and RHO greater than or equal to RHOLS. The asymptotic expansion AE is most useful for AZR less than or equal to 1 for RHO greater than or equal to AELL. For RHO greater than or equal to RHOLC ($=\sqrt{0.5/TOLER}$) a rearrangement of arithmetic operations for the first term of the asymptotic expansion is necessary to maintain the accuracy of EZ2CZ. In the continued fraction and asymptotic expansion regions only, EZ2CZ is first computed; it tends to zero for large $|z|$ and the exponential of $-z^2$ tends to infinity for small AZR. To maintain accuracy here, we compute the exponential of $-Z2R/6$ and do continuous multiplication and testing with appropriate factors to obtain ERFC. The imaginary part of $(AZ)^2$ is tested against ULSC, the upper limit of the sine, cosine routine.

Figure 1 below maps the regions for the various methods.

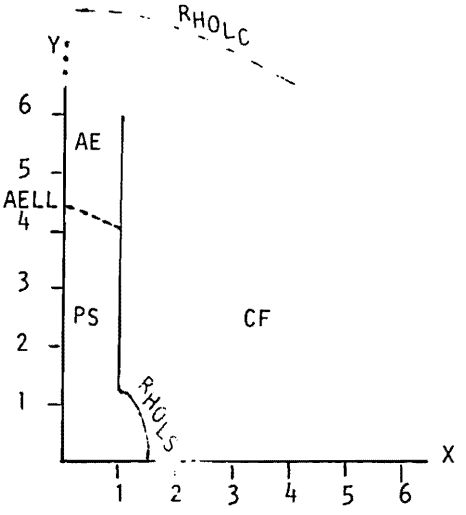


FIGURE 1. Parameter plane.

The dividing line RHO = AELL between the use of the power series and the asymptotic expansion and RHO = RHOLC are the only boundaries subject to the required precision. Single and double precision results, for example on the Univac 1108, are the results of two different methods in the region where RHO roughly lies between 4.3 and 6.4.

This mapping of the region ensures for the required precision that the least number of terms are computed and the loss of significance is kept to a minimum. While the second form of the power series is preferable for real positive z, since all terms are positive, the first form ensures greater accuracy for complex z since the real and imaginary parts of the terms may be positive, negative or zero for any RN($=n$). The power series is there more rapidly convergent; the relative error may then be approximated by the ratio of the term to the sum of terms; comparison of this approximant with the tolerance for the relative error controls the number of terms needed. Since the terms tend to zero through underflow, there will always be convergence even if the tolerance is made considerably smaller than the precision of the machine. The power series is evaluated using recurrence relations in the following form:

$$ERFZ = \frac{TWO}{SQRT(PI)} \sum_{RN=0}^{RNF} SGN(RN)*TM(RN) = \frac{TWO}{RTPI} * SUM$$

where

SGN(0) = 1, SGN(RN + 1) = −SGN(RN)
TM(RN) = ((AZ**(2*RN + 1))/1*2* . . . RN)/(2*RN + 1)
TM(RN) = PTM(RN)/DN(RN)
PTM(0) = AZ, PTM(RN + 1) = (AZ**2)*PTM(RN)/(RN + 1)
DN(0) = 1, DN(RN + 1) = DN(RN) + 2

In determining the terminal value of RN, normalization is necessary to avoid overflows and underflows. The normalization factor TMAX is the maximum of the absolute value of the real and imaginary parts of TM and SUM. If TMAX equals zero or $|TM/TMAX|^2$ underflows, RNF = RN.

If $|SUM/TMAX|^2$ underflows, additional terms are obtained. Otherwise, if $|TM/TMAX|^2/|SUM/TMAX|^2$ is less than $TOLER^2$, then RNF = RN.

The continued fraction expansion starts to converge more slowly as z tends to zero. The "even" form is used since the required number of terms is halved at the expense of very little extra computation for successive numerators and denominators. The continued fraction is evaluated by using the recurrence relations in the "forward" direction. The number of terms needed is determined by checking to see if the relative error of two successive convergents is less than the tolerance. On the other hand, if the relative error remains constant or starts to increase, the recurrence is terminated and the prior convergent taken as the value of the continued fraction. In this way, the process is always terminated when maximum precision is attained.

The "even" form of the continued fraction takes on the following implementation:

$$EZ2CZ = \frac{2*AZ}{RTPI} \prod_{RN=1}^{RNF} \frac{AM(RN)}{BM(RN)}$$

with $AM(1) = 1,\ AM(RN + 1) = -WM(RN + 1)*(WM(RN + 1) + 1)$

$BM(1) = 2*(AZ**2) + 1,\ BM(RN + 1) = BM(RN) + 4$

where $WM(1) = -1,\ WM(RN + 1) = WM(RN) + 2$

$EZ2CZ = (AZ*(FM/GM))*2/RTPI = (AZ*F(RN))*2/RTPI$

where $FM(-1) = 1,\ FM(0) = 0$

$GM(-1) = 0,\ GM(0) = 1$

and $FM(RN) = BM(RN)*FM(RN - 1) + AM(RN)*FM(RN - 2)$

$GM(RN) = BM(RN)*GM(RN - 1) + AM(RN)*GM(RN - 2)$

The relative error may be approximated by $[F(RN) - F(RN - 1)]/F(RN) = RE(RN)$. If the modulus squared of the relative error REM2(RN) is less than the square of the tolerance divided by 8, RNF = RN. If REM2(RN) is greater than or equal to REM2(RN − 1), then RNF = RN − 1. Normalization is likewise necessary here to avoid overflows in computing the relative error and its modulus squared and also in the generation of the successive convergents.

The asymptotic expansion is likewise evaluated using recurrence relations in the following form:

$$EZ2CZ = \frac{1}{SQRT(PI)} \sum_{RN=0}^{RNF} SGN(RN)*TM(RN) = SUM/RTPI$$

where $SGN(0) = 1,\ SGN(RN + 1) = -SGN(RN)$

$TM(RN) = (1/AZ)*(1*3* \ldots (2*RN - 1))/(2*(AZ**2))**RN$

$TM(0) = 1/AZ,\ TM(RN + 1) = DN(RN + 1)*\ TM(RN)*(1/(2*(AZ**2)))$

with $DN(1) = 1,\ DN(RN + 1) = DN(RN) + 2.$

The relative error may be approximated here by the ratio TM/SUM. The convergence test precedes the divergence test and is implemented as REM2 less than (TOLER**2)/8 to attain greater accuracy in both the real and imaginary parts. If the modulus squared of the term remains the same or increases, the prior sum is taken as the final sum.

For z along the imaginary axis, the error function is purely imaginary; the real part of erfc z = 1 and of e^{z^2} erfc $z = e^{-(AZI)^2}$. No difficulties arise in the use of the power series. However, since the asymptotic expansion is given for $|z| \to \infty$, the correction must be applied for AZR $\to$ 0.

The following table gives an indication of the number of terms needed to obtain maximum machine accuracy on the Univac 1108 with the various methods of computation.

Method	Number of Terms	
	Single Precision TOLER = .745E-8	Double Precision TOLER = .867D-18
Power Series	50	112
Continued Fraction	25	99
Asymptotic Expansion	22	45

4. Range

If the real part of z is zero or positive, e^{z^2} erfc z is valid for z throughout the entire machine range. Otherwise, the real part of z^2 is essentially limited by the range of the exponential library subroutine with the imaginary part of z^2 limited by the range of the sine, cosine library subroutine.

5. Accuracy and Precision

The maximum relative error, generally in erfc z, except for regions in the immediate neighborhood of zeros of the real and/or imaginary parts of the functions is 8E-6 for single precision computation on the Univac 1108.

The precision may be varied by changing the value of TOLER.

6. Timing (Univac 1108 Time/Sharing Executive System)

The time estimates given below are highly dependent on the operating system environment and consequently should not be relied on for critical timing measurements.

	Time (Seconds)	
Region	Single Precision TOLER = .745E-8	Double Precision TOLER = .867D-18
ZR = 0(.1)4, ZI = 0(.2)8 (1681 values)	5.94	22.5
Method	Maximum Time/Evaluation	
Power Series	.0101	.038
Continued Fraction	.0088	.052
Asymptotic Expansion	.0035	.0093

7. Testing

The language of the subroutine was checked for conformity with the PFORT VERIFIER.[3] Test arguments were devised and used in the analysis of the subroutine with the PROFILER.[4]

[3] The PFORT Verifier, A. D. Hall and B. G. Ryder, Bell Laboratories, Murray Hill, N.J. Proceedings of the Computer Science and Statistics Eighth Annual Symposium on the Interface, University of California, Los Angeles, February 13-14, 1975.

[4] Program Execution Profiles, G. Sande, World Bank, Washington, D.C. Proceedings of the Computer Science and Statistics Eighth Annual Symposium on the Interface, University of California, Los Angeles, February 13-14, 1975.

The subroutine was used to obtain related functions which were checked against available published tables ([1]-[5], [7]-[9], [13], [15]). Single precision results covering the 9 × 9 grid were compared against double precision results. This precision test particularly verified that the scaling operations were valid and undetected overflows had not occurred.

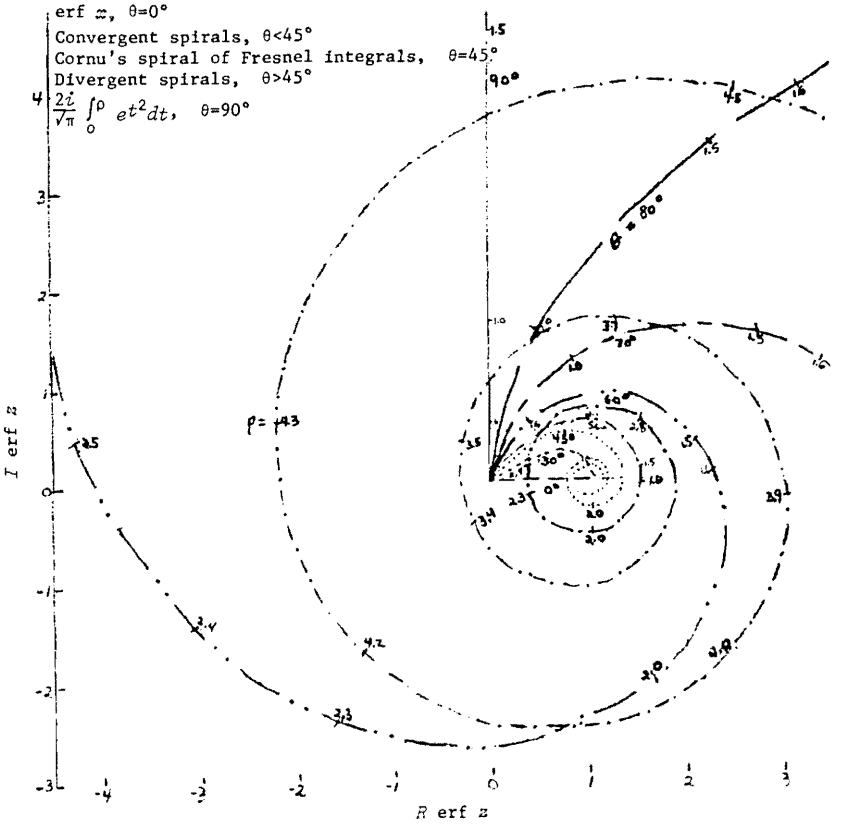
Where applicable, results were obtained by independent methods, for example, the power series and continued fraction, the power series and asymptotic expansion or all three methods. Otherwise, both forms of the power series or continued fraction were used. In addition, numerical integration of various forms of the integral representation was employed.

8. Test Values

Six-significant figure tables of erfc z, e^{z^2} erfc z and e^{z^2} erfc$(-z)$ are included in the appendix. The computations were performed with double precision arithmetic to provide more accurate values for checking purposes. The arguments are in polar coordinate form $z = \varrho e^{i\theta}$ for $\varrho = 0(.02).2(.1)3(.5)9$, $\Theta = 0°(15°)30°$, 37.5°, 45°, 50°(10°)90°. Values of the functions for z in other quadrants are readily obtainable with symmetry relations.

9. Argand Diagram of erf z $= \frac{2}{\sqrt{\pi}} \int_0^{z=\rho e^{i\theta}} e^{-t^2}\, dt$

Since erf z and erfc z are complementary functions, erf z is not tabulated but the argand diagram of the function is included to illustrate divergent and convergent spirals.



10. Special Values

Relevant values are collected here for completeness and ease of modification and checking of the program.

