

## Anomaly detection on spectrograms using data-driven and fixed dictionary representations

M.Abdel-Sayed<sup>1,2,3</sup>, D.Duclos<sup>1</sup>, G.Fay<sup>2</sup>, J.Lacaille<sup>4</sup> and M.Mougeot<sup>3</sup>

1- SafranTech (SAFRAN) - TSI

Rue des Jeunes Bois Châteaufort 78772 Magny-Les-Hameaux – France

2- Ecole CentraleSupélec - MICS

Grande Voie des Vignes 92290 Châtenay-Malabry – France

3- Université Paris Diderot - LPMA

5 rue Thomas Mann 75013 Paris – France

4- Snecma (SAFRAN)

Rond point René Ravaud 77550 Moissy-Cramayel – France

**Abstract.** Spectrograms provide a visual representation of the vibrations of civil aircraft engines. The vibrations contain information relative to damage in the engine, if any. This representation is noisy, high dimensional and the relevant signatures relative to damages concern only a small part of the spectrogram. All these arguments lead to difficulties to automatically detect anomalies in the spectrogram. Adequate lower dimensional representations of the spectrograms are needed. In this paper, we study two types of representations with dictionary, a data-driven one and a non-adaptive one and we show their benefits for automatic anomaly detection.

### 1 Introduction

Each engine manufactured by Snecma is tested on a bench before its delivery to the airline company. Several measures, such as vibrations or performances, are recorded to determine the status of the engine.

Vibrations are one of the most pertinent information to analyze the engine behavior. Each potential defect in an engine component may induce a source of vibrations detectable in the vibration spectrogram rather than in the temporal signal. An expert is able to detect the damaged signatures looking at the spectrogram. However the high dimension of the spectrogram overwhelms this information. In a previous work [1], we discussed the use of a dimension reduction for anomaly detection. In this work, another representation based on an overcomplete fixed dictionary is investigated. The benefits of the decomposition in a dictionary are well known, but the dictionary has to be selected wisely.

In this paper we studied and compared two kinds of dictionaries. The first one is a data-driven dictionary learning, namely the non-negative matrix factorization (NMF) [2] already used in paper [1]. PCA is another example of such unsupervised scheme. The second dictionary is the curvelets frame [3], which atoms are fixed beforehand. Typical representatives of such a dictionary are provided by the Fourier basis and other wavelets base or frames.

For a better understanding of our problematic we describe the vibration analysis and the data in Section 2. The two kinds of dictionaries are defined in Section 3. Finally in Section 4, we present some results and draw conclusions.

## 2 Vibration analysis and data

Vibrations give multiple information concerning the different elements of the engine. Vibration analysis consists in investigating these vibrations data. Those data are generally temporal signals acquired by two accelerometers and subject to noise. Several studies analyze these signals to identify potential anomalies in the system [4].

In the test bench, two signals are acquired, one during the acceleration phase and the second during the deceleration. Indeed, unusual behaviors of the engine may be detected more clearly in non-stationary phases. In our study, we don't use the raw signals but their representation into spectrograms. It consists in the concatenation of short time Fourier transform applied on small temporal windows. The x-axis corresponds to time and the y-axis to frequencies.

Modern engines are characterized by two shafts, the high-pressure shaft with rotation speed denoted N2 and the low-pressure shaft with rotation speed denoted N1. The relation between the two shaft speeds is variable. A sampling of the spectrogram in one of these two shaft speeds (we are working on continuous accelerations or decelerations) instead of the time leads to a better visualization of the vibrations sources. Each vibration source related to the shaft used for the sampling is a straight line (Figure 1), vibrations sources of the other shaft are represented by curves.

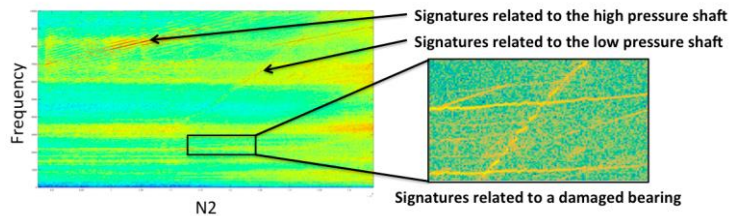


Fig. 1 : Vibrations spectrogram of a damaged engine sampled in N2 with a zoom of the patch containing the damage signatures

This representation is high dimensional and the relevant information related to a damage consists only in a tiny part of the data (Figure 1). Most of the information contained in the spectrogram is noise and expected vibrations signatures. Moreover, since the studied engines come directly from the production line, the number of abnormal engines is extremely low.

