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The 1986 CODATA Recommended Values Of the Fundamental Physical Constants

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This paper gives the values of the basic constants and conversion factors of physics and chemistry resulting from the 1986 least-squares adjustment of the fundamental physical constants as recently published by the CODATA Task Group on Fundamental Constants and as recommended for international use by CODATA. The new, 1986 CODATA set of recommended values replaces its predecessor published by the Task Group and recommended for international use by CODATA in 1973.

Key words: CODATA; conversion factors; fundamental physical constants; least-squares adjustments; recommended values; Task Group on Fundamental Constants.

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CODATA (Committee on Data for Science and Technology[1]) has recently published a report of the CODATA Task Group on Fundamental Constants prepared by the authors [1][2] under the auspices and guidance of the Task Group. The report summarizes the 1986 least-squares adjustment of the fundamental physical constants and gives a set of self-consistent values for the basic constants and conversion factors of physics and chemistry derived from that adjustment. Recommended for international use by CODATA, this 1986 set of values is reprinted here for the convenience of the many readers of the *Journal of Research of the National Bureau of Standards* and to assist in its dissemination throughout the scientific and technological communities. The 1986 CODATA set entirely replaces its immediate predecessor, that recommended for international use by CODATA in 1973. This set was based on the 1973 least-squares adjustment of the fundamental physical constants which was also carried out by the authors under the auspices and guidance of the Task Group [2,3].

As in previous least-squares adjustments of the constants [3,4,5], the data for the 1986 adjustment were divided into two groups: auxiliary constants and stochastic input data. Examples of the 1986 auxiliary constants are the speed of light in vacuum $c \equiv 299792458$ m/s; the permittivity of vacuum $\mu_0 \equiv 4\pi \times 10^{-7}$ N/A^2; the Rydberg constant for infinite mass R_∞; and the quantity $E \equiv 483594.0 \times 10^9$ Hz/V which is equal numerically to the value of the Josephson frequency-voltage ratio $2e/h$ (e is the elementary charge and h is the Planck constant) adopted in 1972 by the Consultative Committee on Electricity of the International Committee of

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[1] CODATA was established in 1966 as an interdisciplinary committee of the International Council of Scientific Unions. It seeks to improve the compilation, critical evaluation, storage, and retrieval of data of importance to science and technology. Dr. David R. Lide, chief of the NBS Office of Standard Reference Data, is the current President of CODATA.

[2] Figures in brackets indicate literature references.

Weights and Measures for defining laboratory representations of the volt [6,7]. Quantities in this category are either defined constants such as c, μ_0, and E with no uncertainty, or constants such as R_∞ with assigned uncertainties sufficiently small in comparison with the uncertainties assigned the stochastic input data with which they are associated in the adjustment that they can be taken as exact (i.e., their values are not subject to adjustment in contrast to the stochastic data). In the 1986 adjustment the uncertainty of each auxiliary constant was no greater than 0.02 parts-per-million or ppm.[3] In contrast, the uncertainties assigned the 38 items of stochastic input data considered in the 1986 adjustment were in the range 0.065 to 9.7 ppm. (The 38 items were of 12 distinct types with the number of items of each type ranging from one to six.) Examples of such data are measurements of the proton gyromagnetic ratio γ'_p (uncertainty in the range 0.24 to 5.4 ppm), the molar volume of silicon $M(\mathrm{Si})/\rho(\mathrm{Si})$ (1.15 ppm), and the quantized Hall resistance $R_H=h/e^2$ (0.12 to 0.22 ppm).

Because new results which can influence a least-squares adjustment of the constants are reported continually, it is always difficult to choose an optimal time at which to carry out a new adjustment and to revise the recommended values of the constants. In the present case, all data available up to 1 January 1986 were considered for inclusion, with the recognition that any additional changes to the 1973 recommended values that might result by taking into account more recent data would be much less than the changes resulting from the data available prior to that date.

Each of the 38 items of stochastic data are expressed (using the auxiliary constants as necessary) in terms of five quantities that serve as the "unknowns" or variables of the 1986 adjustment. These are α^{-1}, the inverse fine-structure constant; K_V, a dimensionless quantity relating the SI (International System of Units) volt V to the unit of voltage $V_{76\text{-}BI}$ maintained at the International Bureau of Weights and Measures (BIPM) using a value of the Josephson frequency-voltage ratio equal numerically to E: $V_{76\text{-}BI}=K_V$ V, and thus $2e/h=E/K_V$; K_Ω, a dimensionless quantity relating the SI ohm to the BIPM as-maintained unit of resistance as it existed on 1 January 1985, Ω_{BI85}, based on the mean resistance of a particular group of wire-wound precision resistors: $\Omega_{BI85}=K_\Omega\ \Omega$; d_{220}, the (220) lattice spacing of a perfect crystal of pure silicon at 22.5 °C in vacuum; and μ_μ/μ_p, the ratio of the magnetic moment of the muon to that of the proton. "Best" values in the least-squares sense for these five quantities, with their variances and covariances, are thus the immediate output of the adjustment.

After a thorough analysis using a number of least-squares algorithms, the initial group of 38 items of stochastic input data was reduced to 22 items by deleting those that were either highly inconsistent with the remaining data or had assigned uncertainties so large that they carried negligible weight. The adjusted values of the five unknowns, and hence all the other 1986 recommended values that were subsequently derived from them (with the aid of the auxiliary constants), are therefore based on a least-squares adjustment with 17 degrees of freedom.