Zeros

	$\operatorname{erf} z_s = 0$
s	z_s
1	1.45061616 + i 1.88094300
2	2.24465927 + i 2.61657514
3	2.83974105 + i 3.17562810

	$w(z_s) = 0$
s	z_s
1	1.99146684 − i 1.35481013
2	2.69114902 − i 2.17704491
3	3.23533087 − i 2.78438761

	$C(z_s) = 0$	$S(z_s) = 0$
s	z_s	z_s
1	1.7437 + i .3057	2.0093 + i .2885
2	2.6515 + i .2529	2.8335 + i .2443
3	3.3204 + i .2240	3.4675 + i .2185

Maxima and Minima of Fresnel Integrals

s	$M_s = C(\sqrt{4s+1})$	$m_s = C(\sqrt{4s+3})$	$M_s^* = S(\sqrt{4s+2})$	$m_s^* = S(\sqrt{4s+4})$
0	.779893	.321056	.713972	.343416
1	.640807	.380391	.628940	.387969
2	.605721	.404261	.600362	.408301
3	.588128	.417922	.584942	.420516

Radius of Univalence ϱ

$\int_0^z e^{-t^2}\,dt \qquad e^{z^2}\int_0^z e^{-t^2}\,dt$

ϱ = 1.5748376 .92413887

Maximum and Inflection Point for Dawson's Integral

$F(.92413\ 88730) = .54104\ 42246$

$F(1.50197\ 52682) = .42768\ 66160$

Related Constants

1° = 1.74532 92519 94329 57692 36907 68488 61271 $(-2)r$
π = 3.14159 26535 89793 23846 26433 83279 50288
$\pi/2$ = 1.57079 63267 94896 61923 13216 91639 75144
$\sqrt{\pi}$ = 1.77245 38509 05516 02729 81674 83341 14518
$\sqrt{\pi}/2$ = .88622 69254 52758 01364 90837 41670 57259
2π = 6.28318 53071 79586 47692 52867 66559 00576
$2\sqrt{\pi}$ = 3.54490 77018 11032 05459 63349 66682 29036
e = 2.71828 18284 59045 23536 02874 71352 66249
$1/\pi$ = .31830 98861 83790 67153 77675 26745 02872
$2/\pi$ = .63661 97723 67581 34307 55350 53490 05744
$1/\sqrt{\pi}$ = .56418 95835 47756 28694 80794 51560 77258
$2/\sqrt{\pi}$ = 1.12837 91670 95512 57389 61589 03121 54517
$1/2\pi$ = .15915 49430 91895 33576 88837 63372 51436
$1/2\sqrt{\pi}$ = .28209 47917 73878 14347 40397 25780 38629
1/e = .36787 94411 71442 32159 55237 70161 46086
erf 1 = .84270 07929 49714 86934 12206 35082 60926

$$\int_0^1 e^{-t^2}dt = (\sqrt{\pi}/2)\,\mathrm{erf}\,1 = \sum_{n=0}^{\infty}\frac{(-1)^n}{n!(2n+1)} = .74682\ 41328\ 12427\ 02539\ 94674\ 36131\ 85300$$

$$(\sqrt{\pi}/2)e^1\mathrm{erf}\,1 = \sum_{n=0}^{\infty}\frac{2^n}{1\cdot 3\ldots(2n+1)} = 2.03007\ 84692\ 78704\ 97553\ 90899\ 25665\ 95044$$

$$\int_0^1 e^{t^2}dt = \sum_{n=0}^{\infty}\frac{1}{n!(2n+1)} = 1.46265\ 17459\ 07181\ 60880\ 40485\ 86856\ 98815$$

$$\sum_{n=0}^{\infty}\frac{1}{(2n)!(4n+1)} = 1.10473\ 79393\ 59804\ 31710\ 17580\ 11494\ 42058$$

$$\sum_{n=0}^{\infty}\frac{1}{(2n+1)!(4n+3)} = .35791\ 38065\ 47377\ 29170\ 22905\ 75362\ 56757$$

$$\sum_{n=0}^{\infty}\frac{2^{2n}}{1\cdot 3\ldots(4n+1)} = 1.28407\ 89880\ 95736\ 69733\ 77386\ 73036\ 75360$$

$$\sum_{n=0}^{\infty}\frac{2^{2n+1}}{1\cdot 3\ldots(4n+3)} = .74599\ 94811\ 82968\ 27820\ 13512\ 52629\ 19684$$

$$e^{-1}\int_0^1 e^{t^2}dt = \sum_{n=0}^{\infty}\frac{(-1)^n 2^n}{1\cdot 3\ldots(2n+1)} = .53807\ 95069\ 12768\ 41913\ 63874\ 20407\ 55675$$

Typical Tolerances and Their Natural Logarithms

2^{-16} = 0.15258 78906 25(−4)
2^{-24} = .59604 64477 53906 25(−7)
2^{-27} = .74505 80596 92382 8125(−8)
2^{-36} = .14551 91522 83668 51806 64062 5(−10)
2^{-48} = .35527 13678 80050 09293 55621 33789 0625(−14)
2^{-56} = .13877 78780 78144 56755 29539 58511 35253 90625(−16)
2^{-60} = .86736 17379 88403 54720 59622 40695 95336 91406 25(−18)
2^{-108} = .30814 87911 01957 73648 89564 70813 58837 09660 96263 71446 21112 38390 20729 06494 14062 5(−32)

$\log_e(2^{-16})$ = −11.09035 48889 59124 95067 57139 43330 82508
$\log_e(2^{-24})$ = −16.63553 23334 38687 42601 35709 14996 23763
$\log_e(2^{-27})$ = −18.71497 38751 18523 35426 52672 79370 76733

$\log_e(2^{-36}) = -24.95329\ 85001\ 58031\ 13902\ 03563\ 72494\ 35645$
$\log_e(2^{-48}) = -33.27106\ 46668\ 77374\ 85202\ 71418\ 29992\ 47526$
$\log_e(2^{-56}) = -38.81624\ 21113\ 56937\ 32736\ 49988\ 01657\ 88781$
$\log_e(2^{-60}) = -41.58883\ 08335\ 96718\ 56503\ 39272\ 87490\ 59408$
$\log_e(2^{-108}) = -74.85989\ 55004\ 74093\ 41706\ 10691\ 17483\ 06935$

Maximum and Minimum Machine Values and Their Natural Logarithms
(Univac 1108 Single and Double Precision Limits)

NBC = Number of binary digits in the (biased) characteristic of a floating point number

$$2^{-(2^{NBC-1}+1)} \leq x < 2^{2^{NBC-1}-1}$$

NBC = 8

$2^{127} = 0.17014\ 11834\ 60469\ 23173\ 16873\ 03715\ 88410(39)$
$2^{-129} = .14693\ 67938\ 52785\ 93849\ 60920\ 67152\ 78070(-38)$
$\log_e(2^{127}) = 88.02969\ 19311\ 13054\ 29598\ 84794\ 25188\ 42414$
$\log_e(2^{-129}) = -89.41598\ 62922\ 32944\ 91482\ 29436\ 68104\ 77728$

NBC = 11

$2^{1023} = 0.89884\ 65674\ 31157\ 95386\ 46525\ 95394\ 51236(308)$
$2^{-1025} = .27813\ 42323\ 13400\ 17288\ 62790\ 89666\ 55050(-308)$
$\log_e(2^{1023}) = 709.08956\ 57128\ 24051\ 53382\ 84602\ 51714\ 62914$
$\log_e(2^{-1025}) = -710.47586\ 00739\ 43942\ 15266\ 29244\ 94630\ 98227$

11. References

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C****                          APPENDIX
C                        IMPLEMENTING PROGRAM
C LANGUAGE. AMERICAN NATIONAL STANDARD FORTRAN
C (SUBROUTINE SUBJECTED TO PFORT VERIFIER*)
C DEFINITIONS. Z. A COMPLEX VARIABLE=ZR+I ZI
C     ERF(Z)=(2/SQRT(PI))*INTEGRAL(EXP(-T**2))DT FROM 0 TO Z
C            =ERFZR+I ERFZI
C     ERFC(Z)=(2/SQRT(PI))*INTEGRAL(EXP(-T**2))DT FROM Z TO
C                                                      INFINITY
C            =1-ERF(Z)
C            =ERFCZR+I ERFCZI
C     EXP(Z**2)*ERFC(Z)=EZ2CZR+I EZ2CZI
C   SYMMETRY RELATIONS
C     ERF(-Z)=-ERF(Z)
C     ERF(Z CONJG)=CONJG(ERF(Z))
C     ERFC(-Z)=2-ERFC(Z)
C     ERFC(Z CONJG)=CONJG(ERFC(Z))
C     EXP(Z**2)*ERFC(-Z)=2*EXP(Z**2)-EXP(Z**2)*ERFC(Z)
C     EXP(Z CONJG**2)*ERFC(Z CONJG)=CONJG(EXP(Z**2)*ERFC(Z))
C   SPECIAL CASE, Z=0
C       ERF(Z)=0
C       ERFC(Z)=1
C       EXP(Z**2)*ERFC(Z)=1
C USAGE. CALL ERRZ (ZR,ZI,ERFZR,ERFZI,ERFCZR,ERFCZI,EZ2CZR,
C                                                 EZ2CZI,IERR)
C       ARGUMENTS
C       (REAL TYPE VARIABLES ARE USED THROUGHOUT TO READILY
C       ALLOW FOR DOUBLE PRECISION COMPUTATION. REAL AND
C       IMAGINARY PARTS OF COMPLEX VARIABLES HAVE R AND
C       I AS FINAL CHARACTERS.)
C         ZR,ZI          REAL(OR DOUBLE PRECISION) TYPE INPUT
C         ERFZR,ERFZI          (SAME TYPE AS Z)         OUTPUT
C         ERFCZR,ERFCZI                "                 OUTPUT
C         EZ2CZR,EZ2CZI                "                 OUTPUT
C         IERR                    INTEGER TYPE           OUTPUT
C               IERR
C                0                                  NORMAL RETURN
C                1 EXP(Z**2)*ERFC(Z)                    INVALID
C                2 ERF(Z),ERFC(Z)                       INVALID
C                3 ERF(Z),ERFC(Z),EXP(Z**2)*ERFC(Z)     INVALID
C                  (Z IN 2ND OR 3RD QUADRANTS(ZR .LT. 0))
C           COMMONLY USED INTERNAL VARIABLES
C             AELL      LOWER LIMIT OF |ZI FOR ASYMPTOTIC
C                        EXPANSION(A.E.) ABS(ZR) .LE. 1
C             AZI       ABS(ZI)
C             AZR       ABS(ZR)
C             AZ2I      ABS(IMAG(Z**2))
C             CMAX      MAXIMUM MACHINE VALUE
C             CMIN      MINIMUM MACHINE VALUE
C             REM2      MODULUS SQUARED OF RELATIVE
C                        ERROR(R.E.)
C                        |R.E.|**2=REM2(N)
C             REPM2     REM2(N-1)
C             RHO       |ZI=SQRT(ZR**2+ZI**2)
C             RHOLC     LOWER LIMIT OF RHO FOR USING ONLY
C                         1ST TERM OF A.E.
C             RHOLS     UPPER LIMIT OF RHO, UNRESTRICTED
```