For these reasons, a representation of the spectrogram in a more suitable domain is needed to perform automatic analysis.

## 3 Decomposition over a dictionary

Experts visually analyze spectrograms by looking successively at different ranges of frequencies. We adapt this approach by subdividing the spectrograms into squared patches, each defined by a range of frequencies and shaft speeds N2. This subdivision

allows to accelerate the algorithm and to perform simultaneous analysis on different patches. Another advantage is the localization of the abnormal signature in the spectrogram. Ideally, only patches containing abnormal signatures trigger an alarm. Moreover, this subdivision can lead to a detection process based on multiple test.

### 3.1 Data-driven dictionary – Non-negative matrix factorization

#### 3.1.1 Non-negative matrix factorization (NMF)

In this first approach, the dictionary and the representations are learnt from the database. In our study, we use the non-negative matrix factorization (NMF) [2] to learn the dictionary. This method consists in the decomposition of a positive matrix  $V$  into the product of two positives matrices  $W$  and  $H$ . The following optimization problem summarizes it:

$$(W^*, H^*) = \underset{W > 0, H > 0}{\operatorname{argmin}} \|V - WH\|_2^2 \text{ st } \forall i, \|W(\cdot, i)\|_2^2 = 1$$

with  $W(\cdot, i)$  the columns of  $W$ .  $W$  represents the dictionary and  $H$  the coefficients.  $V$  is a matrix where each column represents a spectrogram for a given engine. For more details, we refer to our previous study using this approach in paper [1].

#### 3.1.2 Anomaly scores based on the NMF representation

Various scores allow to compare the different spectrograms in order to discriminate the patches containing some signature of damage. The dictionary  $W$  of the NMF is learnt on a database  $D_{model}$  containing patches without such signatures.

These scores (Table 1) are explained in [1], we refer to this article for more details. It consists in the distance to nearest neighbor spectrogram (NMF dnn), the Mahalanobis distance (NMF Mahal) and the reconstruction error (NMF RE).

Scores (Y)	NMF dnn	NMF Mahal	NMF RE
Formula	$\min_{X \in D_{model}} \ H_Y - H_X\ _2^2$	$(H_Y - \bar{H})^T \Sigma^{-1} (H_Y - \bar{H})$	$\ Y - WH_Y\ _2^2$

Table 1: Anomaly scores for the NMF representation

$H_X$  refers to the representation in the dictionary of spectrogram  $X$ ,  $\bar{H}$  and  $\Sigma$  are respectively the mean and the covariance of the coefficients learnt on safe engines.

### 3.2 Non adaptive dictionary – Curvelet transform

The curvelets are chosen as a way to represent the spectrograms. This choice is based on two facts. The first one is the perception of vibrations as curves in the spectrogram; the second one is the efficiency of the curvelets to represent curves with a small number of coefficients. We only give here the main elements to understand how the curvelets work. For more precision, we refer to [3].

#### 3.2.1 Curvelet transform

The curvelet transform [3] is based on the ridgelet transform [5], which allows the characterization of linear singularities. The ridgelet coefficients are indexed by three parameters: the scale, the localization and the orientation, so are the curvelet coefficients. The curvelet coefficients are then in a way local.

However, the vibrations are represented by curves in the spectrogram, hence the ridgelet transform is not sufficient to characterize the vibrations; but a curve can be approximated by a succession of small straight lines. Therefore by applying the ridgelet transform on dyadic squares at fine scale, it is possible to characterize a curve. This is the idea on which the curvelet transform is based.

For the curvelet transform, the orthonormal ridgelets [6] are used. The construction of the curvelet transform is based on the process in figure 2.

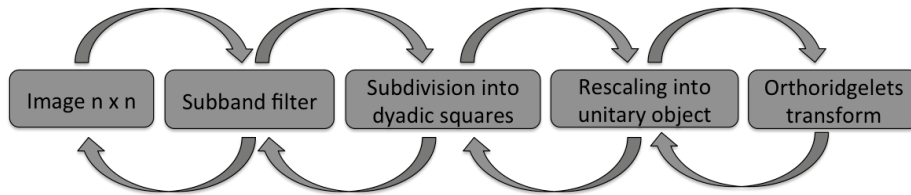


Fig. 2 : Construction of the curvelet transform and its inverse

With this construction, the curvelet transform is invertible and respect the Parseval relation.