The 1986 adjustment represents a major advance over its 1973 counterpart; the uncertainties of the recommended values have been reduced by roughly an order of magnitude due to the enormous advances made throughout the precision measurement-fundamental constants field in the last dozen years. This can be seen from the following comparison of the 1973 and 1986 recommended values for the inverse fine-structure constant α^{-1}, the elementary charge e, the Planck constant h, the electron mass m_e, the Avogadro constant N_A, the proton electron mass ratio m_p/m_e, the Faraday constant F, and the Josephson frequency-voltage ratio $2e/h$:

	Uncertainty of recommended value in ppm		Change in 1973 recommended value in ppm resulting from 1986 adjustment
Quantity	1973	1986	
α^{-1}	0.82	0.045	−0.37
e	2.9	0.30	−7.4
h	5.4	0.60	−15.2
m_e	5.1	0.59	−15.8
N_A	5.1	0.59	+15.2
m_p/m_e	0.38	0.020	+0.64
F	2.8	0.30	+7.8
$2e/h$	2.6	0.30	+7.8

It is also clear from this comparison that unexpectedly large changes have occurred in the 1973 recommended values of a number of these constants (i.e., a change which is large relative to the uncertainty assigned the 1973 value). These changes are a direct consequence of the 7.8 ppm decrease from 1973 to 1986 in the quantity K_V and the high correlation between K_V and the calculated values of e, h, m_e, N_A, and F. Since $2e/h=E/K_V$, the 1986 value of K_V also implies that the value of the Josephson frequency-voltage ratio adopted by the Consultative Committee on Electricity in 1972, which was believed to be consistent with the SI value and which most national standards laborato-

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[3] Throughout, all uncertainties are one standard deviation estimates.

ries adopted to define and maintain their laboratory unit of voltage, is actually 7.8 ppm smaller than the SI value. This unsatisfactory situation should be rectified in the near future [8,9].

The large change in K_V and hence in many other quantities between 1973 and 1986 would have been avoided if two determinations of F which seemed to be discrepant with the remaining data had not been deleted in the 1973 adjustment. In retrospect, the disagreement was comparatively mild. In view of this experience it is important to recognize that there are no similar disagreements in the 1986 adjustment; the measurements which were deleted were so discrepant that they obviously could not be correct, or of such low weight that if retained the adjusted values of the five unknowns would change negligibly. Thus, it is unlikely that any alternate evaluation of the data considered in the 1986 least-squares adjustment could lead to significant changes in the 1986 recommended values. Moreover, the quality of the 1986 data and its redundancy would seem to preclude future changes in the 1986 recommended values relative to their uncertainties comparable to the changes which occurred in the 1973 values.

The 1986 recommended values of the fundamental physical constants are given in five tables. Table 1 is an abbreviated list containing the quantities which should be of greatest interest to most users. Table 2 is a much more complete compilation.

Table 1. Summary of the 1986 recommended values of the fundamental physical constants.

An abbreviated list of the fundamental constants of physics and chemistry based on a least-squares adjustment with 17 degrees of freedom. The digits in parentheses are the one-standard-deviation uncertainty in the last digits of the given value. Since the uncertainties of many of these entries are correlated, the full covariance matrix must be used in evaluating the uncertainties of quantities computed from them.

Quantity	Symbol	Value	Units	Relative Uncertainty (ppm)
speed of light in vacuum	c	299 792 458	$\mathrm{m\,s^{-1}}$	(exact)
permeability of vacuum	μ_0	$4\pi\times10^{-7}$	$\mathrm{N\,A^{-2}}$	
		=12.566 370 614...	$10^{-7}\,\mathrm{N\,A^{-2}}$	(exact)
permittivity of vacuum	ϵ_0	$1/\mu_0 c^2$		
		=8.854 187 817...	$10^{-12}\,\mathrm{F\,m^{-1}}$	(exact)
Newtonian constant of gravitation	G	6.672 59(85)	$10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$	128
Planck constant	h	6.626 0755(40)	$10^{-34}\,\mathrm{J\,s}$	0.60
$h/2\pi$	$\hbar$	1.054 572 66(63)	$10^{-34}\,\mathrm{J\,s}$	0.60
elementary charge	e	1.602 177 33(49)	$10^{-19}\,\mathrm{C}$	0.30
magnetic flux quantum, $h/2e$	Φ_0	2.067 834 61(61)	$10^{-15}\,\mathrm{Wb}$	0.30
electron mass	m_e	9.109 3897(54)	$10^{-31}\,\mathrm{kg}$	0.59
proton mass	m_p	1.672 6231(10)	$10^{-27}\,\mathrm{kg}$	0.59
proton-electron mass ratio	m_p/m_e	1836.152 701(37)		0.020
fine-structure constant, $\mu_0 ce^2/2h$	α	7.297 353 08(33)	10^{-3}	0.045
inverse fine-structure constant	α^{-1}	137.035 9895(61)		0.045
Rydberg constant, $m_e c\alpha^2/2h$	R_∞	10 973 731.534(13)	$\mathrm{m^{-1}}$	0.0012
Avogadro constant	N_A, L	6.022 1367(36)	$10^{23}\,\mathrm{mol^{-1}}$	0.59
Faraday constant, $N_A e$	F	96 485.309(29)	$\mathrm{C\,mol^{-1}}$	0.30
molar gas constant	R	8.314 510(70)	$\mathrm{J\,mol^{-1}\,K^{-1}}$	8.4
Boltzmann constant, R/N_A	k	1.380 658(12)	$10^{-23}\,\mathrm{J\,K^{-1}}$	8.5
Stefan–Boltzmann constant, $(\pi^2/60)k^4/\hbar^3c^2$	σ	5.670 51(19)	$10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$	34
		Non-SI units used with SI		
electron volt, $(e/\mathrm{C})\,\mathrm{J} = \{e\}\,\mathrm{J}$	eV	1.602 177 33(49)	$10^{-19}\,\mathrm{J}$	0.30
(unified) atomic mass unit, $1\ \mathrm{u} = m_u = \frac{1}{12}m(^{12}\mathrm{C})$	u	1.660 5402(10)	$10^{-27}\,\mathrm{kg}$	0.59

Table 2. The 1986 recommended values of the fundamental physical constants.

