EXPERIENCE WITH GPS-SUPPORTED AERIAL TRIANGULATION

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Abstract

Airborne kinematic GPS positioning provides the eccentric determination of the position of the aerial camera at the individual moments of exposure. These positions are introduced in the combined photogrammetric block adjustment as additional observations. The observation equations take into account the eccentricity of the GPS antenna with respect to the aerial camera and unknown parameters for modelling linear drift or datum corrections. The paper briefly reviews the mathematical model of GPS-supported aerial triangulation. Empirical results of a controlled photogrammetric test flight with GPS camera positioning are presented and discussed.

KEY WORDS: Aerotriangulation, GPS, Accuracy

1. Introduction

Photogrammetric block adjustment with bundles or independent models are well established numerical procedures in aerial triangulation for photogrammetric point determination as well as for providing orientation data of stereo-models for mapping. For carrying out an aerial triangulation, terrestrial control points are necessary. Terrestrial control points are required for the determination of the geodetic datum, for the geometric stability of the photogrammetric block to avoid weakly conditioned or singular normal equation systems and especially for controlling the propagation of errors in photogrammetric blocks. Number, accuracy and configuration of the terrestrial control points are correspondingly planned to meet the required accuracy specification of the respective project. The establishment of the necessary ground control by terrestrial survey is, however, time-consuming and often constitutes the major financial costs in aerial triangulation projects, especially in non-accessible areas.

The application of GPS positioning for aerial triangulation aims at precisely determining the position of the aircraft at the individual moments of the camera exposure. These positions can be introduced in a combined block adjustment as eccentric observations of the positions of the camera projection centre. The prime effect is the substantial reduction of necessary terrestrial control points. Control points are then merely required for providing the geodetic datum. Thus, GPS camera positions as additional observations provide an improvement in practicability and economy of aerial triangulation projects. The impact of the GPS camera positions on the accuracy characteristics of combined block adjustment and on the reduction of terrestrial control are well known on the basis of computer simulations and theoretical propagation of errors.

This paper presents results of an empirical analysis of GPS supported aerial triangulation based on the controlled photogrammetric test flight "Glandorf". The aim of the test flight is the empirical examination of the theoretical expectations of combined block adjustment with regard to accuracy and reduction of terrestrial control.

2. GPS-Positioning for Aerial Triangulation

(1) GPS airborne kinematic positioning for aerial triangulation is usually based on GPS carrier phase observations processed in relative mode (single- or double differences). The accuracy potential of airborne kinematic GPS positioning, especially on the basis of carrier phase observations, has been proven by several independent investigations of photogrammetrically controlled test flights (VAN DER VEGT 1989, DORRER/SCHWIERTZ 1990, FRIESS 1990). The empirical investigations have shown, that beside the high internal precision of GPS aircraft positions ($\sigma \approx 2$ cm), drift errors can occur. The drift seems to be linear in first approximation, at least for time intervals of up to 15 min.

The drift can be attributed to remaining uncertainties in the a priori corrections (e.g. atmospheric refraction), and to unmodelled error effects (e.g. satellite orbit errors), in spite of applying differencing techniques. The sensitivity of kinematic positioning with respect to an uncertainty of the initial ambiguity parameter has to be mentioned in particular. Special analyses have shown, that incorrect ambiguities cause noticable drifts in the subsequent kinematic positioning.

(2) The question, whether drift errors can remain or whether complete error modelling is possible was controversially discussed on different occasions among photogrammetrists and geodesists. It is not put in doubt, that it is basically possible to avoid drift errors in kinematic positioning. But it is still in question, whether it is possible under the operational conditions of photogrammetric flight missions.

The causes for drift effects are firstly remaining systematic errors of the GPS observations in spite of the use of differencing techniques. As the reduction of unmodelled errors by differencing techniques decreases with time and distance, systematic GPS positioning errors will increase with time and distance. In case of photogrammetric flight missions observation time spans of up to 5 hours or more and distances of several hundred km are to be expected, since it will not always be possible to place the reference receiver within the flying area. Also, a flight mission may combine several smaller projects quite some distance apart. It would not be economical to place a reference receiver in every area.