```
58      C                          ABS(ZR), FOR POWER SERIES
59      C             RTPI         SQRT(PI=3.14...)
60      C             SUMM2        !SUMI**2=SUMR**2+SUMI**2
61      C                          OR !SUM/TMAX!**2
62      C             TMAX         NORMALIZATION FACTOR
63      C             TMM2         !TMI**2=TMR**2+TMI**2
64      C                          OR !TM/TMAX!**2
65      C             TOLER        UPPER LIMIT FOR RELATIVE ERRORS
66      C             TOLER2       TOLER**2
67      C             TOL2         TOLER2/8
68      C             ULSC         MAXIMUM ARGUMENT FOR SIN/COS ROUTINE
69      C MODIFICATIONS.
70      C    THE CODE IS SET UP FOR SINGLE PRECISION COMPUTATION
71      C    WITH SINGLE PRECISION FUNCTION REFERENCES AND SINGLE
72      C    PRECISION MACHINE DEPENDENT CONSTANTS. FOR THE UNIVAC
73      C    1108, CMAX APPROX. 2**127,CMIN=2**(-129),ULSC=2**20 AND
74      C    TOLER=.745E-8. RTPI IS GIVEN IN DOUBLE
75      C    PRECISION FORMAT TO 19 SIGNIFICANT FIGURES.
76      C  DOUBLE PRECISION RESULTS ARE OBTAINED BY INSERTING
77      C    (1) THE DOUBLE PRECISION TYPE STATEMENT
78      C    (2) DOUBLE PRECISION INTRINSIC FUNCTION REFERENCES -
79      C        DABS,DMAX1 AND DMIN1
80      C    (3) DOUBLE PRECISION EXTERNAL FUNCTION REFERENCES -
81      C        DCOS,DEXP,DLOG,DSIN AND DSQRT AND
82      C    (4) FOR THE UNIVAC 1108 ADJUSTING THE CONSTANTS
83      C        CMAX APPROX. 2**1023,CMIN=2**(-1025),ULSC=2**56 AND
84      C        TOLER=.667D-18.
85      C   THE DETAILED METHODS SHOULD WORK FOR ANY PRECISION
86      C   IF THE MACHINE DEPENDENT CONSTANTS ARE CHANGED
87      C   WITH RTPI GIVEN TO THE REQUIRED NUMBER OF SIGNIFICANT
88      C   FIGURES.
89      C METHOD.    Z=ZR+I ZI =RHO*EXP(I*ARCTAN(ZI/ZR))
90      C              ALL METHODS APPLY TO AZ=ABS(ZR)+I ABS(ZI)=AZR+
91      C              I AZI. USE IS THEN MADE OF SYMMETRY RELATIONS.
92      C     POWER SERIES
93      C              RHO .LT. RHOLS(=1.5)
94      C              AZR .LE. 1. RHOLS .LE. RHO .LT. AELL
95      C                                      AELL=SQRT(-LOG(TOLER))
96      C         ERF(AZ)=(2/SQRT(PI))*SUM(SGN(RN)*TM(RN))
97      C                                              RN=0,1,....,RNF
98      C                    SGN(0)=1
99      C                    SGN(RN+1)=-SGN(RN)
100     C                    TM(RN)=((AZ**(2*RN+1))/1*2...RN)/(2*RN+1)
101     C                    TM(RN)=PTM(RN)/DN(RN)
102     C                         PTM(0)=AZ
103     C                         PTM(RN+1)=(AZ**2)*PTM(RN)/(RN+1)
104     C                         DN(0)=1
105     C                         DN(RN+1)=DN(RN)+2
106     C            RNF=RN IF TM=0 AND SUM=0, IF !TM/TMAX!**2(=TMM2)
107     C            =0 OR IF !SUM/TMAX!**2(=SUMM2) .NE. 0 AND
108     C            REM2(=TMM2/SUMM2) .LT. TOLER2
109     C     CONTINUED FRACTION
110     C              AZR .GT. 1. RHOLS .LE. RHO .LT. RHOLC
111     C                                   RHOLC=SQRT(ONE/(TWO*TOLER))
112     C         EXP(AZ**2)*ERFC(AZ)=
113     C                    (2*AZ/SQRT(PI))*(1 I/I (2*(AZ**2)+1)-
114     C                                     1*2 I/I (2*(AZ**2)+5)-
115     C                                     3*4 I/I (2*(AZ**2)+9)-...)
```