This list of the fundamental constants of physics and chemistry is based on a least-squares adjustment with 17 degrees of freedom. The digits in parentheses are the one-standard-deviation uncertainty in the last digits of the given value. Since the uncertainties of many of these entries are correlated, the full covariance matrix must be used in evaluating the uncertainties of quantities computed from them.

Quantity	Symbol	Value	Units	Relative Uncertainty (ppm)
		GENERAL CONSTANTS		
		Universal Constants		
speed of light in vacuum	c	299 792 458	$\mathrm{m\,s^{-1}}$	(exact)
permeability of vacuum	μ_o	$4\pi \times 10^{-7}$	$\mathrm{N\,A^{-2}}$	
		=12.566 370 614...	$10^{-7}\,\mathrm{N\,A^{-2}}$	(exact)
permittivity of vacuum	ϵ_o	$1/\mu_o c^2$		
		=8.854 187 817...	$10^{-12}\,\mathrm{F\,m^{-1}}$	(exact)
Newtonian constant of gravitation	G	6.672 59(85)	$10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$	128
Planck constant	h	6.626 0755(40)	$10^{-34}\,\mathrm{J\,s}$	0.60
in electron volts, $h/\{e\}$		4.135 6692(12)	$10^{-15}\,\mathrm{eV\,s}$	0.30
$h/2\pi$	$\hbar$	1.054 572 66(63)	$10^{-34}\,\mathrm{J\,s}$	0.60
in electron volts, $\hbar/\{e\}$		6.582 1220(20)	$10^{-16}\,\mathrm{eV\,s}$	0.30
Planck mass, $(\hbar c/G)^{\frac{1}{2}}$	m_{P}	2.176 71(14)	$10^{-8}\,\mathrm{kg}$	64
Planck length, $\hbar/m_{\mathrm{P}}c = (\hbar G/c^3)^{\frac{1}{2}}$	l_{P}	1.616 05(10)	$10^{-35}\,\mathrm{m}$	64
Planck time, $l_{\mathrm{P}}/c = (\hbar G/c^5)^{\frac{1}{2}}$	t_{P}	5.390 56(34)	$10^{-44}\,\mathrm{s}$	64
		Electromagnetic Constants		
elementary charge	e	1.602 177 33(49)	$10^{-19}\,\mathrm{C}$	0.30
	e/h	2.417 988 36(72)	$10^{14}\,\mathrm{A\,J^{-1}}$	0.30
magnetic flux quantum, $h/2e$	Φ_o	2.067 834 61(61)	$10^{-15}\,\mathrm{Wb}$	0.30
Josephson frequency–voltage ratio	$2e/h$	4.835 9767(14)	$10^{14}\,\mathrm{Hz\,V^{-1}}$	0.30
quantized Hall conductance	e^2/h	3.874 046 14(17)	$10^{-5}\,\mathrm{S}$	0.045
quantized Hall resistance, $h/e^2 = \mu_o c/2\alpha$	R_{H}	25 812.8056(12)	Ω	0.045
Bohr magneton, $e\hbar/2m_e$	μ_{B}	9.274 0154(31)	$10^{-24}\,\mathrm{J\,T^{-1}}$	0.34
in electron volts, $\mu_{\mathrm{B}}/\{e\}$		5.788 382 63(52)	$10^{-5}\,\mathrm{eV\,T^{-1}}$	0.089
in hertz, μ_{B}/h		1.399 624 18(42)	$10^{10}\,\mathrm{Hz\,T^{-1}}$	0.30
in wavenumbers, μ_{B}/hc		46.686 437(14)	$\mathrm{m^{-1}\,T^{-1}}$	0.30
in kelvins, μ_{B}/k		0.671 7099(57)	$\mathrm{K\,T^{-1}}$	8.5
nuclear magneton, $e\hbar/2m_{\mathrm{p}}$	μ_{N}	5.050 7866(17)	$10^{-27}\,\mathrm{J\,T^{-1}}$	0.34
in electron volts, $\mu_{\mathrm{N}}/\{e\}$		3.152 451 66(28)	$10^{-8}\,\mathrm{eV\,T^{-1}}$	0.089
in hertz, μ_{N}/h		7.622 5914(23)	$\mathrm{MHz\,T^{-1}}$	0.30
in wavenumbers, μ_{N}/hc		2.542 622 81(77)	$10^{-2}\,\mathrm{m^{-1}\,T^{-1}}$	0.30
in kelvins, μ_{N}/k		3.658 246(31)	$10^{-4}\,\mathrm{K\,T^{-1}}$	8.5

Table 2. The 1986 recommended values of the fundamental physical constants (continued).