Secondly, incorrect carrier phase ambiguities contribute to drift errors in the positions. In airborne applications the problem of determining carrier phase ambiguities may arise several times. It will for instance not be possible to start with a low take-off angle at all airports. It may also be necesseray to take a turn directly after take-off. As a consequence, individual or even all satellite signals are interrupted and the problem of determining the phase ambiguities re-occurs. Further signal interruption can arise due to shadowing of satellites during the turns by the wings of the aircraft, even if attention is payed to flying with low banking-angles. Flying with minimal banking-angles leads moreover to a considerable increase in flying time.

The determination of carrier phase ambiguities of a moving receiver is not an unsolvable problem. There exist different approaches. Multi-channel receiver for example are able to observe simultaneously all visible satellites, which result in redundant observations. As long as at least 4 satellites are continuously observed, continuous positioning is feasible and the ambiguities of interrupted observations can be re-assessed. Situations where only 3 satellites or less are observed whilst all other satellite signals are interrupted over short intervals of time (< 3 sec) can be handled by prediction methods for the receiver clock error and/or for the aircraft trajectory. But it has to be taken into consideration that the satellite geometry changes due to the interruption of individual satellites. The observation geometry can considerably deteriorate, even if 4 satellites are received. In that time span, the influence of sytematic errors onto the position determination increases and consequently drift errors can occur.

Recent developments in kinematic positioning are directed towards algorithms for rapid carrier phase ambiguity resolution (e.g. FREI-/BEUTLER 1990, HATCH 1990). To what extent these algorithms may improve the situation in airborne kinematic positioning is to be shown by future investigations.

(3) At the moment, it can not be precluded, in view of the operational conditions of photogrammetric flight missions, that drift errors occur in photogrammetric applications, due to satellite shadowing, large distances between reference station and aircraft, and flights over several hours including changing satellite geometry. Whether, and to which extent, drift errors are acceptable depends on the intended use of GPS data and on the possibility of subsequent correction. It is important to notice, in the context, that in combination with aerial triangulation linear GPS drift errors can be corrected. If the drift errors can be taken into account in this way, the accuracy of the GPS positions can reach a level of a few centimeters (FRIESS 1990).

Therefore it can be recommended for GPS-supported aerial triangulation, to evaluate the GPS data independently for each photo strip, instead of considering one total flight mission as one processing unit. The phase ambiguities can, in this case, be computed approximately via a differential pseudorange solution for the first position of each strip. The resulting drift errors are acceptable as they can be corrected during the combined block adjustment. It is emphasized, that stripwise processing of GPS data is only recommended in connection with aerial triangulation i.e. with combined block adjustment.

The method of treating flight strips separately has certain advantages from the operational point of view. The data recording can start at the beginning of the first photo strip and, moreover, the flight maneuvers during take-off and during the turns can be carried out as usual, without paying attention to satellites.

3. GPS-Supported Block Adjustment

(1) In application to aerial triangulation it is assumed that the positions of the GPS aircraft antenna are interpolated onto the individual times of camera exposure. The interpolated GPS antenna coordinates are introduced into the combined block adjustment as additional observations for each camera position via appropriate observation equations.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{AC} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{PC} + R(\omega,\varphi,\kappa) \cdot \begin{bmatrix} x_{PC}^{AC} \\ y_{PC}^{AC} \\ z_{PC}^{AC} \end{bmatrix} + \begin{bmatrix} a_X \\ a_Y \\ a_Z \end{bmatrix} + \begin{bmatrix} b_X \\ b_Y \\ b_Z \end{bmatrix} \cdot dt$$