```
C                        =(2*AZ/RTPI)*II(AM(RN) I/I EM(RN))
C                                                    RN=1,2,...,RNF
C                             AM(1)=1
C                             AM(RN+1)=-WM(RN+1)*(WM(RN+1)+1)
C                             EM(1)=2*(AZ**2)+1
C                             BM(RN+1)=BM(RN)+4
C                                  WM(1)=-1
C                                  WM(RN+1)=WM(RN)+2
C                        =(AZ*(FM/GM))*2/RTPI
C                        =(AZ*F(FN))*2/RTPI
C                     FM(-1)=1
C                     GM(-1)=0
C                      FM(0)=0
C                      GM(0)=1
C                     FM(RN)=BM(RN)*FM(RN-1)+AM(RN)*FM(RN-2)
C                     GM(RN)=BM(RN)*GM(RN-1)+AM(RN)*GM(RN-2)
C               RNF=RN IF REM2(FOR R.E.=(F(RN)-F(RN-1))/F(RN))
C                   .LT. TOL2 OR
C               RNF=RN-1 IF REM2(RN) .GE. REM2(RN-1)
C     ASYMPTOTIC EXPANSION
C               AZR .LE. 1. AELL .LE. RHO .LT. RHOLC
C               (FOR RHOLC .LE. RHO .LE. CMAX, TO PRESERVE
C                  ACCURACY AN ALTERNATIVE COMPUTATION OF
C                  THE FIRST TERM OF THE A.E. IS EMPLOYED.)
C          EXP(AZ**2)*ERFC(AZ)=(SUM(SGN(RN)*TM(RN)))/SQRT(PI)
C                                                    RN=0,1,...,RNF
C                SGN(0)=1
C                SGN(RN+1)=-SGN(RN)
C                TM(RN)=(1/AZ)*(1*3...(2*RN-1))/
C                                                 (2*(AZ**2))**RN
C                TM(0)=1/AZ
C                TM(RN+1)=DN(RN+1)*(TM(RN)*(1/(2*(AZ**2))))
C                     DN(1)=1
C                     DN(RN+1)=DN(RN)+2
C                RNF=RN IF REM2(FOR R.E.=TM/SUM) .LT. TOL2
C                RNF=RN-1 IF TMM2(RN) .GE. TMPM2(=TMM2(RN-1))
C                                                 (DIVERGENCE)
C RANGE.
C     EXP(Z**2)*ERFC(Z) IS VALID FOR ZR .GE. 0 THROUGHOUT
C     THE ENTIRE MACHINE RANGE. ERF(Z),ERFC(Z) AND
C     EXP(Z**2)*ERFC(Z)(FOR ZR .LT. 0) ARE LIMITED BY THE
C     RANGE AND ACCURACY OF THE SINE,COSINE AND/OR THE
C     EXPONENTIAL LIBRARY ROUTINES.
C ACCURACY. THE MAXIMUM RELATIVE ERROR (GENERALLY IN ERFC)
C           EXCEPT IN THE IMMEDIATE NEIGHBORHOOD OF ZEROS.
C           IS B(-E) ON THE UNIVAC 1108 FOR
C           SINGLE PRECISION COMPUTATION.  THE REAL
C           AND IMAGINARY PARTS INDEPENDENTLY AS WELL
C           AS THEIR ZEROS ENTER INTO CONSIDERATION.
C PRECISION. VARIABLE - BY SETTING A PREDETERMINED VALUE OF
C                       TOLER
C MAXIMUM      UNIVAC 1108 TIME/SHARING EXECUTIVE SYSTEM
C TIMING.        S.P.             D.P.
C (SECONDS)      .0101            .052
C STORAGE. 1171 WORDS REQUIRED BY THE UNIVAC 1108 COMPILER
C           (313 FORTRAN STATEMENTS, 95 VARIABLES)
C* THE PFORT VERIFIER,A.C.HALL AND B.G.RYDER
C (BELL LABORATORIES,MURRAY HILL, N.J.) PROC.OF THE COMPUTER
```

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      C SCIENCE AND STATISTICS EIGHTH ANNUAL SYMPOSIUM ON THE
      C INTERFACE, UNIV. OF CALIF., LOS ANGELES, FEB.13-14,1975.
      C*****
            SUBROUTINE ERRZ(ZR,ZI,ERFZR,ERFZI,ERFCZR,ERFCZI,
           1                                  EZ2CZR,EZ2CZI,IERR)
      C           MACHINE DEPENDENT CONSTANTS
            CMAX=.17014116E39
            CMIN=.14693679E-38
            ULSC=.104857EE7
            TOLER=.745E-8
      C NOTE TOLER IS SET TO THE PRECISION OF
      C THE UNIVAC 1108 SINGLE PRECISION ARITHMETIC.
            RTPI=1.77245385090551602D0
      C           OTHER CONSTANTS
            ZERO=0
            ONE=1
            TWO=2
            THREE=3
            FOUR=4
            ONPTFV=THREE/TWO
      C           INITIALIZATION OF ERROR INDICATORS
            IERR=0
            IQ=0
      C           FUNCTION REFERENCES
      C NOTE FUNCTION REFERENCES OCCUR IN THE REGIONS OF STATEMENT
      C LABELS 5,15 AND 85 AND IN STATEMENT LABELS 110,315,515,
      C 517 AND 955.
      C           SET UP FOR Z IN FIRST QUADRANT    AZ=AZR+I AZI
          5 AZR=ABS(ZR)
            AZI=ABS(ZI)
            ARMN=AMIN1(AZR,AZI)
            ARMX=AMAX1(AZR,AZI)
            IF (ARMX .GT. ZERO) GO TO 10
      C           SPECIAL CASE, Z=0
            ERFZR=ZERO
            ERFZI=ZERO
            ERFCZR=ONE
            ERFCZI=ZERO
            EZ2CZR=ONE
            EZ2CZI=ZERO
            RETURN
      C           CONTROL VARIABLES
         10 TOLER2=TOLER*TOLER
            TOL2=TOLER2/FOUR/TWO
            RHOLS=ONPTFV
            RMNMX=ARMN/ARMX
         15 CMAXLN=ALOG(CMAX)
            CMINLN=ALOG(CMIN)
            AELL=SQRT(-ALOG(TOLER))
            RHOLC=SQRT(ONE/(TWO*TOLER))
            PRHO=SQRT(RMNMX*RMNMX+ONE)
      C           COMPUTATION OF AUXILIARY QUANTITIES
      C                COMPUTATION OF AZ**2=Z2R+I AZ2I
            IF (ARMX .LT. ONE) GO TO 60
      C                     OVERFLOW CHECK ON Z2RN=-Z2R
            TEMPB=((ONE-RMNMX)*ARMX)*(ONE+RMNMX)
            IF (TEMPB .LT. CMAX/ARMX) GO TO 20
            TEMPC=CMAX
```

```
232            GO TO 30
233         20 TEMPC=TEMPE*ARIMX
234         30 IF (AZR-ARIMX) 50,40,50
235         40 Z2RN=-TEMPC
236            GO TO 70
237         50 Z2RN=TEMPC
238            GO TO 70
239         60 Z2RN=(AZI+AZR)*(AZI-AZR)
240            GO TO 80
241       C                       OVERFLOW CHECK ON AZ2I
242         70 IF (ARIMN .LT. (CMAX/TWO)/ARIMX) GO TO 80
243            AZ2I=CMAX
244            GO TO 90
245         80 AZ2I=(TWO*ARIMN)*ARIMX
246       C                 PRELIMINARY COMPUTATIONS FOR EXP(AZ**2) AND
247       C                   EXP(-AZ**2)
248       C                       CHECK IF VALID ARGUMENT FOR SIN/COS
249            IF (AZ2I .GE. ULSC) GO TO 90
250         85 COAZ2I=COS(AZ2I)
251            SIAZ2I=SIN(AZ2I)
252       C                       EXTENDING RANGE OF EXP ROUTINE
253         90 TEMP=Z2RN/THREE
254            Z2R=-Z2RN
255       C                       OVERFLOW CHECK
256            IF (TEMP .LT. CMAXLN) GO TO 100
257            EMZ2D3=CMAX
258            EZ2R=ZERO
259            GO TO 190
260       C                       UNDERFLOW CHECK
261        100 IF (TEMP .GT. CMINLN) GO TO 110
262            EMZ2D3=ZERO
263       C                       EXP(AZ**2) OVERFLOWS (IQ=1)
264            IQ=1
265            GO TO 160
266        110 EMZ2D6=EXP(TEMP/TWO)
267            EMZ2D3=EMZ2D6*EMZ2D6
268            EZ2D6=ONE/EMZ2D6
269            IF (EZ2D6 .LE. ONE) GO TO 180
270            J=1
271            PEXP=ZERO
272            TEMP=EZ2D6
273        120   IF (TEMP .GE. CMAX/EZ2D6) GO TO 150
274              TEMP=TEMP*EZ2D6
275              J=J+1
276              IF (J-5) 120,130,140
277        130 PEXP=TEMP
278            GO TO 120
279        140 EZ2R=TEMP
280            GO TO 190
281        150 IF (PEXP) 170,160,170
282        160 PEXP=CMAX
283        170 EZ2R=CMAX
284            GO TO 190
285        180 PEXP=EZ2D6**5
286            EZ2R=PEXP*EZ2D6
287            TEZ2=(PEXP*TWO)*EZ2D6
288       C                 COMPUTATION OF RHO
289       C                       OVERFLOW CHECK
```

```
      190 IF (ARIMX .LT. CMAX/PRHO) GO TO 200
          RHO=CMAX
          GO TO 210
      200 RHO=ARIMX*PRHO
C           METHOD SELECTION
      210 IF (RHO .LT. RHOLC) GO TO 220
C                 IMPROVE ACCURACY FOR LARGE RHO
          FA=AZR/ARIMX
          FB=AZI/ARIMX
          FC=RTPI*(RMNMX*RMNMX+ONE)
          EZ2CZR=(FA/ARIMX)/FC
          EZ2CZI=-(FB/ARIMX)/FC
          GO TO 800
      220 IF (RHO .LT. RHOLS) GO TO 300
          IF (AZR .GT. ONE) GO TO 500
          IF (RHO-AELL) 300,700,700
C                 POWER SERIES FOR ERF(AZ)
C                       INITIALIZATION
      300 SUMR=ZERO
          SUMI=ZERO
          SGN=ONE
          RN=ZERO
          DN=ONE
          PTMR=AZR
          PTMI=AZI
C                       COMPUTING SUM
      310   TMR=PTMR/DN
            TMI=PTMI/DN
            SUMR=SUMR+TMR*SGN
            SUMI=SUMI+TMI*SGN
C                       SCALING TO AVOID OVERFLOW OR UNDER-
C                         FLOW IN APPROXIMATING R.E.
      315   TMAX=AMAX1(ABS(TMR),ABS(TMI),ABS(SUMR),ABS(SUMI))
            IF (TMAX) 320,360,320
      320   TMM2=(TMR/TMAX)**2+(TMI/TMAX)**2
            SUMM2=(SUMR/TMAX)**2+(SUMI/TMAX)**2
            IF (TMM2) 330,360,330
      330   IF (SUMM2) 340,350,340
      340   REM2=TMM2/SUMM2
C                       TOLERANCE CHECK
            IF (REM2 .LT. TOLER2) GO TO 360
C                       ADDITIONAL TERMS
      350   DN=DN+TWO
            RN=RN+ONE
            SGN=-SGN
            TEMP=(PTMR*Z2R-PTMI*AZ2I)/RN
            PTMI=(PTMR*AZ2I+PTMI*Z2R)/RN
            PTMR=TEMP
            GO TO 310
C                       FUNCTIONS EVALUATED IN FIRST QUADRANT
      360 ERFZR=SUMR*TWO/RTPI
          ERFZI=SUMI*TWO/RTPI
          ERFCZR=ONE-ERFZR
          ERFCZI=-ERFZI
          EZ2CZR=(COAZ2I*ERFCZR-SIAZ2I*ERFCZI)*EZ2R
          EZ2CZI=(SIAZ2I*ERFCZR+COAZ2I*ERFCZI)*EZ2R
          GO TO 840
C                 CONTINUED FRACTION FOR EXP(AZ**2)*ERFC(AZ)
```

```
C                       INITIALIZATION
  500 WM=-CNE
      BMR=TWC*Z2R+CNE
      BMI=TWC*AZ2I
      AM=CNE
      FMM2R=CNE
      FMM2I=ZERO
      GMM2R=ZERO
      GMM2I=ZERO
      FMM1R=ZERO
      FMM1I=ZERO
      GMM1R=CNE
      GMM1I=ZERO
      REPM2=CMAX
      FPR=ZERO
      FPI=ZERO
C                       RECURRENCE RELATION
  510   FMR=BMR*FMM1R-BMI*FMM1I+AM*FMM2R
        FMI=BMI*FMM1R+BMR*FMM1I+AM*FMM2I
        GMR=BMR*GMM1R-BMI*GMM1I+AM*GMM2R
        GMI=BMI*GMM1R+BMR*GMM1I+AM*GMM2I
C                       CONVERGENT F=FM/GM
C                         SCALING TO AVOID OVERFLOW IN
C                           COMPUTING CONVERGENT
  515   TMAX=AMAX1(ABS(FMR),ABS(FMI),ABS(GMR),ABS(GMI))
        SFMR=FMR/TMAX
        SFMI=FMI/TMAX
        SGMR=GMR/TMAX
        SGMI=GMI/TMAX
        TEMP=SGMR*SGMR+SGMI*SGMI
        FR=(SFMR*SGMR+SFMI*SGMI)/TEMP
        FI=(SFMI*SGMR-SFMR*SGMI)/TEMP
C                       APPROXIMATING R.E.
        TEMP=FR*FR+FI*FI
        TEMPA=FR-FPR
        TEMPB=FI-FPI
        REM2=(TEMPA*TEMPA+TEMPB*TEMPB)/TEMP
C                       TOLERANCE CHECK
        IF (REM2 .LT. TOL2) GO TO 530
        IF (REM2 .GE. REPM2) GO TO 520
C                       ADDITIONAL CONVERGENTS
        WM=WM+TWC
        BMR=BMR+FCUR
        AM=-WM*(WM+CNE)
        FMM2R=FMM1R
        FMM2I=FMM1I
        GMM2R=GMM1R
        GMM2I=GMM1I
        FMM1R=FMR
        FMM1I=FMI
        GMM1R=GMR
        GMM1I=GMI
        FPR=FR
        FPI=FI
        REPM2=REM2
C                       SCALING
C SCALING SHOULD NOT BE DELETED AS THE VALUES OF FMR,FMI
C FMR,GMI MAY OVERFLOW FOR SMALL VALUES OF REAL OF Z
```

```
      517   ABM=AMAX1(ABS(BMR),ABS(BMI))
            IF (TMAX .LT. (CMAX/FCUR)/(TWO*ABM-AM)) GO TO 510
            FMM2R=FMM2R/TMAX
            FMM2I=FMM2I/TMAX
            GMM2R=GMM2R/TMAX
            GMM2I=GMM2I/TMAX
            FMM1R=FMM1R/TMAX
            FMM1I=FMM1I/TMAX
            GMM1R=GMM1R/TMAX
            GMM1I=GMM1I/TMAX
            GO TO 510
C                       RELATIVE ERROR INCREASED-ROUNDOFFS
C                         ACCEPT PRIOR CONVERGENT
      520 FR=FPR
          FI=FPI
C                       EVALUATE EXP(AZ**2)*ERFC(AZ)
      530 EZ2CZR=(AZR*FR-AZI*FI)*TWC/RTPI
          EZ2CZI=(AZI*FR+AZR*FI)*TWC/RTPI
          GO TO 800
C                 ASYMPTOTIC EXPANSION FOR EXP(AZ**2)*ERFC(AZ)
C                       INITIALIZATION
      700 TZ2R=TWC*Z2R
          TZ2I=TWC*AZ2I
          TEMP=TZ2R*TZ2R+TZ2I*TZ2I
          RTZ2R=TZ2R/TEMP
          RTZ2I=-TZ2I/TEMP
          TMM1R=(AZR/RHC)/RHC
          TMM1I=-(AZI/RHC)/RHC
          TMPM2=TMM1R*TMM1R+TMM1I*TMM1I
          SUMR=TMM1R
          SUMI=TMM1I
          DN=ONE
          SGN=-ONE
C                       COMPUTING SUM
      710   TMR=DN*(TMM1R*RTZ2R-TMM1I*RTZ2I)
            TMI=DN*(TMM1I*RTZ2R+TMM1R*RTZ2I)
            SUMR=SGN*TMR+SUMR
            SUMI=SGN*TMI+SUMI
C                       APPROXIMATING R.E.
            SUMM2=SUMR*SUMR+SUMI*SUMI
            TMM2=TMR*TMR+TMI*TMI
            REM2=TMM2/SUMM2
C                       TOLERANCE CHECK
            IF (REM2 .LT. TOL2) GO TO 730
            IF (TMM2 .LT. TMPM2) GO TO 720
C                       DIVERGENT PATH
            SUMR=SUMR-SGN*TMR+SGN*TMM1R
            SUMI=SUMI-SGN*TMI+SGN*TMM1I
            GO TO 730
C                       ADDITIONAL TERMS
      720   SGN=-SGN
            DN=DN+TWO
            TMM1R=TMR
            TMM1I=TMI
            TMPM2=TMM2
            GO TO 710
C                       EVALUATE EXP(AZ**2)*ERFC(AZ)
      730 EZ2CZR=SUMR/RTPI
```

```
      EZ2CZI=SUMI/RTPI
C                        MAINTAINING ACCURACY IN EZ2CZR
C                          FOR SMALL AZR
      TEMP=EZ2CZR*TCLER
      IF (TEMP .GT. EZ2R) GO TO 800
      TEMP=CMIN*RHOLC
      IF (EZ2CZR .GT. TEMP) GO TO 750
      IF (EZ2R .GT. TEMP) GO TO 750
      IF (AZR) 740,750,740
C                        INDICATE RESULTING ERRORS IN ERFC
C                          (AND ERF)(IERR=2)
  740 IERR=2
  750 EZ2CZR=EZ2CZR+EZ2R*COAZ2I
C                   EVALUATE ERFC AND ERF FOR AZ IN CONTINUED
C                     FRACTION AND ASYMPTOTIC EXPANSION REGIONS
  800 IF (AZ2I .LT. ULSC) GO TO 830
C                       INVALID ARGUMENT FOR SIN/COS
C                         ERFC(AND ERF) INVALID (IERR=2)
      IF (AZR-ARIMX) 820,810,820
C                           AZR .GE. AZI
  810 ERFCZR=ZERO
      ERFCZI=ZERO
      GO TO 910
C                           AZR .LT. AZI
  820 ERFCZR=CMAX
      ERFCZI=CMAX
      GO TO 910
C                      VALID ARGUMENT FOR SIN/COS
  830 IF (RFC .GE. RHOLC) GO TO 840
C                           RHO .LT. RHOLC
      TEMPA=EZ2CZR*COAZ2I+EZ2CZI*SIAZ2I
      TEMPB=-EZ2CZR*SIAZ2I+EZ2CZI*COAZ2I
      TEMPC=EMZ2D3
      GO TO 850
C                           RFC .GE. RHOLC
  840 TEMPA=(FA*COAZ2I-FB*SIAZ2I)/FC
      TEMPB=(-FA*SIAZ2I-FB*COAZ2I)/FC
      TEMP=EMZ2D3/ARIMX
      TEMPA=TEMPA*TEMP
      TEMPB=TEMPB*TEMP
      TEMPC=CNE
  850 IF (EMZ2D3 .LE. CNE) GO TO 920
C                           EVALUATE ERFC(AZ)(AZI .GT. AZR)
      I=1
      TEMP=TEMPA
  860   J=1
        SGN=CNE
        FD=TEMPC
        IF (TEMP .GE. ZERO) GO TO 870
        SGN=-SGN
        TEMP=-TEMP
  870     IF (TEMP .LT. CMAX/FD) GO TO 880
          TEMP=CMAX
          IC=2
          GO TO 890
  880     TEMP=TEMP*FD
          J=J+1
          IF (J .GT. 3) GO TO 890
```

```
522                FD=EMZ2D3
523                GO TO 870
524          890   IF (I .GE. 2) GO TO 900
525                ERFCZR=TEMP*SGN
526                I=I+1
527                TEMP=TEMPB
528                GO TO 860
529          900 ERFCZI=TEMP*SGN
530              IF (IQ .NE. 2) GO TO 930
531        C                            ERFC(AND ERF) INVALID (IERR=2)
532        C                                OVERFLOW OF ERFC(AZ)(IQ=2)
533          910 IERR=2
534              GO TO 930
535        C                          EVALUATE ERFC(AZ)(AZI .LE. AZR)
536          920 ERFCZR=(TEMPA*EMZ2D3)*EMZ2D3*TEMPC
537              ERFCZI=(TEMPB*EMZ2D3)*EMZ2D3*TEMPC
538        C                         SPECIAL CASE (AZR=0)
539          930 IF (AZR .LE. ZERO) ERFCZR=ONE
540        C                          EVALUATE ERF(AZ)
541              ERFZR=ONE-ERFCZR
542              ERFZI=-ERFCZI
543        C           SYMMETRY RELATIONS APPLIED
544          940 IF (ZR-AZR) 950,1000,950
545        C                   REAL OF Z .LT. 0
546          950 ERFZR=-ERFZR
547              ERFCZR=TWO-ERFCZR
548              IF (AZ2I .GE. ULSC) GO TO 960
549              IF (IQ .EQ. 1) GO TO 960
550        C                   MAINTAINING ACCURACY IN 2*EXP(Z**2)
551              IF (AZR .LE. AZI) GO TO 980
552          955 TEMP=AMAX1(ABS(SIAZ2I), ABS(COAZ2I))
553              IF (TEMP .LT. ((CMAX/TWO)/PEXP)/EZ2D6) GO TO 970
554        C                   EXP(Z**2)*ERFC(Z) INVALID (IERR+1)
555          960 IERR=IERR+1
556              EZ2CZR=CMAX
557              EZ2CZI=CMAX
558              GO TO 1000
559          970 TEZ2R=((PEXP*COAZ2I)*EZ2D6)*TWO
560              TEZ2I=((PEXP*SIAZ2I)*EZ2D6)*TWO
561              GO TO 990
562          980 TEZ2R=TEZ2*COAZ2I
563              TEZ2I=TEZ2*SIAZ2I
564        C                   EVALUATE EXP(Z**2)*ERFC(Z)
565          990 EZ2CZR=TEZ2R-EZ2CZR
566              EZ2CZI=EZ2CZI-TEZ2I
567         1000 IF (ZI-AZI) 1010,1020,1010
568        C                   IMAGINARY OF Z .LT. 0
569         1010 ERFZI=-ERFZI
570              ERFCZI=-ERFCZI
571              EZ2CZI=-EZ2CZI
572         1020 RETURN
573              END
```