Quantity	Symbol	Value	Units	Relative Uncertainty (ppm)
		ATOMIC CONSTANTS		
fine-structure constant, $\mu_0 ce^2/2h$	α	7.297 353 08(33)	10^{-3}	0.045
inverse fine-structure constant	α^{-1}	137.035 9895(61)		0.045
Rydberg constant, $m_e c\alpha^2/2h$	R_∞	10 973 731.534(13)	m^{-1}	0.0012
in hertz, $R_\infty c$		3.289 841 9499(39)	10^{15} Hz	0.0012
in joules, $R_\infty hc$		2.179 8741(13)	10^{-18} J	0.60
in eV, $R_\infty hc/\{e\}$		13.605 6981(40)	eV	0.30
Bohr radius, $\alpha/4\pi R_\infty$	a_0	0.529 177 249(24)	10^{-10} m	0.045
Hartree energy, $e^2/4\pi\epsilon_0 a_0 = 2R_\infty hc$	E_h	4.359 7482(26)	10^{-18} J	0.60
in eV, $E_\mathrm{h}/\{e\}$		27.211 3961(81)	eV	0.30
quantum of circulation	$h/2m_e$	3.636 948 07(33)	$10^{-4}\ \mathrm{m}^2\,\mathrm{s}^{-1}$	0.089
	h/m_e	7.273 896 14(65)	$10^{-4}\ \mathrm{m}^2\,\mathrm{s}^{-1}$	0.089
		Electron		
electron mass	m_e	9.109 3897(54)	10^{-31} kg	0.59
		5.485 799 03(13)	10^{-4} u	0.023
in electron volts, $m_e c^2/\{e\}$		0.510 999 06(15)	MeV	0.30
electron–muon mass ratio	m_e/m_μ	4.836 332 18(71)	10^{-3}	0.15
electron–proton mass ratio	m_e/m_p	5.446 170 13(11)	10^{-4}	0.020
electron–deuteron mass ratio	m_e/m_d	2.724 437 07(6)	10^{-4}	0.020
electron–α-particle mass ratio	m_e/m_α	1.370 933 54(3)	10^{-4}	0.021
electron specific charge	$-e/m_e$	−1.758 819 62(53)	$10^{11}\ \mathrm{C\,kg}^{-1}$	0.30
electron molar mass	$M(\mathrm{e}), M_e$	5.485 799 03(13)	10^{-7} kg/mol	0.023
Compton wavelength, $h/m_e c$	λ_C	2.426 310 58(22)	10^{-12} m	0.089
$\lambda_\mathrm{C}/2\pi = \alpha a_0 = \alpha^2/4\pi R_\infty$	$\bar{\lambda}_\mathrm{C}$	3.86 159 323(35)	10^{-13} m	0.089
classical electron radius, $\alpha^2 a_0$	r_e	2.817 940 92(38)	10^{-15} m	0.13
Thomson cross section, $(8\pi/3)r_e^2$	σ_e	0.665 246 16(18)	$10^{-28}\ \mathrm{m}^2$	0.27
electron magnetic moment	μ_e	928.477 01(31)	$10^{-26}\ \mathrm{J\,T}^{-1}$	0.34
in Bohr magnetons	μ_e/μ_B	1.001 159 652 193(10)		1×10^{-5}
in nuclear magnetons	μ_e/μ_N	1838.282 000(37)		0.020
electron magnetic moment anomaly, $\mu_e/\mu_\mathrm{B} - 1$	a_e	1.159 652 193(10)	10^{-3}	0.0086
electron g-factor, $2(1+a_e)$	g_e	2.002 319 304 386(20)		1×10^{-5}
electron–muon magnetic moment ratio	μ_e/μ_μ	206.766967(30)		0.15
electron–proton magnetic moment ratio	μ_e/μ_p	658.210 6881(66)		0.010
		Muon		
muon mass	m_μ	1.883 5327(11)	10^{-28} kg	0.61
		0.113 428 913(17)	u	0.15
in electron volts, $m_\mu c^2/\{e\}$		105.658 389(34)	MeV	0.32
muon–electron mass ratio	m_μ/m_e	206.768 262(30)		0.15
muon molar mass	$M(\mu), M_\mu$	1.134 289 13(17)	10^{-4} kg/mol	0.15
muon magnetic moment	μ_μ	4.490 4514(15)	$10^{-26}\ \mathrm{J\,T}^{-1}$	0.33
in Bohr magnetons,	μ_μ/μ_B	4.841 970 97(71)	10^{-3}	0.15
in nuclear magnetons,	μ_μ/μ_N	8.890 5981(13)		0.15

Table 2. The 1986 recommended values of the fundamental physical constants (continued).