$[X,Y,Z]_{AC}^T$	coordinates of the GPS antenna
$[X,Y,Z]_{PC}^T$	coordinates of the projection centre
$[x_{PC}^{AC},y_{PC}^{AC},z_{PC}^{AC}]^T$	GPS antenna eccentricity
$R(\omega,\varphi,\kappa)$	orthogonal rotation matrix
$[a_X,a_Y,a_Z]^T$	GPS drift parameter (constant terms)
$[b_X,b_Y,b_Z]^T$	GPS drift parameter (linear terms)
dt	time interval

The observation equations take into account the eccentricity of the GPS antenna with respect to the arial camera. The eccentricity is assumed to be measured with respect to the camera coordinate system by terrestrial surveying techniques (e.g. SCHWIERTZ/-DORRER 1991). In the block adjustment, the eccentricity is treated as known.

The observation equations include also linear drift parameters for each coordinate as additional unknowns. They approximate and correct the drift errors of the GPS antenna positions in the combined block adjustment. Linear drift parameter can also be interpreted as corrections to the datum transformation. The drift modelling has to be flexible. Depending on the case the drift parameters may be chosen stripwise or common for several strips. If the GPS observations are continuous for a complete photo-flight, without any interruption, one set of drift parameters may be sufficient for a complete block. If stripwise GPS data processing is to be applied, as recommended above, one set of drift parameters has to be introduced for each of the individual photo strips. The mathematical model of GPS-supported aerial triangulation has been described and discussed in several papers (COLOMINA 1989, ACKERMANN 1990, FRIESS 1990A).

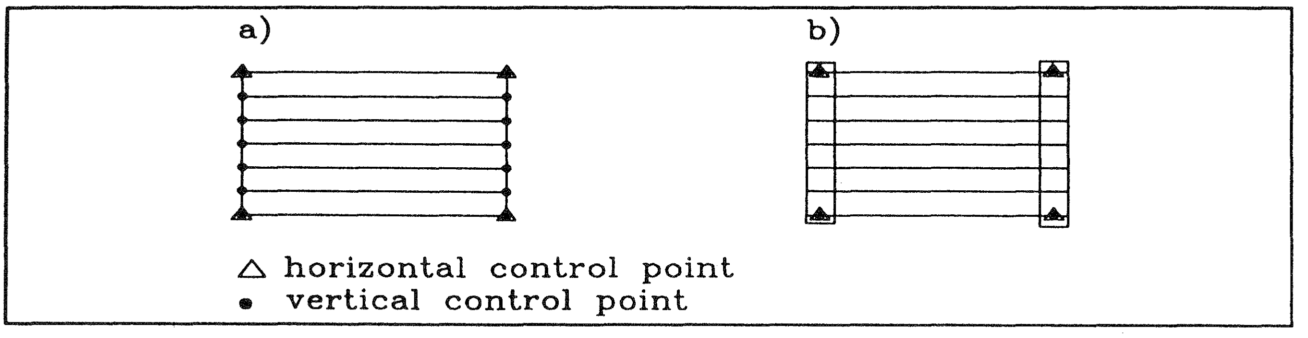

Fig. 1 : Minimum terrestrial control required for GPS blocks

(2) The unkown drift parameters require some additional consideration. Their determinability must be guaranteed by a corresponding ground control configuration and/or flight pattern. For GPS supported aerial triangulation the photogrammetric block is assumed to be geometrically determined in the conventional sense, i.e. with standard overlap and standard tie-point distribution. Two different ground control configurations are recommended (Fig 1).

With either configuration the linear drift parameters can be safely determined, and the block adjustment can be carried out. The suggested use of 4 ground control points is considered necessary for solving the datum problem. Aerial triangulation without any ground control is possible in principle. It is normally not applicable, however, as the results refer to the WGS 84 rectangular coordinate system.