TABLE 1

ERFC(Z)

RHO\THETA	0°	15°	30°	37.5°	45°
.00	.100000+01 .0	.100000+01 .000000	.100000+01 .000000	.100000+01 .000000	.100000+01 .000000
.02	.977435+00 .0	.978204+00 -.583879-02	.980456+00 -.112808-01	.982095+00 -.137355-01	.984040+00 -.159556-01
.04	.954889+00 .0	.956420+00 -.116648-01	.960912+00 -.225435-01	.964183+00 -.274543-01	.968068+00 -.318984-01
.06	.932378+00 .0	.934662+00 -.174654-01	.941368+00 -.337702-01	.946257+00 -.411398-01	.952070+00 -.478156-01
.08	.909922+00 .0	.912942+00 -.232279-01	.921824+00 -.449428-01	.928310+00 -.547751-01	.936033+00 -.636943-01
.10	.887537+00 .0	.891273+00 -.289397-01	.902280+00 -.560434-01	.910337+00 -.683437-01	.919946+00 -.795217-01
.12	.865242+00 .0	.869667+00 -.345886-01	.882738+00 -.670542-01	.892330+00 -.818288-01	.903796+00 -.952846-01
.14	.843053+00 .0	.848138+00 -.401625-01	.863197+00 -.779575-01	.874283+00 -.952136-01	.887571+00 -.110970+00
.16	.820988+00 .0	.826697+00 -.456493-01	.843657+00 -.887357-01	.856190+00 -.108481+00	.871258+00 -.126564+00
.18	.799064+00 .0	.805358+00 -.510376-01	.824121+00 -.993713-01	.838045+00 -.121615+00	.854845+00 -.142053+00
.20	.777297+00 .0	.784132+00 -.563161-01	.804590+00 -.109847+00	.819844+00 -.134598+00	.838321+00 -.157424+00
.30	.671373+00 .0	.680129+00 -.806921-01	.707071+00 -.159241+00	.727824+00 -.196662+00	.753652+00 -.231995+00
.40	.571608+00 .0	.580740+00 -.100871+00	.610081+00 -.202202+00	.633845+00 -.252418+00	.664672+00 -.301347+00
.50	.479500+00 .0	.487320+00 -.115988+00	.514270+00 -.236937+00	.537866+00 -.299764+00	.570447+00 -.363359+00
.60	.396144+00 .0	.401058+00 -.125565+00	.420627+00 -.261983+00	.440342+00 -.336667+00	.470518+00 -.415645+00
.70	.322199+00 .0	.322926+00 -.129533+00	.330478+00 -.276308+00	.342317+00 -.361274+00	.365070+00 -.455585+00
.80	.257899+00 .0	.253631+00 -.128213+00	.245436+00 -.279418+00	.245465+00 -.372052+00	.255106+00 -.480414+00
.90	.203092+00 .0	.193577+00 -.122258+00	.167296+00 -.271437+00	.152078+00 -.367979+00	.142621+00 -.487404+00
1.00	.157299+00 .0	.142847+00 -.112577+00	.978859-01 -.253165+00	.649787-01 -.348749+00	.307358-01 -.474148+00
1.10	.119795+00 .0	.101201+00 -.100228+00	.388720-01 -.226077+00	-.126659-01 -.314972+00	-.762502-01 -.438939+00
1.20	.896860-01 .0	.680972-01 -.863111-01	-.844458-02 -.192259+00	-.776217-01 -.268341+00	-.172938+00 -.381252+00
1.30	.659921-01 .0	.427427-01 -.718743-01	-.433244-01 -.154271+00	-.126958+00 -.211694+00	-.253111+00 -.302268+00
1.40	.477149-01 .0	.241557-01 -.578252-01	-.657413-01 -.114934+00	-.158478+00 -.148963+00	-.310259+00 -.205369+00
1.50	.338949-01 .0	.112438-01 -.448760-01	-.764609-01 -.770708-01	-.171150+00 -.849352-01	-.338390+00 -.965018-01
1.60	.236516-01 .0	.288326-02 -.335145-01	-.770202-01 -.432136-01	-.165475+00 -.248347-01	-.333094+00 .157600-01
1.70	.162095-01 .0	-.200643-02 -.240031-01	-.696000-01 -.153368-01	-.143694+00 .262601-01	-.292775+00 .120601+00
1.80	.109095-01 .0	-.440212-02 -.164025-01	-.567976-01 .535338-02	-.109748+00 .641075-01	-.219843+00 .205946+00
1.90	.720957-02 .0	-.513475-02 -.106123-01	-.413301-01 .185261-01	-.689521-01 .859866-01	-.121580+00 .260096+00
2.00	.467773-02 .0	-.486777-02 -.641967-02	-.257133-01 .247077-01	-.273484-01 .912368-01	-.103117-01 .273926+00
2.10	.297947-02 .0	-.409504-02 -.354979-02	-.119733-01 .251347-01	.916428-02 .815039-01	.974865-01 .243355+00
2.20	.186285-02 .0	-.315468-02 -.170957-02	-.144281-02 .215037-01	.358034-01 .605788-01	.183390+00 .171536+00
2.30	.114318-02 .0	-.225377-02 -.621965-03	.531949-02 .156622-01	.497620-01 .337876-01	.230486+00 .700421-01
2.40	.688514-03 .0	-.149843-02 -.481389-04	.847425-02 .930477-02	.507367-01 .700451-02	.227631+00 -.417236-01
2.50	.406952-03 .0	-.924217-03 .201683-03	.872881-02 .372996-02	.409438-01 -.145264-01	.173531+00 -.139462+00
2.60	.236034-03 .0	-.523179-03 .266848-03	.709056-02 -.293708-03	.245557-01 -.272819-01	.792740-01 -.199249+00
2.70	.134333-03 .0	-.265334-03 .241454-03	.460569-02 -.253794-02	.664754-02 -.301636-01	-.321336-01 -.204295+00
2.80	.750132-04 .0	-.113783-03 .183127-03	.214179-02 -.321372-02	-.809354-02 -.245881-01	-.130194+00 -.151392+00
2.90	.410979-04 .0	-.339435-04 .122861-03	.256793-03 -.279917-02	-.165397-01 -.138997-01	-.185124+00 -.545715-01
3.00	.220905-04 .0	.198248-05 .741051-04	-.830403-03 -.184961-02	-.178141-01 -.225516-02	-.178018+00 .564096-01
3.50	.743098-06 .0	.383634-05 -.331302-06	.361438-04 .343118-03	.663124-02 .774278-03	.146013+00 -.667835-01
4.00	.154173-07 .0	-.506480-07 -.121829-06	-.100583-04 -.454411-04	-.206887-02 .808579-03	-.704373-01 .121816+00
4.50	.196616-09 .0	-.172664-08 .241877-08	.342245-05 .358814-05	.149447-03 -.641621-03	-.697054-01 -.103987+00
5.00	.153746-11 .0	.431344-10 -.812606-11	-.410390-06 .690194-07	.163226-03 .593711-04	.909031-01 -.666628-01
5.50	.735785-14 .0	-.401550-12 -.137042-12	-.988539-10 -.274562-07	.529439-06 .406179-04	.945377-01 .396363-01
6.00	.215197-16 .0	.222076-14 .150084-14	.136924-08 -.383336-09	-.550781-05 .636094-05	.563713-01 .752047-01
6.50	.384215-19 .0	-.905982-17 -.633184-17	.477805-10 .324007-10	-.126107-05 .886874-06	.498547-01 .710156-01
7.00	.418383-22 .0	.276523-19 .109112-19	.941894-12 .157585-11	-.164519-06 .187753-06	.710858-01 .379415-01
7.50	.277665-25 .0	-.517996-22 .562585-23	.233092-13 .393021-13	.525818-10 .356932-07	.667858-01 -.345871-01
8.00	.112243-28 .0	.396093-25 -.443759-25	.731135-15 .506747-15	.420606-08 .160997-08	-.258232-01 -.656143-01
8.50	.276232-32 .0	.111560-28 .427748-28	.130972-16 -.343079-17	.123912-09 -.485650-09	-.475602-01 .462887-01
9.00	.413703-36 .0	-.213063-31 -.174915-32	.313042-20 -.160996-18	-.462157-10 .168052-10	.623821-01 -.612171-02

ERFC(Z)