Quantity	Symbol	Value	Units	Relative Uncertainty (ppm)
muon magnetic moment anomaly, $[\mu_\mu/(e\hbar/2m_\mu)]-1$	a_μ	1.165 9230(84)	10^{-3}	7.2
muon g-factor, $2(1+a_\mu)$	g_μ	2.002 331 846(17)		0.0084
muon–proton magnetic moment ratio	μ_μ/μ_p	3.183 345 47(47)		0.15
		Proton		
proton mass	m_p	1.672 6231(10)	10^{-27} kg	0.59
		1.007 276 470(12)	u	0.012
in electron volts, $m_\mathrm{p}c^2/\{e\}$		938.272 31(28)	MeV	0.30
proton–electron mass ratio	$m_\mathrm{p}/m_\mathrm{e}$	1836.152 701(37)		0.020
proton–muon mass ratio	m_p/m_μ	8.880 2444(13)		0.15
proton specific charge	e/m_p	9.578 8309(29)	10^{7} C kg^{-1}	0.30
proton molar mass	$M(\mathrm{p}), M_\mathrm{p}$	1.007 276 470(12)	10^{-3} kg/mol	0.012
proton Compton wavelength, $h/m_\mathrm{p}c$	$\lambda_\mathrm{C,p}$	1.321 410 02(12)	10^{-15} m	0.089
$\lambda_\mathrm{C,p}/2\pi$	$\bar{\lambda}_\mathrm{C,p}$	2.103 089 37(19)	10^{-16} m	0.089
proton magnetic moment	μ_p	1.410 607 61(47)	10^{-26} J T^{-1}	0.34
in Bohr magnetons	$\mu_\mathrm{p}/\mu_\mathrm{B}$	1.521 032 202(15)	10^{-3}	0.010
in nuclear magnetons	$\mu_\mathrm{p}/\mu_\mathrm{N}$	2.792 847 386(63)		0.023
diamagnetic shielding correction for protons in pure water, spherical sample, 25 °C, $1-\mu_\mathrm{p}'/\mu_\mathrm{p}$	$\sigma_{\mathrm{H_2O}}$	25.689(15)	10^{-6}	–
shielded proton moment (H_2O, sph., 25 °C)	μ_p'	1.410 571 38(47)	10^{-26} J T^{-1}	0.34
in Bohr magnetons	$\mu_\mathrm{p}'/\mu_\mathrm{B}$	1.520 993 129(17)	10^{-3}	0.011
in nuclear magnetons	$\mu_\mathrm{p}'/\mu_\mathrm{N}$	2.792 775 642(64)		0.023
proton gyromagnetic ratio	γ_p	26 752.2128(81)	10^{4} s^{-1} T^{-1}	0.30
	$\gamma_\mathrm{p}/2\pi$	42.577 469(13)	MHz T^{-1}	0.30
uncorrected (H_2O, sph., 25 °C)	γ_p'	26 751.5255(81)	10^{4} s^{-1} T^{-1}	0.30
	$\gamma_\mathrm{p}'/2\pi$	42.576 375(13)	MHz T^{-1}	0.30
		Neutron		
neutron mass	m_n	1.674 9286(10)	10^{-27} kg	0.59
		1.008 664 904(14)	u	0.014
in electron volts, $m_\mathrm{n}c^2/\{e\}$		939.565 63(28)	Mev	0.30
neutron–electron mass ratio	$m_\mathrm{n}/m_\mathrm{e}$	1838.683 662(40)		0.022
neutron–proton mass ratio	$m_\mathrm{n}/m_\mathrm{p}$	1.001 378 404(9)		0.009
neutron molar mass	$M(\mathrm{n}), M_\mathrm{n}$	1.008 664 904(14)	10^{-3} kg/mol	0.014
neutron Compton wavelength, $h/m_\mathrm{n}c$	$\lambda_\mathrm{C,n}$	1.319 591 10(12)	10^{-15} m	0.089
$\lambda_\mathrm{C,n}/2\pi$	$\bar{\lambda}_\mathrm{C,n}$	2.100 194 45(19)	10^{-16} m	0.089
neutron magnetic moment *	μ_n	0.966 237 07(40)	10^{-26} J T^{-1}	0.41
in Bohr magnetons	$\mu_\mathrm{n}/\mu_\mathrm{B}$	1.041 875 63(25)	10^{-3}	0.24
in nuclear magnetons	$\mu_\mathrm{n}/\mu_\mathrm{N}$	1.913 042 75(45)		0.24
neutron–electron magnetic moment ratio	$\mu_\mathrm{n}/\mu_\mathrm{e}$	1.040 668 82(25)	10^{-3}	0.24
neutron–proton magnetic moment ratio	$\mu_\mathrm{n}/\mu_\mathrm{p}$	0.684 979 34(16)		0.24

Table 2. The 1986 recommended values of the fundamental physical constants (continued).

Quantity	Symbol	Value	Units	Relative Uncertainty (ppm)
Deuteron				
deuteron mass	m_d	3.343 5860(20)	10^{-27} kg	0.59
		2.013 553 214(24)	u	0.012
in electron volts, $m_d c^2/\{e\}$		1875.613 39(57)	MeV	0.30
deuteron–electron mass ratio	m_d/m_e	3670.483 014(75)		0.020
deuteron–proton mass ratio	m_d/m_p	1.999 007 496(6)		0.003
deuteron molar mass	$M(d), M_d$	2.013 553 214(24)	10^{-3} kg/mol	0.012
deuteron magnetic moment *	μ_d	0.433 073 75(15)	10^{-26} J T^{-1}	0.34
in Bohr magnetons,	μ_d/μ_B	0.466 975 4479(91)	10^{-3}	0.019
in nuclear magnetons,	μ_d/μ_N	0.857 438 230(24)		0.028
deuteron–electron magnetic moment ratio	μ_d/μ_e	0.466 434 5460(91)	10^{-3}	0.019
deuteron–proton magnetic moment ratio	μ_d/μ_p	0.307 012 2035(51)		0.017
PHYSICO-CHEMICAL CONSTANTS				
Avogadro constant	N_A, L	6.022 1367(36)	10^{23} mol^{-1}	0.59
atomic mass constant, $\frac{1}{12}m(^{12}C)$	m_u	1.660 5402(10)	10^{-27} kg	0.59
in electron volts, $m_u c^2/\{e\}$		931.494 32(28)	MeV	0.30
Faraday constant	F	96 485.309(29)	C mol^{-1}	0.30
molar Planck constant	$N_A h$	3.990 313 23(36)	10^{-10} J s mol^{-1}	0.089
	$N_A hc$	0.119 626 58(11)	J m mol^{-1}	0.089
molar gas constant	R	8.314 510(70)	J mol^{-1} K^{-1}	8.4
Boltzmann constant, R/N_A	k	1.380 658(12)	10^{-23} J K^{-1}	8.5
in electron volts, $k/\{e\}$		8.617 385(73)	10^{-5} eV K^{-1}	8.4
in hertz, k/h		2.083 674(18)	10^{10} Hz K^{-1}	8.4
in wavenumbers, k/hc		69.503 87(59)	m^{-1} K^{-1}	8.4
molar volume (ideal gas), RT/p				
$T = 273.15\,K,\ p = 101\,325\,Pa$	V_m	22.414 10(19)	L/mol	8.4
Loschmidt constant, N_A/V_m	n_o	2.686 763(23)	10^{25} m^{-3}	8.5
$T = 273.15\,K,\ p = 100\,kPa$	V_m	22.711 08(19)	L/mol	8.4
Sackur-Tetrode constant (absolute entropy constant), ** $\frac{5}{2} + \ln\{(2\pi m_u kT_1/h^2)^{\frac{3}{2}} kT_1/p_o\}$				
$T_1 = 1\ K,\quad p_o = 100\ kPa$	S_o/R	−1.151 693(21)		18
$p_o = 101\,325\ Pa$		−1.164 856(21)		18
Stefan–Boltzmann constant, $(\pi^2/60)k^4/\hbar^3c^2$	σ	5.670 51(19)	10^{-8} W m^{-2} K^{-4}	34
first radiation constant, $2\pi hc^2$	c_1	3.741 7749(22)	10^{-16} W m^2	0.60
second radiation constant, hc/k	c_2	0.014 387 69(12)	m K	8.4
Wien displacement law constant, $b = \lambda_{max}T = c_2/4.965\,114\,23\ldots$	b	2.897 756(24)	10^{-3} m K	8.4