### 3.2.2 Anomaly scores based on the curvelet transform

Most of the coefficients of the curvelet transform applied to the spectrograms have low values. We set up a threshold on the coefficients to keep the 10% largest ones. This threshold gives a good reconstruction with a minimal number of coefficients.

#### Distance to nearest neighbor spectrogram in the coefficients domain (Curvelets dnn)

This score computes the minimal distance between the representation of the spectrogram and the representation in the database  $D_{model}$ .

$$score_{dnn}(Y) = \min_{X \in D_{model}} \|T_Y - T_X\|_2^2$$

$T_X$  and  $T_Y$  correspond to the curvelet transform of respectively spectrograms  $X$  and  $Y$ .

#### Reconstruction error (Curvelets RE)

In order to learn the structure of normal spectrogram, we use the locality of the curvelets by learning the support of the coefficients of the spectrogram.

$$support_i = \{index \mid T_i(index) \neq 0\}$$

We define then the global support of the learning database  $D_{model}$  by :

$$support = \bigcup_{index} \left\{ index \mid \sum_{i \in D_{model}} \mathbb{I}\{index \in support_i\} \geq Q\% |D_{model}| \right\} \quad (1)$$

We conserve the  $Q\%$  indices appearing in the majority of spectrograms. This support is learnt on normal spectrograms, so it characterizes only normal behaviors.

The reconstruction error compares the spectrogram with its reconstruction restricted to the support.

$$score_{RE}(Y) = \|Y - T_{|support}^{-1}(T_Y)\|_2^2$$

with  $T_{|support}^{-1}(\cdot)$  the restriction of the inverse curvelet transform to the support (1).

## 4 Experimentations

### 4.1 Probability of anomaly

The scores of the methods defined above have various orders of magnitude. In order to compare them, we compute the probability of anomaly. For this purpose, we compute the different anomaly scores on another set  $D_{fit}$  with no damaged engines.  $D_{model}$  and  $D_{fit}$  are disjoint and define the learning dataset  $D_{learning}$ . A Gamma distribution representing the normal behaviors of the scores is fitted on each score. This distribution is chosen empirically and approved by a Kolmogorov-Smirnoff test.

We consider now a third set  $D_{test}$ , with  $D_{learning} \cap D_{test} = \emptyset$ . The different anomaly scores are also computed on this set. We perform then a statistical test in order to accept or reject the hypothesis  $H_0$ : “The studied engine has no damage”. The result of this test is determined by the p-value, the probability of wrongly rejecting the null hypothesis under its probability. The lower this value, the higher the suspicion on the engine. Under the null hypothesis, the score is assumed to be distributed as the estimated Gamma distribution, and the p-value writes

$$pvalue(Y) = \mathbb{P}(Z > score(Y)) = 1 - F_Z(score(Y))$$

with  $Z$  a random variable following the estimated Gamma distribution,  $F_Z$  its distribution function and  $score(\cdot)$  is any of the anomaly scores defined above.

### 4.2 Results

Our database contains 564 engines among which one engine is damaged (some zones contain abnormal signatures) and a second shows corrupt data (the whole spectrogram is abnormal). These 2 engines and 48 normal ones form the test database  $D_{test}$ .

In order to verify the consistency of our method, we randomly select a hundred times, among the learning database  $D_{learning}$ , 400 engines that will take part of the learning of the model  $D_{model}$  and 114 engines used for the distribution  $D_{proba}$ .

Scores	Normal engines	Damaged engine	Corrupt data engine
NMF dnn	$0.55 \pm 0.07$	$(2.25 \pm 0.13) \times 10^{-9}$	$(4.4 \pm 5.0) \times 10^{-3}$
NMF mahal	$0.57 \pm 0.06$	$(6.03 \pm 0.6) \times 10^{-11}$	$(1.06 \pm 1.7) \times 10^{-4}$
NMF RE	$0.52 \pm 0.03$	$< 10^{-16}$	$(2.12 \pm 2.66) \times 10^{-4}$
Curvelets dnn	$0.54 \pm 0.03$	$< 10^{-16}$	$(1.05 \pm 1.86) \times 10^{-4}$
Curvelets RE	<b><math>0.57 \pm 0.03</math></b>	<b><math>&lt; 10^{-16}</math></b>	<b><math>&lt; 10^{-16}</math></b>