(3) In ACKERMANN 1991 the accuracy of adjusted GPS-supported photogrammetric blocks is analyzed theoretically, based on the inversion of the normal equation matrix. These studies have shown, that the accuracy properties of the adjusted GPS blocks are extremely favourable. The GPS camera positions control a block very well, suppressing essentially any propagation of errors. The accuracy of GPS blocks depends (within reasonable limits) very little on ground control, on block size, and on GPS accuracy. However, the accuracy depends markedly on whether and how many drift parameters have to be applied in the adjustment, as they weaken the geometry of the block noticeably. The resulting block accuracy, expressed as r.m.s values $\mu_{X,Y}$ and μ_Z of the horizontal and vertical standard deviations, respectively, of all adjusted tie-points, are summarized in fig. 2.

The above theoretical results are directly valid for a block of 6 strips with 21 photographs and combined bundle block adjustment. The precision of the ground control point coordinates (σ_{PP}) and of the image coordinates of the ground control points ($\sigma_{PP'}$) has been assumed to be equal to σ_o and $\sigma_o \cdot s$, (s = image scale number) respectively. The accuracy of the GPS camera positions is assumed to be $\sigma_{GPS} \leq \sigma_o \cdot s$, i.e. to correspond to the photogrammetric measuring accuracy. The combined block adjustment can be done equally well with the independent model method. The accuracy results are expected to be very similar to the bundle method.

The above results are not restricted to the particular assumptions which have been made. The block size has only a slight influence. According to ACKERMANN 1991, for smaller blocks the accuracy deteriorates within 10%, for larger blocks the accuracy improves within 10%. The most important result of these studies is the fact,

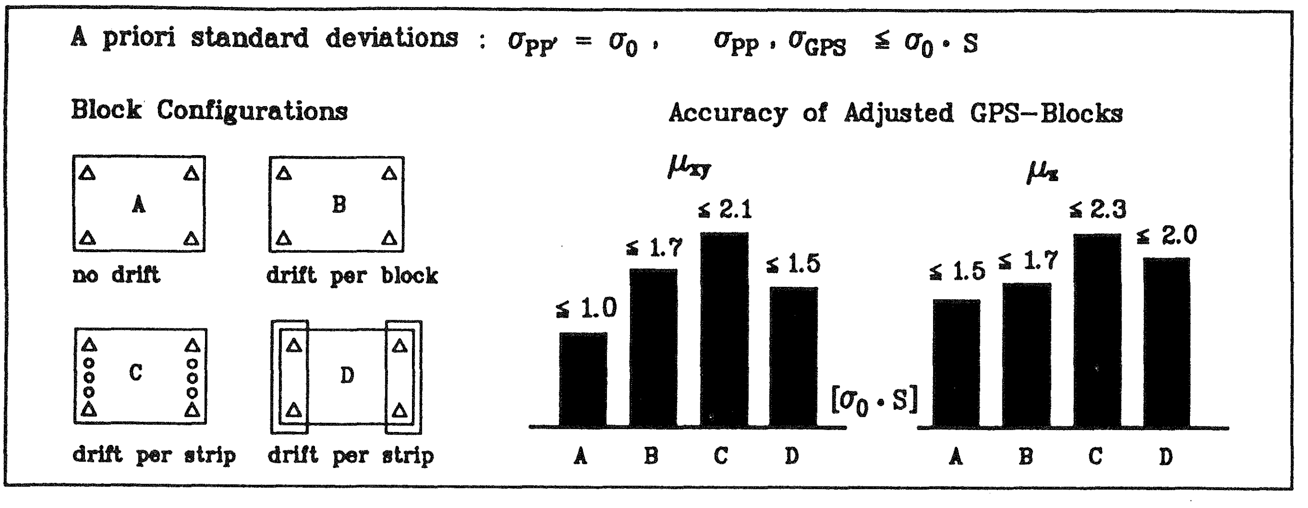

Fig. 2 : Theoretical accuracy of GPS supported blocks (results of simulations)