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*The scalar magnitude of the neutron moment is listed here. The neutron magnetic dipole is directed oppositely to that of the proton, and corresponds to the dipole associated with a spinning negative charge distribution. The vector sum, $\mu_d = \mu_p + \mu_n$, is approximately satisfied.

**The entropy of an ideal monatomic gas of relative atomic weight A_r is given by $S = S_o + \frac{3}{2}R \ln A_r - R \ln(p/p_o) + \frac{5}{2}R \ln(T/K)$.

Table 3 is a list of related "maintained units and standard values," while table 4 contains a number of scientifically, technologically, and metrologically useful energy conversion factors. Finally, table 5 is an extended covariance matrix containing the variances, covariances, and correlation coefficients of the unknowns and a number of different constants (included for convenience) from which the like quantities for other constants may be readily calculated.[4] Such a matrix is necessary, of course, because the variables in a least-squares adjustment are statistically correlated. Thus, with the exception of quantities which depend only on aux-

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[4] The variable d_{220} is omitted from table 5 because there is little need for its correlations with other quantities. Moreover, since the more significant and related quantity N_A is included (note that $N_A \sim d_{220}^{-3}$), there is no loss of information by omitting d_{220}.

Table 3. Maintained units and standard values.

A summary of "maintained" units and "standard" values and their relationship to SI units, based on a least-squares adjustment with 17 degrees of freedom. The digits in parentheses are the one-standard-deviation uncertainty in the last digits of the given value. Since the uncertainties of many of these entries are correlated, the full covariance matrix must be used in evaluating the uncertainties of quantities computed from them.

Quantity	Symbol	Value	Units	Relative Uncertainty (ppm)
electron volt, $(e/\mathrm{C})\,\mathrm{J} = \{e\}\,\mathrm{J}$	eV	1.602 177 33(49)	10^{-19} J	0.30
(unified) atomic mass unit,	u	1.660 5402(10)	10^{-27} kg	0.59
$1\ \mathrm{u} = m_\mathrm{u} = \frac{1}{12}m(^{12}\mathrm{C})$				
standard atmosphere	atm	101 325	Pa	(exact)
standard acceleration of gravity	g_n	9.806 65	$\mathrm{m\,s^{-2}}$	(exact)
		'As-Maintained' Electrical Units		
BIPM maintained ohm, $\Omega_{69-\mathrm{BI}}$				
$\Omega_{\mathrm{BI85}} \equiv \Omega_{69-\mathrm{BI}}(1\ \mathrm{Jan}\ 1985)$	Ω_{BI85}	$1 - 1.563(50)\times10^{-6}$	Ω	
		= 0.999 998 437(50)	Ω	0.050
Drift rate of $\Omega_{69-\mathrm{BI}}$	$\dfrac{d\Omega_{69-\mathrm{BI}}}{dt}$	−0.0566(15)	μΩ/a	—
BIPM maintained volt,	$\mathrm{V}_{76-\mathrm{BI}}$	$1 - 7.59(30)\times10^{-6}$	V	
$\mathrm{V}_{76-\mathrm{BI}} \equiv 483\,594\ \mathrm{GHz}(h/2e)$		= 0.999 992 41(30)	V	0.30
BIPM maintained ampere,	$\mathrm{A}_{\mathrm{BI85}}$	$1 - 6.03(30)\times10^{-6}$	A	
$\mathrm{A}_{\mathrm{BIPM}} = \mathrm{V}_{76-\mathrm{BI}}/\Omega_{69-\mathrm{BI}}$		= 0.999 993 97(30)	A	0.30
		X-Ray Standards		
Cu x-unit : $\lambda(\mathrm{CuK}\alpha_1) \equiv 1537.400\,\mathrm{xu}$	$\mathrm{xu(CuK}\alpha_1)$	1.002 077 89(70)	10^{-13} m	0.70
Mo x-unit : $\lambda(\mathrm{MoK}\alpha_1) \equiv 707.831\,\mathrm{xu}$	$\mathrm{xu(MoK}\alpha_1)$	1.002 099 38(45)	10^{-13} m	0.45
Å* : $\lambda(\mathrm{WK}\alpha_1) \equiv 0.209\,100$ Å*	Å*	1.000 014 81(92)	10^{-10} m	0.92
lattice spacing of Si (in vacuum, 22.5 °C), [+]	a	0.543 101 96(11)	nm	0.21
$d_{220} = a/\sqrt{8}$	d_{220}	0.192 015 540(40)	nm	0.21
molar volume of Si, $M(\mathrm{Si})/\rho(\mathrm{Si}) = N_\mathrm{A}a^3/8$	$V_\mathrm{m}(\mathrm{Si})$	12.058 8179(89)	$\mathrm{cm^3/mol}$	0.74

[+] The lattice spacing of single-crystal Si can vary by parts in 10^7 depending on the preparation process. Measurements at PTB indicate also the possibility of distortions from exact cubic symmetry of the order of 0.2 ppm.

iliary constants, the uncertainty associated with a quantity calculated from other constants in general can be found only with the use of the full covariance matrix.