RHO\THETA	50°		60°		70°		80°		90°
.00	.100000+01	.000000	.100000+01	.000000	.100000+01	.000000	.100000+01	.000000	.1+01	.000000
.02	.985491+00	-.172863-01	.988713+00	-.195441-01	.992279+00	-.212081-01	.996080+00	-.222273-01	.1+01	-.225706-01
.04	.970967+00	-.345635-01	.977408+00	-.390882-01	.984542+00	-.424252-01	.992150+00	-.444703-01	.1+01	-.451593-01
.06	.956411+00	-.518226-01	.966067+00	-.586322-01	.976774+00	-.636604-01	.988203+00	-.667446-01	.1+01	-.677841-01
.08	.941809+00	-.690545-01	.954672+00	-.781761-01	.968959+00	-.849226-01	.984228+00	-.890659-01	.1+01	-.904633-01
.10	.927144+00	-.862497-01	.943204+00	-.977195-01	.961080+00	-.106221+00	.980217+00	-.111450+00	.1+01	-.113215+00
.12	.912401+00	-.103399+00	.931646+00	-.117262+00	.953123+00	-.127564+00	.976160+00	-.133913+00	.1+01	-.136058+00
.14	.897565+00	-.120493+00	.919978+00	-.136803+00	.945070+00	-.148961+00	.972048+00	-.156471+00	.1+01	-.159011+00
.16	.882621+00	-.137521+00	.908183+00	-.156342+00	.936906+00	-.170421+00	.967870+00	-.179140+00	.1+01	-.182093+00
.18	.867552+00	-.154473+00	.896242+00	-.175878+00	.928612+00	-.191952+00	.963617+00	-.201936+00	.1+01	-.205323+00
.20	.852345+00	-.171339+00	.884135+00	-.195409+00	.920173+00	-.213564+00	.959279+00	-.224877+00	.1+01	-.228721+00
.30	.773712+00	-.253982+00	.820454+00	-.292919+00	.875153+00	-.323124+00	.935924+00	-.342344+00	.1+01	-.348949+00
.40	.689468+00	-.332643+00	.749698+00	-.389843+00	.823615+00	-.435932+00	.908660+00	-.466099+00	.1+01	-.476625+00
.50	.598041+00	-.405420+00	.669242+00	-.485367+00	.762712+00	-.552889+00	.875613+00	-.598673+00	.1+01	-.614952+00
.60	.498195+00	-.469950+00	.576286+00	-.578076+00	.688562+00	-.674665+00	.834330+00	-.742997+00	.1+01	-.767853+00
.70	.389205+00	-.523342+00	.467898+00	-.665719+00	.598036+00	-.801524+00	.781501+00	-.902533+00	.1+01	-.940283+00
.80	.271098+00	-.562163+00	.341135+00	-.744933+00	.484501+00	-.933057+00	.712565+00	-.108143+01	.1+01	-.113867+01
.90	.144933+00	-.582512+00	.193283+00	-.810912+00	.341532+00	-.106775+01	.621138+00	-.128468+01	.1+01	-.13715+01
1.00	.131385-01	-.580211+00	.222736-01	-.857061+00	.160625+00	-.120233+01	.498184+00	-.151831+01	.1+01	-.165043+01
1.10	-.120165+00	-.551166+00	-.172655+00	-.874670+00	-.686592-01	-.133079+01	.330784+00	-.178947+01	.1+01	-.199117+01
1.20	-.248882+00	-.491928+00	-.389989+00	-.852699+00	-.358812+00	-.144298+01	.100318+00	-.210652+01	.1+01	-.241591+01
1.30	-.364829+00	-.400481+00	-.624572+00	-.777828+00	-.724161+00	-.152248+01	-.220245+00	-.247879+01	.1+01	-.295609+01
1.40	-.457890+00	-.277246+00	-.865837+00	-.635009+00	-.118000+01	-.154380+01	-.670390+00	-.291589+01	.1+01	-.365696+01
1.50	-.516623+00	-.126215+00	-.109565+01	-.408856+00	-.174051+01	-.146840+01	-.130809+01	-.342593+01	.1+01	-.458473+01
1.60	-.529455+00	.439353-01	-.128600+01	-.863300-01	-.241452+01	-.123975+01	-.221880+01	-.401186+01	.1+01	-.583773+01
1.70	-.486589+00	.219059+00	-.139708+01	.338770+00	-.319777+01	-.777391+00	-.352885+01	-.466405+01	.1+01	-.756418+01
1.80	-.382619+00	.379468+00	-.137659+01	.859011+00	-.405945+01	.291660-01	-.542509+01	-.534607+01	.1+01	-.999112+01
1.90	-.219614+00	.501081+00	-.116196+01	.144402+01	-.492009+01	.132454+01	-.818351+01	-.596797+01	.1+01	-.134718+02
2.00	-.101646-01	.558267+00	-.687539+00	.202914+01	-.561710+01	.328529+01	-.122098+02	-.633691+01	.1+01	-.185648+02
2.10	.220530+00	.528666+00	.100155+00	.250402+01	-.585370+01	.610278+01	-.180944+02	-.606686+01	.1+01	-.261677+02
2.20	.434783+00	.399806+00	.121649+01	.270506+01	-.512891+01	.993582+01	-.266833+02	-.441546+01	.1+01	-.377471+02
2.30	.586112+00	.176582+00	.260024+01	.241951+01	-.265168+01	.148074+02	-.391538+02	.862691-02	.1+01	-.557397+02
2.40	.627560+00	-.112358+00	.406480+01	.141207+01	.274334+01	.204062+02	-.570679+02	.976572+01	.1+01	-.842631+02
2.50	.524677+00	-.412231+00	.524907+01	-.511462+00	.126293+02	.257378+02	-.823162+02	.294995+02	.1+01	-.130396+03
2.60	.271303+00	-.647141+00	.559076+01	-.339050+01	.288616+02	.285642+02	-.116747+03	.675244+02	.1+01	-.206519+03
2.70	-.961516-01	-.735197+00	.436233+01	-.694257+01	.530807+02	.245883+02	-.161012+03	.138532+03	.1+01	-.334671+03
2.80	-.492661+00	-.613886+00	.827238+00	-.103663+02	.854677+02	.644245+01	-.211587+03	.268092+03	.1+01	-.554777+03
2.90	-.794447+00	-.271241+00	-.542122+01	-.121876+02	.122167+03	-.371981+02	-.253771+03	.499853+03	.1+01	-.940470+03
3.00	-.870118+00	.227570+00	-.139065+02	-.102944+02	.150602+03	-.121463+03	-.246068+03	.906353+03	.1+01	-.162999+04
3.50	.128375+01	-.438817+00	.436770+02	.609546+02	-.185185+04	-.696951+03	.126726+05	.110011+05	.1+01	-.352823+05
4.00	-.143233+01	.176970+01	-.286672+03	-.315634+03	.142504+05	.268505+05	.414164+06	-.267047+06	.1+01	-.129696+07
4.50	-.153046+01	-.394580+01	.303475+04	.899044+03	-.581405+05	-.694896+06	-.104472+08	-.211627+08	.1+01	-.801975+08
5.00	.818979+01	-.289861+01	-.236016+05	.194203+05	-.301677+05	.237891+08	-.159723+10	.902680+09	.1+01	-.829827+10
5.50	.141149+02	.136673+02	-.192298+06	-.331209+06	-.279467+09	-.117057+10	.155714+12	.170313+12	.1+01	-.143210+13
6.00	.892289+01	.480481+02	.434927+07	-.444020+07	.640472+11	.633707+11	.196750+14	-.425327+14	.1+01	-.411275+15
6.50	.767743+01	.133290+03	.130111+09	.950077+07	-.992019+13	.967322+12	-.152018+17	.205488+16	.1+01	-.196225+18
7.00	.164446+03	.364872+03	.308919+10	.172358+10	.442269+15	-.156615+16	.619376+19	.519553+19	.1+01	-.155349+21
7.50	.126058+04	.375323+03	.107926+12	.606848+11	.371118+18	.125665+18	-.143772+22	-.670033+22	.1+01	-.203882+24
8.00	.231199+04	-.413300+04	.557097+13	.462008+12	-.785527+19	.138783+21	-.257275+25	.897512+25	.1+01	-.443245+27
8.50	-.181429+05	-.441024+04	.231401+15	-.228725+15	-.624892+23	.370071+23	.115166+29	-.168785+29	.1+01	-.159530+31
9.00	.404091+05	.697050+05	-.117844+17	-.213680+17	-.554262+26	-.688648+25	-.488030+32	.526592+32	.1+01	-.950078+34

TABLE 2

EXP(Z**2)*ERFC(Z)

RHO\THETA	0°	15°	30°	37.5°	45°
.00	.100000+01 .0	.100000+01 .000000	.100000+01 .000000	.100000+01 .000000	.100000+01 .000000
.02	.977826+00 .0	.978544+00 -.564511-02	.980656+00 -.109433-01	.982202+00 -.133574-01	.984046+00 -.155619-01
.04	.956418+00 .0	.957755+00 -.109148-01	.961711+00 -.212290-01	.964623+00 -.259749-01	.968117+00 -.303494-01
.06	.935741+00 .0	.937610+00 -.158323-01	.943165+00 -.308907-01	.947276+00 -.378843-01	.952236+00 -.443878-01
.08	.915764+00 .0	.918086+00 -.204192-01	.925014+00 -.399605-01	.930171+00 -.491167-01	.936422+00 -.577025-01
.10	.896457+00 .0	.899159+00 -.246960-01	.907257+00 -.484692-01	.913315+00 -.597018-01	.920696+00 -.703184-01
.12	.877791+00 .0	.880810+00 -.286817-01	.889890+00 -.564458-01	.896717+00 -.696688-01	.905075+00 -.822605-01
.14	.859740+00 .0	.863016+00 -.323942-01	.872908+00 -.639183-01	.880383+00 -.790455-01	.889575+00 -.935532-01
.16	.842277+00 .0	.845758+00 -.358502-01	.856308+00 -.709132-01	.864318+00 -.878590-01	.874212+00 -.104221+00
.18	.825378+00 .0	.829017+00 -.390656-01	.840085+00 -.774557-01	.848526+00 -.961353-01	.858998+00 -.114286+00
.20	.809020+00 .0	.812775+00 -.420551-01	.824233+00 -.835698-01	.833010+00 -.103900+00	.843946+00 -.123774+00
.30	.734559+00 .0	.738441+00 -.540691-01	.750340+00 -.108476+00	.759640+00 -.135855+00	.771453+00 -.163319+00
.40	.670788+00 .0	.674177+00 -.621855-01	.684812+00 -.125660+00	.693269+00 -.158259+00	.704192+00 -.191604+00
.50	.615690+00 .0	.618355+00 -.674592-01	.626815+00 -.137032+00	.633647+00 -.173298+00	.642609+00 -.210932+00
.60	.567805+00 .0	.569635+00 -.706620-01	.575515+00 -.144060+00	.580342+00 -.182718+00	.586778+00 -.223250+00
.70	.525930+00 .0	.526911+00 -.723596-01	.530125+00 -.147863+00	.532829+00 -.187900+00	.536524+00 -.230166+00
.80	.489101+00 .0	.489272+00 -.729681-01	.489917+00 -.149296+00	.490547+00 -.189926+00	.491521+00 -.232989+00
.90	.456532+00 .0	.455962+00 -.727939-01	.454239+00 -.149005+00	.452940+00 -.189631+00	.451358+00 -.232766+00
1.00	.427584+00 .0	.426354+00 -.720633-01	.422513+00 -.147478+00	.419480+00 -.187662+00	.415588+00 -.230320+00
1.10	.401730+00 .0	.399923+00 -.709434-01	.394231+00 -.145081+00	.389679+00 -.184510+00	.383760+00 -.226295+00
1.20	.378537+00 .0	.376233+00 -.695576-01	.368549+00 -.142050+00	.363094+00 -.180548+00	.355440+00 -.221186+00
1.30	.357643+00 .0	.354916+00 -.679971-01	.346284+00 -.138710+00	.339332+00 -.176056+00	.330223+00 -.215369+00
1.40	.338744+00 .0	.335662+00 -.663291-01	.325905+00 -.135093+00	.318044+00 -.171244+00	.307738+00 -.209128+00
1.50	.321585+00 .0	.318209+00 -.646029-01	.307525+00 -.131350+00	.298923+00 -.166267+00	.287653+00 -.202672+00
1.60	.305953+00 .0	.302334+00 -.628545-01	.290896+00 -.127564+00	.281701+00 -.161236+00	.269672+00 -.196156+00
1.70	.291663+00 .0	.287848+00 -.611100-01	.275808+00 -.123794+00	.266147+00 -.156235+00	.253535+00 -.189690+00
1.80	.278560+00 .0	.274589+00 -.593879-01	.262075+00 -.120081+00	.252058+00 -.151319+00	.239013+00 -.183351+00
1.90	.266509+00 .0	.262415+00 -.577015-01	.249540+00 -.116455+00	.239260+00 -.146528+00	.225908+00 -.177191+00
2.00	.255396+00 .0	.251208+00 -.560596-01	.238065+00 -.112935+00	.227600+00 -.141889+00	.214048+00 -.171246+00
2.10	.245119+00 .0	.240863+00 -.544681-01	.227532+00 -.109534+00	.216948+00 -.137417+00	.203282+00 -.165535+00
2.20	.235593+00 .0	.231289+00 -.529306-01	.217837+00 -.106258+00	.207188+00 -.133122+00	.193480+00 -.160068+00
2.30	.226742+00 .0	.222408+00 -.514490-01	.208891+00 -.103112+00	.198222+00 -.129008+00	.184530+00 -.154849+00
2.40	.218499+00 .0	.214150+00 -.500239-01	.200615+00 -.100095+00	.189963+00 -.125073+00	.176334+00 -.149876+00
2.50	.210806+00 .0	.206456+00 -.486549-01	.192941+00 -.972068-01	.182336+00 -.121315+00	.168808+00 -.145143+00
2.60	.203613+00 .0	.199271+00 -.473413-01	.185809+00 -.944437-01	.175275+00 -.117730+00	.161878+00 -.140642+00
2.70	.196874+00 .0	.192549+00 -.460815-01	.179166+00 -.918020-01	.168723+00 -.114311+00	.155479+00 -.136364+00
2.80	.190549+00 .0	.186248+00 -.448740-01	.172967+00 -.892774-01	.162630+00 -.111051+00	.149556+00 -.132298+00
2.90	.184602+00 .0	.180331+00 -.437168-01	.167168+00 -.868650-01	.156950+00 -.107943+00	.144061+00 -.128434+00
3.00	.179001+00 .0	.174766+00 -.426079-01	.161735+00 -.845599-01	.151645+00 -.104980+00	.138950+00 -.124761+00
3.50	.155294+00 .0	.151281+00 -.377161-01	.139036+00 -.744669-01	.129658+00 -.920841-01	.117989+00 -.108897+00
4.00	.136999+00 .0	.133241+00 -.337346-01	.121846+00 -.663430-01	.113196+00 -.817946-01	.102526+00 -.963790-01
4.50	.122485+00 .0	.118978+00 -.304579-01	.108404+00 -.597166-01	.100432+00 -.734593-01	.906646-01 -.863253-01
5.00	.110705+00 .0	.107435+00 -.277278-01	.976147-01 -.542350-01	.902530-01 -.666012-01	.812804-01 -.781076-01
5.50	.100962+00 .0	.979088-01 -.254257-01	.887699-01 -.496392-01	.819492-01 -.608758-01	.736702-01 -.712820-01
6.00	.927766-01 .0	.899189-01 -.234628-01	.813899-01 -.457386-01	.750469-01 -.560327-01	.673730-01 -.655313-01
6.50	.858057-01 .0	.831244-01 -.217720-01	.751399-01 -.423911-01	.692192-01 -.518878-01	.620749-01 -.606252-01
7.00	.798001-01 .0	.772774-01 -.203020-01	.697797-01 -.394900-01	.642331-01 -.483034-01	.575547-01 -.563934-01
7.50	.745737-01 .0	.721940-01 -.190134-01	.651322-01 -.369532-01	.599186-01 -.451749-01	.536520-01 -.527075-01
8.00	.699852-01 .0	.677345-01 -.178753-01	.610646-01 -.347176-01	.561483-01 -.424219-01	.502479-01 -.494694-01
8.50	.659251-01 .0	.637913-01 -.168633-01	.574747-01 -.327332-01	.528254-01 -.399814-01	.472522-01 -.466031-01
9.00	.623077-01 .0	.602800-01 -.159579-01	.542832-01 -.309606-01	.498745-01 -.378036-01	.445953-01 -.440484-01