Table 2: Mean p-values of the engines in  $D_{test}$  on a patch [256 x 256] containing an abnormal signature. The normal engines correspond to the mean of the safe engines in  $D_{test}$

Scores	Normal engines	Damaged engine	Corrupt data engine
NMF dnn	$0.57 \pm 0.09$	$0.5 \pm 0.11$	$0.06 \pm 0.03$
NMF mahal	$0.5 \pm 0.09$	$0.29 \pm 0.09$	$0.01 \pm 0.01$
NMF RE	$0.49 \pm 0.04$	$0.34 \pm 0.05$	$(1.3 \pm 1.2) \times 10^{-3}$
Curvelets dnn	$0.53 \pm 0.03$	$0.51 \pm 0.07$	$(1.7 \pm 1.6) \times 10^{-3}$
Curvelets RE	<b><math>0.44 \pm 0.03</math></b>	<b><math>0.27 \pm 0.04</math></b>	<b><math>(1.2 \pm 4.42) \times 10^{-11}</math></b>

Table 3: Mean p-values of the  $D_{test}$  engines on a patch [256 x 256] without abnormal signatures.

Table 2 shows that the damaged engine is quite well detected and rejected with p-value of order at least  $10^{-3}$  for every method in the zone containing the signature. Therefore using another representation of the normal spectrogram vibrations enables the detection of some signatures. However, the results for the corrupt data engine are variable between the different methods. We can see that the reconstruction error applied with the curvelets provides p-values lower than the other methods. This difference is due to the fact that by learning the support of the curvelet coefficients, we learn the structure of the normal spectrogram. This structure is not present in the corrupt spectrogram, which leads to a higher score.

Table 3 gives the results in a patch without abnormal signature. The damaged engine is no more detected in this patch whereas the corrupt data engine is still rejected. The reconstruction error applied to the curvelets still discriminates it with higher performance than the other methods.

## 5 Conclusion

Representing a spectrogram in a dictionary allows to discriminate damaged engines. In this paper, we tested two types of dictionaries with anomaly scores based on them, the NMF and the curvelets. The two methods detect the damaged engine. The fixed dictionary representation combined with a data-driven selection of some atoms gives the best results. The selection of a support for the inverse curvelet transform accounts to learn the structure of the different lines in the spectrogram.

The subdivision into patches enables detection only in a zone with an abnormal signature leading to a rough localization of the signature in the spectrogram.

This type of algorithm for the detection of anomaly in spectrograms shows some encouraging results. Further analyzes of the process and more advanced comparisons are required. Ongoing works consist in the detection of weak signatures on shifted windows and on a detection procedure based on multiple testing theory.

## References

- [1] M. Abdel-Sayed, D. Duclos, G. Faÿ, J. Lacaille and M. Mougeot, NMF-based Decomposition for Anomaly Detection applied to Vibration Analysis. Proceedings of the 12<sup>th</sup> international Conference on Condition Monitoring and Machinery Failure Prevention Technologies (CM-MFPT2015), June 9-11, Oxford (United Kingdom), 2015.
- [2] D. D. Lee and H. S. Seung, Algorithms for Non-negative Matrix Factorization. In Advances in neural information processing systems, pages 556-562, 2001.
- [3] E. J. Candès and D. L. Donoho, Curvelets – a surprisingly effective nonadaptive representation for objects with edges. In A. Cohen, C. Rabut and L. L. Schumaker, editors, Curve and Surface Fitting: Saint-Malo, eds., pages 105-120, Vanderbilt University Press, Nashville, TN, 2000.
- [4] R. Klein, E. Rudyk and E. Masad, Methods for diagnostics of bearings in non-stationary environment. International journal of Condition monitoring 2, pages 2-7, 2012.
- [5] E. J. Candès and D. L. Donoho, Ridgelets: The key to higher-dimensional intermittency? Philosophical transactions-Royal Society. Mathematical, physical and engineering sciences, 357, 1760: 2495-2509, 1999.
- [6] D. L. Donoho, Orthonormal Ridgelets and Linear Singularities. SIAM Journal of Mathematical Analysis 31, 5: 1062-1099, 2000.