To use table 5, note that the covariance between two quantities Q_k and Q_s which are functions of a common set of variables $x_i (i=1,\ldots,N)$ is given by

$$v_{ks}=\sum_{i,j=1}^{N}\frac{\partial Q_k}{\partial x_i}\frac{\partial Q_s}{\partial x_j}v_{ij} \tag{1}$$

where v_{ij} is the covariance of x_i and x_j. In this general form, the units of v_{ij} are the product of the units of x_i and x_j and the units of v_{ks} are the product of the units of Q_k and Q_s. For most cases of interest

Table 4. Energy conversion factors.

To use this table note that all entries on the same line are equal; the unit at the top of a column applies to all of the values beneath it.

Example: 1 eV=806544.10 m^{-1}

	J	**kg**	**m^{-1}**	**Hz**
1 J =	1	$1/\{c^2\}$ $1.112\,650\,06\times10^{-17}$	$1/\{hc\}$ $5.034\,1125(30)\times10^{24}$	$1/\{h\}$ $1.509\,188\,97(90)\times10^{33}$
1 kg =	$\{c^2\}$ $8.987\,551\,787\times10^{16}$	1	$\{c/h\}$ $4.524\,4347(27)\times10^{41}$	$\{c^2/h\}$ $1.356\,391\,40(81)\times10^{50}$
1 m^{-1} =	$\{hc\}$ $1.986\,4475(12)\times10^{-25}$	$\{h/c\}$ $2.210\,2209(13)\times10^{-42}$	1	$\{c\}$ $299\,792\,458$
1 Hz =	$\{h\}$ $6.626\,0755(40)\times10^{-34}$	$\{h/c^2\}$ $7.372\,5032(44)\times10^{-51}$	$1/\{c\}$ $3.335\,640\,952\times10^{-9}$	1
1 K =	$\{k\}$ $1.380\,658(12)\times10^{-23}$	$\{k/c^2\}$ $1.536\,189(13)\times10^{-40}$	$\{k/hc\}$ $69.503\,87(59)$	$\{k/h\}$ $2.083\,674(18)\times10^{10}$
1 eV =	$\{e\}$ $1.602\,177\,33(49)\times10^{-19}$	$\{e/c^2\}$ $1.782\,662\,70(54)\times10^{-36}$	$\{e/hc\}$ $806\,554.10(24)$	$\{e/h\}$ $2.417\,988\,36(72)\times10^{14}$
1 u =	$\{m_{\mathrm{u}}c^2\}$ $1.492\,419\,09(88)\times10^{-10}$	$\{m_{\mathrm{u}}\}$ $1.660\,5402(10)\times10^{-27}$	$\{m_{\mathrm{u}}c/h\}$ $7.513\,005\,63(67)\times10^{14}$	$\{m_{\mathrm{u}}c^2/h\}$ $2.252\,342\,42(20)\times10^{23}$
1 hartree =	$\{2R_\infty hc\}$ $4.359\,7482(26)\times10^{-18}$	$\{2R_\infty h/c\}$ $4.850\,8741(29)\times10^{-35}$	$\{2R_\infty\}$ $21\,947\,463.067(26)$	$\{2R_\infty c\}$ $6.579\,683\,8999(78)\times10^{15}$

	K	**eV**	**u**	**hartree**
1 J =	$1/\{k\}$ $7.242\,924(61)\times10^{22}$	$1/\{e\}$ $6.241\,5064(19)\times10^{18}$	$1/\{m_{\mathrm{u}}c^2\}$ $6.700\,5308(40)\times10^{9}$	$1/\{2R_\infty hc\}$ $2.293\,7104(14)\times10^{17}$
1 kg =	$\{c^2/k\}$ $6.509\,616(55)\times10^{39}$	$\{c^2/e\}$ $5.609\,5862(17)\times10^{35}$	$1/\{m_{\mathrm{u}}\}$ $6.022\,1367(36)\times10^{26}$	$\{c/2R_\infty h\}$ $2.061\,4841(12)\times10^{34}$
1 m^{-1} =	$\{hc/k\}$ $0.014\,387\,69(12)$	$\{hc/e\}$ $1.239\,842\,44(37)\times10^{-6}$	$\{h/m_{\mathrm{u}}c\}$ $1.331\,025\,22(12)\times10^{-15}$	$1/\{2R_\infty\}$ $4.556\,335\,2672(54)\times10^{-8}$
1 Hz =	$\{h/k\}$ $4.799\,216(41)\times10^{-11}$	$\{h/e\}$ $4.135\,6692(12)\times10^{-15}$	$\{h/m_{\mathrm{u}}c^2\}$ $4.439\,822\,24(40)\times10^{-24}$	$1/\{2R_\infty c\}$ $1.519\,829\,8508(18)\times10^{-16}$
1 K =	1	$\{k/e\}$ $8.617\,385(73)\times10^{-5}$	$\{k/m_{\mathrm{u}}c^2\}$ $9.251\,140(78)\times10^{-14}$	$\{k/2R_\infty hc\}$ $3.166\,829(27)\times10^{-6}$
1 eV =	$\{e/k\}$ $11\,604.45(10)$	1	$\{e/m_{\mathrm{u}}c^2\}$ $1.073\,543\,85(33)\times10^{-9}$	$\{e/2R_\infty hc\}$ $0.036\,749\,309(11)$
1 u =	$\{m_{\mathrm{u}}c^2/k\}$ $1.080\,9478(91)\times10^{13}$	$\{m_{\mathrm{u}}c^2/e\}$ $931.494\,32(28)\times10^{6}$	1	$\{m_{\mathrm{u}}c/2R_\infty h\}$ $3.423\,177\,25(31)\times10^{7}$
1 hartree =	$\{2R_\infty hc/k\}$ $3.157\,733(27)\times10^{5}$	$\{2R_\infty hc/e\}$ $27.211\,3961(81)$	$\{2R_\infty h/m_{\mathrm{u}}c\}$ $2.921\,262\,69(26)\times10^{-8}$	1

Table 5. Expanded covariance and correlation coefficient matrix for the 1986 recommended set of fundamental physical constants.