EXP(Z**2)*ERFC(Z)

RHO\THETA	50°		60°		70°		80°		90°	
.00	.100000+01	.000000	.100000+01	.000000	.100000+01	.000000	.100000+01	.000000	.100000+01	.000000
.02	.985430+00	-.168969-01	.988522+00	-.191978-01	.991980+00	-.209465-01	.995708+00	-.220828-01	.999600+00	-.225616-01
.04	.970750+00	-.330243-01	.976680+00	-.377036-01	.983379+00	-.413619-01	.990684+00	-.438614-01	.998401+00	-.450871-01
.06	.955991+00	-.484013-01	.964508+00	-.555200-01	.974228+00	-.612308-01	.984947+00	-.653065-01	.996406+00	-.675405-01
.08	.941179+00	-.630470-01	.952039+00	-.726507-01	.964559+00	-.805399-01	.978518+00	-.863902-01	.993620+00	-.898862-01
.10	.926338+00	-.769813-01	.939307+00	-.891010-01	.954404+00	-.992775-01	.971421+00	-.107086+00	.990050+00	-.112089+00
.12	.911494+00	-.902242-01	.926342+00	-.104877+00	.943794+00	-.117434+00	.963679+00	-.127369+00	.985703+00	-.134113+00
.14	.896667+00	-.102796+00	.913175+00	-.119987+00	.932761+00	-.135001+00	.955317+00	-.147215+00	.980591+00	-.155925+00
.16	.881878+00	-.114717+00	.899834+00	-.134439+00	.921336+00	-.151972+00	.946360+00	-.166602+00	.974725+00	-.177491+00
.18	.867146+00	-.126007+00	.886348+00	-.148243+00	.909551+00	-.168342+00	.936834+00	-.185511+00	.968119+00	-.198777+00
.20	.852490+00	-.136687+00	.872742+00	-.161410+00	.897434+00	-.184108+00	.926768+00	-.203921+00	.960789+00	-.219753+00
.30	.780858+00	-.181638+00	.803774+00	-.218107+00	.832921+00	-.253864+00	.869300+00	-.287964+00	.913931+00	-.318916+00
.40	.713036+00	-.214294+00	.735131+00	-.260834+00	.764351+00	-.308806+00	.802581+00	-.357672+00	.852144+00	-.406153+00
.50	.649979+00	-.236926+00	.668830+00	-.291462+00	.694712+00	-.349876+00	.730181+00	-.412479+00	.778801+00	-.478925+00
.60	.592156+00	-.251548+00	.606260+00	-.311920+00	.626410+00	-.378484+00	.655426+00	-.452678+00	.697676+00	-.535713+00
.70	.539686+00	-.259892+00	.548281+00	-.324060+00	.561255+00	-.396308+00	.581214+00	-.479263+00	.612626+00	-.576042+00
.80	.492447+00	-.263412+00	.495329+00	-.329575+00	.500496+00	-.405125+00	.509895+00	-.493727+00	.527292+00	-.600412+00
.90	.450161+00	-.263301+00	.447520+00	-.329960+00	.444890+00	-.406682+00	.443222+00	-.497872+00	.444858+00	-.610142+00
1.00	.412460+00	-.260519+00	.404741+00	-.326488+00	.394786+00	-.402602+00	.382368+00	-.493623+00	.367879+00	-.607158+00
1.10	.378934+00	-.255830+00	.366725+00	-.320215+00	.350217+00	-.394324+00	.327976+00	-.482883+00	.298197+00	-.593761+00
1.20	.349161+00	-.249826+00	.333111+00	-.311994+00	.310985+00	-.383080+00	.280243+00	-.467415+00	.236928+00	-.572397+00
1.30	.322730+00	-.242965+00	.303494+00	-.302503+00	.276740+00	-.369883+00	.239021+00	-.448772+00	.184520+00	-.545456+00
1.40	.299256+00	-.235593+00	.277452+00	-.292262+00	.247038+00	-.355543+00	.203909+00	-.428255+00	.140858+00	-.515113+00
1.50	.278384+00	-.227969+00	.254575+00	-.281666+00	.221396+00	-.340682+00	.174350+00	-.406909+00	.105399+00	-.483227+00
1.60	.259793+00	-.220280+00	.234476+00	-.271004+00	.199324+00	-.325767+00	.149701+00	-.385528+00	.773047-01	-.451284+00
1.70	.243198+00	-.212662+00	.216798+00	-.260483+00	.180352+00	-.311128+00	.129300+00	-.364688+00	.555762-01	-.420388+00
1.80	.228349+00	-.205210+00	.201224+00	-.250247+00	.164045+00	-.296992+00	.112501+00	-.344773+00	.391639-01	-.391291+00
1.90	.215025+00	-.197986+00	.187470+00	-.240389+00	.150014+00	-.283502+00	.987098-01	-.326022+00	.270518-01	-.364437+00
2.00	.203034+00	-.191033+00	.175289+00	-.230965+00	.137912+00	-.270739+00	.873971-01	-.308553+00	.183156-01	-.340026+00
2.10	.192211+00	-.184372+00	.164467+00	-.222006+00	.127441+00	-.258737+00	.781048-01	-.292400+00	.121552-01	-.318073+00
2.20	.182411+00	-.178015+00	.154819+00	-.213521+00	.118345+00	-.247499+00	.704459-01	-.277541+00	.790705-02	-.298468+00
2.30	.173510+00	-.171964+00	.146185+00	-.205507+00	.110409+00	-.237007+00	.641001-01	-.263913+00	.504176-02	-.281026+00
2.40	.165400+00	-.166214+00	.138430+00	-.197951+00	.103450+00	-.227227+00	.588065-01	-.251431+00	.315111-02	-.265522+00
2.50	.157988+00	-.160758+00	.131439+00	-.190836+00	.973162-01	-.218118+00	.543547-01	-.240002+00	.193045-02	-.251723+00
2.60	.151195+00	-.155584+00	.125111+00	-.184138+00	.918812-01	-.209636+00	.505775-01	-.229528+00	.115923-02	-.239403+00
2.70	.144950+00	-.150680+00	.119364+00	-.177833+00	.870395-01	-.201733+00	.473424-01	-.219915+00	.682328-03	-.228355+00
2.80	.139193+00	-.146031+00	.114124+00	-.171899+00	.827037-01	-.194364+00	.445450-01	-.211075+00	.393669-03	-.218399+00
2.90	.133871+00	-.141624+00	.109330+00	-.166309+00	.788013-01	-.187486+00	.421033-01	-.202926+00	.222630-03	-.209377+00
3.00	.128940+00	-.137445+00	.104930+00	-.161041+00	.752717-01	-.181058+00	.399530-01	-.195396+00	.123410-03	-.201157+00
3.50	.108892+00	-.119509+00	.874505-01	-.138780+00	.617088-01	-.154443+00	.321214-01	-.165001+00	.478512-05	-.168830+00
4.00	.942797-01	-.105481+00	.750909-01	-.121741+00	.525024-01	-.134607+00	.270928-01	-.143006+00	.112535-06	-.145954+00
4.50	.831655-01	-.942896-01	.658787-01	-.108356+00	.458048-01	-.119295+00	.235275-01	-.126308+00	.160523-08	-.128735+00
5.00	.744259-01	-.851875-01	.587346-01	-.975919-01	.406890-01	-.107125+00	.208412-01	-.113170+00	.138879-10	-.115246+00
5.50	.673700-01	-.776560-01	.530232-01	-.887581-01	.366399-01	-.972205-01	.187325-01	-.102546+00	.728772-13	-.104367+00
6.00	.615511-01	-.713293-01	.483467-01	-.813834-01	.333477-01	-.890010-01	.170271-01	-.937703-01	.231952-15	-.953962-01
6.50	.566679-01	-.659441-01	.444434-01	-.751362-01	.306137-01	-.820692-01	.156161-01	-.863935-01	.447773-18	-.878644-01
7.00	.525101-01	-.613075-01	.411335-01	-.697775-01	.283042-01	-.761436-01	.144276-01	-.801029-01	.524289-21	-.814475-01
7.50	.489261-01	-.572750-01	.382898-01	-.651309-01	.263257-01	-.710192-01	.134115-01	-.746733-01	.372336-24	-.759126-01
8.00	.458040-01	-.537368-01	.358190-01	-.610637-01	.246107-01	-.665434-01	.125322-01	-.699379-01	.160381-27	-.710891-01
8.50	.430594-01	-.506078-01	.336516-01	-.574741-01	.231091-01	-.626001-01	.117632-01	-.657709-01	.419009-31	-.668445-01
9.00	.406273-01	-.478214-01	.317343-01	-.542828-01	.217828-01	-.590993-01	.110848-01	-.620752-01	.663968-35	-.630821-01

TABLE 3

EXP(Z**2)*ERFC(-Z)