The elements of the covariance matrix appear on and above the major diagonal in (parts in 10^9)^2; correlation coefficients appear in *italics* below the diagonal. The values are given to as many as six digits only as a matter of consistency.

The correlation coefficient between m_e and N_A appears as -1.000 in this table because the auxiliary constants were considered to be exact in carrying out the least-squares adjustment. When the uncertainties of m_p/m_e and M_p are properly taken into account, the correlation coefficient is -0.999 and the variances of m_e and N_A are slightly increased.

	α^{-1}	K_V	K_Ω	μ_μ/μ_p	e	h	m_e	N_A	F
α^{-1}	1997	−1062	925	3267	−3059	−4121	−127	127	−2932
K_V	*−0.080*	87988	90	−1737	89050	177038	174914	−174914	−85864
K_Ω	*0.416*	*0.006*	2477	1513	−835	−744	1105	−1105	−1939
μ_μ/μ_p	*0.498*	*−0.040*	*0.207*	21523	−5004	−6742	−208	208	−4796
e	*−0.226*	*0.989*	*−0.055*	*−0.112*	92109	181159	175042	−175042	−82933
h	*−0.154*	*0.997*	*−0.025*	*−0.077*	*0.997*	358197	349956	−349956	−168797
m_e	*−0.005*	*0.997*	*0.038*	*−0.002*	*0.975*	*0.989*	349702	−349702	−174660
N_A	*0.005*	*−0.997*	*−0.038*	*0.002*	*−0.975*	*−0.989*	*−1.000*	349702	174660
F	*−0.217*	*−0.956*	*−0.129*	*−0.108*	*−0.902*	*−0.931*	*−0.975*	*0.975*	91727

involving the fundamental constants, the variables x_i may be taken to be the fractional change in the physical quantity from some fiducial value, and the quantities Q can be expressed as powers of physical constants Z_j according to

$$Q_k = q \prod_{j=1}^{N} Z_j^{Y_{kj}}, \qquad (2)$$

where q is a numerical factor. If the variances and covariances are then expressed in relative units, eq (1) becomes

$$v_{ks} = \sum_{i,j=1}^{N} Y_{ki} Y_{sj} v_{ij}, \qquad (3)$$

where the v_{ij} are to be expressed, for example, in (parts in 10^9)^2. Equation (3) is the basis for the expansion of the covariance matrix to include e, h, m_e, N_A, and F.

In terms of correlation coefficients defined by $r_{ij} \equiv v_{ij}(v_{ii}v_{jj})^{-1/2} \equiv v_{ij}/\epsilon_i\epsilon_j$, where ϵ_i is the standard deviation ($\epsilon_i^2 = v_{ii}$), we may write, from eq (3),

$$\epsilon_k^2 = \sum_{i=1}^{N} Y_{ki}^2 \epsilon_i^2 + 2\sum_{j<i}^{N} Y_{ki} Y_{kj} r_{ij} \epsilon_i \epsilon_j, \qquad (4)$$

where the standard deviations are to be expressed in relative units.

As an example of the use of table 5, consider the calculation of the uncertainty of the Bohr magneton $\mu_B = e\hbar/2m_e$ ($\hbar = h/2\pi$). In terms of the variables of the 1986 adjustment this ratio is given by

$$\mu_B = [2\pi\mu_0 R_\infty \, E]^{-1} (\alpha^{-1})^{-3} K_V, \qquad (5)$$

where the quantities in brackets are auxiliary constants taken to be exact. Using eq (3) and letting α^{-1} correspond to $i=1$ and K_V to $i=2$ gives[5]

$$\epsilon_{\mu_B}^2 = Y_1^2 v_{11} + 2Y_1Y_2v_{12} + Y_2^2 v_{22}. \qquad (6)$$

Comparing eq (5) with eq (2) yields $Y_1 = -3$ and $Y_2 = 1$. Thus eq (6) and table 5 lead to

$$\epsilon_{\mu_B}^2 = [9\,(1997) - 6(-1062) + 1(87988)] \times (10^{-9})^2 \qquad (7)$$

or $\epsilon_{\mu_B} = 0.335$ ppm. An alternate approach is to evaluate $e\hbar/2m_e$ directly from table 5; then e corresponds to $i=5$, h to $i=6$, and m_e to $i=7$ with $Y_5 = Y_6 = 1$ and $Y_7 = -1$. Then

$$\epsilon_{\mu_B}^2 = Y_5^2 v_{55} + 2Y_5Y_6v_{56} + Y_6^2 v_{66} + 2Y_5Y_7v_{57} + 2Y_6Y_7v_{67} + Y_7^2 v_{77} \qquad (8a)$$

$$= [1(92109) + 2(181159) + 1(358197) - 2(175042) - 2(349956) + 1(349702)] \times (10^{-9})^2 \qquad (8b)$$

which also yields $\epsilon_{\mu_B} = 0.335$ ppm.

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[5] Note that in using eq (3), we set $s = k$, $\epsilon_k^2 = v_{kk}$, suppress k as a subscript on Y, and replace k with μ_B.

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