RHO\THETA	0°	15°		30°		37.5°		45°	
.00	.100000+01 .0	.100000+01	.000000	.100000+01	.000000	.100000+01	.000000	.100000+01	.000000
.02	.102297+01 .0	.102215+01	.604525-02	.101974+01	.116363-01	.101801+01	.141302-01	.101595+01	.163619-01
.04	.104678+01 .0	.104502+01	.125170-01	.103989+01	.240025-01	.103620+01	.290671-01	.103188+01	.335494-01
.06	.107147+01 .0	.106863+01	.194435-01	.106043+01	.371373-01	.105458+01	.448455-01	.104775+01	.515878-01
.08	.109708+01 .0	.109302+01	.268548-01	.108137+01	.510811-01	.107311+01	.615C09-01	.106354+01	.705024-01
.10	.112364+01 .0	.111821+01	.347829-01	.110269+01	.658763-01	.109177+01	.790701-01	.107920+01	.903181-01
.12	.115122+01 .0	.114424+01	.432622-01	.112441+01	.815669-01	.111056+01	.975904-01	.109472+01	.111059+00
.14	.117985+01 .0	.117112+01	.523294-01	.114650+01	.981992-01	.112943+01	.117100+00	.111004+01	.132751+00
.16	.120958+01 .0	.119891+01	.620234-01	.116896+01	.115821+00	.114836+01	.137638+00	.112513+01	.155415+00
.18	.124048+01 .0	.122763+01	.723862-01	.119178+01	.134483+00	.116733+01	.159244+00	.113995+01	.179075+00
.20	.127260+01 .0	.125731+01	.834622-01	.121495+01	.154237+00	.118629+01	.181958+00	.115445+01	.203753+00
.30	.145375+01 .0	.142149+01	.151332+00	.133536+01	.271371+00	.127976+01	.313595+00	.122045+01	.343076+00
.40	.167623+01 .0	.161572+01	.245769+00	.146100+01	.424910+00	.136645+01	.479143+00	.127026+01	.510240+00
.50	.195236+01 .0	.184573+01	.377084+00	.158657+01	.623876+00	.143813+01	.683551+00	.129522+01	.705740+00
.60	.229885+01 .0	.211791+01	.559713+00	.170349+01	.878536+00	.148357+01	.930808+00	.128502+01	.927798+00
.70	.273870+01 .0	.243899+01	.813903+00	.179848+01	.119994+01	.148800+01	.122283+01	.122814+01	.117142+01
.80	.330386+01 .0	.281529+01	.116807+01	.185201+01	.159889+01	.143293+01	.155787+01	.111267+01	.142738+01
.90	.403928+01 .0	.325121+01	.166206+01	.183636+01	.208415+01	.129634+01	.192846+01	.927639+00	.168134+01
1.00	.500898+01 .0	.374645+01	.235168+01	.171377+01	.265934+01	.105379+01	.231879+01	.665017+00	.191326+01
1.10	.630524+01 .0	.429102+01	.331474+01	.143482+01	.331817+01	.680682+00	.270192+01	.322278+00	.209753+01
1.20	.806285+01 .0	.485657+01	.465908+01	.938062+00	.403754+01	.156290+00	.303701+01	-.945930-01	.220410+01
1.30	.104813+02 .0	.538160+01	.653258+01	.151941+00	.476793+01	-.530066+00	.326754+01	-.568066+00	.220118+01
1.40	.138599+02 .0	.574679+01	.913501+01	-.998816+00	.542135+01	-.137052+01	.332164+01	-.106664+01	.205955+01
1.50	.186539+02 .0	.573426+01	.127298+02	-.257974+01	.585743+01	-.232809+01	.311621+01	-.154400+01	.175882+01
1.60	.255657+02 .0	.496175+01	.176520+02	-.462254+01	.587039+01	-.332543+01	.256681+01	-.194085+01	.129487+01
1.70	.356950+02 .0	.277771+01	.243017+02	-.708673+01	.518191+01	-.423536+01	.160541+01	-.219057+01	.687583+00
1.80	.507889+02 .0	-.190182+01	.331040+02	-.980423+01	.344908+01	-.487786+01	.206798+00	-.222934+01	-.131466-01
1.90	.736656+02 .0	-.108404+02	.443947+02	-.124081+02	.301777+00	-.502963+01	-.157716+01	-.201049+01	-.725740+00
2.00	.108941+03 .0	-.268411+02	.581561+02	-.142543+02	-.457055+01	-.445375+01	-.358054+01	-.152134+01	-.134236+01
2.10	.164294+03 .0	-.542404+02	.734660+02	-.143607+02	-.112629+02	-.295578+01	-.549415+01	-.798885+00	-.174372+01
2.20	.252703+03 .0	-.995202+02	.874162+02	-.114096+02	-.194032+02	-.468263+00	-.686146+01	.610500-01	-.182367+01
2.30	.396460+03 .0	-.171913+03	.930880+02	-.389141+01	-.278220+02	.284504+01	-.712231+01	.907518+00	-.152069+01
2.40	.634478+03 .0	-.283609+03	.759226+02	.950566+01	-.341808+02	.649023+01	-.572745+01	.155613+01	-.849408+00
2.50	.103581+04 .0	-.448599+03	.748935+01	.291411+02	-.347103+02	.959591+01	-.233541+01	.183009+01	.787847-01
2.60	.172508+04 .0	-.677955+03	-.164667+03	.532363+02	-.243320+02	.109818+02	.292473+01	.161505+01	.105855+01
2.70	.293094+04 .0	-.967019+03	-.532419+03	.763516+02	.239913+01	.941085+01	.919036+01	.913632+00	.182663+01
2.80	.508022+04 .0	-.126560+04	-.124780+04	.879744+02	.489857+02	.405857+01	.147289+02	-.121594+00	.213210+01
2.90	.898334+04 .0	-.141486+04	-.254465+04	.722456+02	.112885+03	-.485116+01	.171054+02	-.119568+01	.182716+01
3.00	.162060+05 .0	-.102319+04	-.474402+04	.105895+02	.179798+03	-.154399+02	.138266+02	-.196121+01	.948998+00
3.50	.417962+06 .0	.759689+05	-.127565+05	-.345002+03	-.846681+03	.352504+02	-.318118+02	.178275+01	-.513342+00
4.00	.177722+08 .0	-.303148+06	.206132+07	.165184+04	.572854+04	-.121850+03	.315751+02	-.201785+01	-.479428+00
4.50	.124593+10 .0	-.632047+08	-.532606+08	.127488+05	-.482630+05	.286262+03	.246430+03	.248394+00	.205738+01
5.00	.144010+12 .0	.504457+10	-.335304+09	-.505861+06	.179234+06	.714413+03	-.107588+04	.190113+01	-.186596+00
5.50	.274434+14 .0	-.398070+12	.262492+12	.359254+07	.647923+07	-.294447+04	-.407345+04	.714128+00	-.176703+01
6.00	.862246+16 .0	.457872+14	-.520744+14	.127587+09	-.310891+08	-.217456+05	-.476709+04	-.323300+00	-.191803+01
6.50	.446655+19 .0	-.100716+17	.118452+17	.133025+10	-.267648+10	-.112172+06	.340422+04	-.383674+00	-.191335+01
7.00	.381469+22 .0	.433489+19	-.317887+19	.207375+10	-.873217+11	-.630212+06	-.132005+06	.543630+00	-.185111+01
7.50	.537149+25 .0	-.283374+22	.426347+21	.630673+11	-.327704+13	-.252634+07	-.336114+07	.185781+01	-.535787+00
8.00	.124703+29 .0	.196497+25	.129886+25	.683728+14	-.142358+15	.165524+08	-.265076+08	.733467+00	.188952+01
8.50	.477316+32 .0	-.989764+25	-.298524+28	.943832+16	-.252614+16	.206701+09	.164806+09	-.204721+01	.598650-01
9.00	.301219+36 .0	-.549838+31	.194934+31	.397576+18	.666613+18	-.243169+10	.751742+09	.150878+01	-.121573+01

EXP(Z**2)*ERFC(-Z)

RHO\THETA	50°		60°		70°		80°		90°	
.00	.100000+01	.000000	.100000+01	.000000	.100000+01	.000000	.100000+01	.000000	.100000+01	.000000
.02	.101443+01	.176847-01	.101108+01	.198904-01	.100741+01	.214606-01	.100354+01	.223563-01	.999600+00	.225616-01
.04	.102869+01	.361748-01	.102172+01	.404727-01	.101417+01	.434163-01	.100631+01	.449542-01	.998401+00	.450871-01
.06	.104275+01	.554874-01	.103189+01	.617442-01	.102026+01	.658462-01	.100830+01	.677607-01	.996406+00	.675405-01
.08	.105656+01	.756385-01	.104154+01	.837004-01	.102564+01	.887273-01	.100949+01	.907418-01	.993620+00	.898862-01
.10	.107009+01	.966430-01	.105064+01	.106335+00	.103029+01	.112035+00	.100986+01	.113862+00	.990050+00	.112089+00
.12	.108331+01	.118515+00	.105916+01	.129639+00	.103418+01	.135743+00	.100942+01	.137086+00	.985703+00	.134113+00
.14	.109617+01	.141267+00	.106704+01	.153603+00	.103728+01	.159822+00	.100814+01	.160377+00	.980591+00	.155925+00
.16	.110862+01	.164910+00	.107424+01	.178212+00	.103956+01	.184242+00	.100603+01	.183697+00	.974725+00	.177491+00
.18	.112062+01	.189453+00	.108074+01	.203453+00	.104100+01	.208971+00	.100307+01	.207609+00	.968119+00	.198777+00
.20	.113213+01	.214906+00	.108648+01	.229307+00	.104157+01	.233974+00	.999272+00	.230273+00	.960789+00	.219753+00
.30	.118040+01	.355926+00	.110242+01	.366982+00	.103071+01	.361797+00	.967640+00	.344526+00	.913931+00	.318916+00
.40	.120806+01	.519531+00	.109341+01	.515837+00	.995593+00	.490450+00	.915655+00	.451794+00	.852144+00	.406153+00
.50	.120731+01	.703662+00	.105496+01	.670616+00	.935432+00	.614113+00	.845306+00	.547520+00	.778801+00	.478925+00
.60	.116980+01	.903774+00	.983747+00	.824347+00	.851095+00	.726620+00	.759756+00	.627812+00	.697676+00	.535713+00
.70	.108743+01	.111228+01	.878285+00	.968587+00	.745231+00	.821976+00	.663105+00	.689772+00	.612626+00	.576042+00
.80	.953336+00	.131817+01	.739549+00	.109394+01	.622230+00	.894945+00	.560035+00	.731740+00	.527292+00	.600412+00
.90	.763292+00	.150697+01	.571471+00	.119083+01	.487972+00	.941619+00	.455412+00	.753397+00	.444858+00	.610142+00
1.00	.517275+00	.166123+01	.381152+00	.125055+01	.349369+00	.959889+00	.353863+00	.755730+00	.367879+00	.607158+00
1.10	.221086+00	.176167+01	.178694+00	.126642+01	.213743+00	.949748+00	.259408+00	.740868+00	.298197+00	.593761+00
1.20	-.112292+00	.178923+01	-.234440-01	.123493+01	.881139-01	.913353+00	.175175+00	.711800+00	.236928+00	.572397+00
1.30	-.462011+00	.172780+01	-.211562+00	.115668+01	-.215044-01	.854822+00	.103231+00	.672034+00	.184520+00	.545456+00
1.40	-.799794+00	.156770+01	-.372237+00	.103688+01	-.110704+00	.779790+00	.445420-01	.625237+00	.140858+00	.515113+00
1.50	-.109192+01	.130926+01	-.494065+00	.885190+00	-.177075+00	.694764+00	-.945684-03	.574901+00	.105399+00	.483227+00
1.60	-.130302+01	.965812+00	-.569332+00	.714952+00	-.220337+00	.606395+00	-.341317-01	.524075+00	.773047-01	.451284+00
1.70	-.140154+01	.565275+00	-.595323+00	.541594+00	-.242190+00	.520751+00	-.565267-01	.475193+00	.555762-01	.420388+00
1.80	-.136640+01	.149150+00	-.574932+00	.380624+00	-.245914+00	.442725+00	-.699973-01	.429992+00	.391639-01	.391291+00
1.90	-.119347+01	-.231426+00	-.516381+00	.245402+00	-.235801+00	.375650+00	-.765256-01	.389521+00	.270518-01	.364437+00
2.00	-.900424+00	-.523642+00	-.432005+00	.145177+00	-.216510+00	.321167+00	-.780102-01	.354223+00	.183156-01	.340026+00
2.10	-.527958+00	-.682837+00	-.336258+00	.837718-01	-.192467+00	.279344+00	-.761241-01	.324056+00	.121552-01	.318073+00
2.20	-.135761+00	-.683750+00	-.243313+00	.592582-01	-.167392+00	.248596+00	-.722347-01	.298640+00	.790705-02	.298468+00
2.30	.207215+00	-.529536+00	-.164751+00	.647151-01	-.144014+00	.228112+00	-.673774-01	.277393+00	.504176-02	.281026+00
2.40	.437243+00	-.255605+00	-.107845+00	.899283-01	-.123586+00	.214327+00	-.622737-01	.259650+00	.315111-02	.265522+00
2.50	.512073+00	.744257-01	-.748106-01	.123641+00	-.107586+00	.205321+00	-.573770-01	.244750+00	.193045-02	.251723+00
2.60	.424377+00	.381557+00	-.631827-01	.155822+00	-.959278-01	.199114+00	-.529309-01	.232099+00	.115923-02	.239403+00
2.70	.207358+00	.591072+00	-.671444-01	.179408+00	-.872383-01	.194224+00	-.490308-01	.221154+00	.682328-03	.228355+00
2.80	-.711724-01	.654101+00	-.794229-01	.191148+00	-.811203-01	.189696+00	-.456768-01	.211636+00	.393669-03	.218399+00
2.90	-.326684+00	.563991+00	-.932090-01	.191421+00	-.767654-01	.185036+00	-.428169-01	.203120+00	.222630-03	.209377+00
3.00	-.483674+00	.360592+00	-.103604+00	.183219+00	-.734511-01	.180089+00	-.403769-01	.195423+00	.123410-03	.201157+00
3.50	.999905-01	.472332-02	-.891007-01	.134728+00	-.617122-01	.154611+00	-.321314-01	.164984+00	.478512-05	.168830+00
4.00	-.218410+00	.993987-01	-.749050-01	.122386+00	-.525086-01	.134600+00	-.270923-01	.143006+00	.112535-06	.145954+00
4.50	-.558352-01	.147044+00	-.658583-01	.108279+00	-.458045-01	.119295+00	-.235275-01	.126308+00	.160523-08	.128735+00
5.00	-.517307-01	.724167-01	-.587417-01	.975944-01	-.406890-01	.107125+00	-.208412-01	.113170+00	.138879-10	.115246+00
5.50	-.679421-01	.672066-01	-.530230-01	.887586-01	-.366399-01	.972205-01	-.187325-01	.102546+00	.728772-13	.104367+00
6.00	-.639612-01	.683196-01	-.483467-01	.813834-01	-.333477-01	.890010-01	-.170271-01	.937703-01	.231952-15	.953962-01
6.50	-.576053-01	.650398-01	-.444434-01	.751362-01	-.306137-01	.820692-01	-.156161-01	.863935-01	.447773-18	.878644-01
7.00	-.526816-01	.609424-01	-.411335-01	.697775-01	-.283042-01	.761436-01	-.144276-01	.801029-01	.524289-21	.814475-01
7.50	-.488796-01	.571703-01	-.382898-01	.651309-01	-.263257-01	.710192-01	-.134115-01	.746733-01	.372336-24	.759126-01
8.00	-.457747-01	.537426-01	-.358190-01	.610637-01	-.246107-01	.665434-01	-.125322-01	.699379-01	.160381-27	.710881-01
8.50	-.430626-01	.506142-01	-.336516-01	.574741-01	-.231091-01	.626001-01	-.117632-01	.657709-01	.419009-31	.668445-01
9.00	-.406279-01	.478200-01	-.317343-01	.542828-01	-.217828-01	.590993-01	-.110848-01	.620752-01	.663968-35	.630821-01