that with decreasing accuracy of the GPS camera positions ($\sigma_{GPS} \geq \sigma_o \cdot s$) the block accuracy deteriorates with a much slower rate than σ_{GPS} itself. If, for example, the accuracy of the GPS positions amount to $\sigma_{GPS} = 10 \cdot \sigma_o \cdot s$, the above values for the block accuracy $\mu_{X,Y}$ and μ_Z decrease only by the factor 2. Taking into account that the required block accuracy for medium and small scale mapping is only in the order of $2 \cdot \sigma_o \cdot s$ to $5 \cdot \sigma_o \cdot s$, there exists a considerable margin with respect to the required accuracy of the GPS camera positions. It means that the requirements for the GPS positioning accuracy are not stringent at all, in case of aerial triangulation for mapping purposes. It can generally be stated, that GPS-supported aerial triangulation with the specified minimum of ground control can meet the accuracy demands for the complete spectrum of photogrammetric mapping tasks, even if drift parameters per strip (Fig. 2, case C, D) have to be applied (with the possible exception of very large scale applications).

4. Test Flight Glandorf

(1) The test flight Glandorf was carried out in August 1990 by HANSA Luftbild GmbH. The State Survey Authority of Niedersachsen was responsible for the signalization and for the coordinates of the check points. The participation of the Institute for Photogrammetry of Stuttgart University took place within the scope of the Special Collaborative Programme "High Precision Navigation" of the German Research Foundation (Sonderforschungsbereich 228, Deutsche Forschungsgemeinschaft).

The extension of the test flight area is approximately 6 km to 8 km. The photogrammetric block (fig. 3) covers 5 strips in N-S with 14 photographs each and 7 cross strips in E-W with 10 photographs each. The photo scale is 1 : 8000, the forward overlap 60 % and the side overlap 20 %. The used aerial camera Zeiss RMK TOP is able to generate a pulse at the moments of exposure, which triggers the GPS receiver to record the internal time (ZÜGGE 1989). In this way it is possible to assign the times of exposure to the GPS observation epochs. The GPS observation were carried out at a data rate of 0.5 sec with two Ashtech L-XII GPS receiver, one receiver in the aircraft and the second receiver placed at a reference point in the test area. The test area contains 15 signalized ground points of which the X,Y and Z coordinates are known and in addition 20 signalized ground points of which the Z-coordinates are given. These signalized points are applied in the following investigations as control and/or check points.

(2) The photogrammetric measurements as well as the block adjustment was carried out by the Institute for Photogrammetry of Stuttgart University. The GPS data processing was done by the author.

The GPS data of the test flight were processed independently for each of the photo strips by carrier phase single differences. The initial ambiguities are derived from a pseudorange single difference solution for the respective first position within each strip. The relation in time of the GPS antenna positions and the camera positions was established
by a Legrange interpolation of third order.

(3) Two topics have been investigated : the empirical accuracy of stripwise GPS carrier phase processing and in particular the accuracy characteristics of the GPS supported photogrammetric block adjustment with linear GPS dift modelling per photo strip.

(3.1) The accuracy analysis of the GPS positioning is based on the comparison of the GPS antenna positions and the independently determined positions of the projection centre of the aerial camera. To this, the complete photogrammetric block Glandorf (5 strips N-S + 7 strips E-W, fig. 3) was conventionally adjusted according to the bundle method, supported by all available terrestrial control points (15 signalized XYZ control points, 20 signalized Z control points). The coordinates of the projection centres could thus be determined with a theoretical accuracy of 8.2 cm for X,Y and 4.4 cm in Z. They serve as check values for the testing of the GPS antenna coordinates.

For the accuracy analysis, the spatial distance between the GPS antenna centre and the camera projection centre is calculated at the individual moments of exposure from the coordinates of the antenna centre and the coordinates of the projection centre. The distance between the two centres is not affected by the variations of the aircraft attitude. The variations of this distance within the individual photo strips can be taken as criteria for the assessment of the accuracy of the GPS aircraft antenna positions.

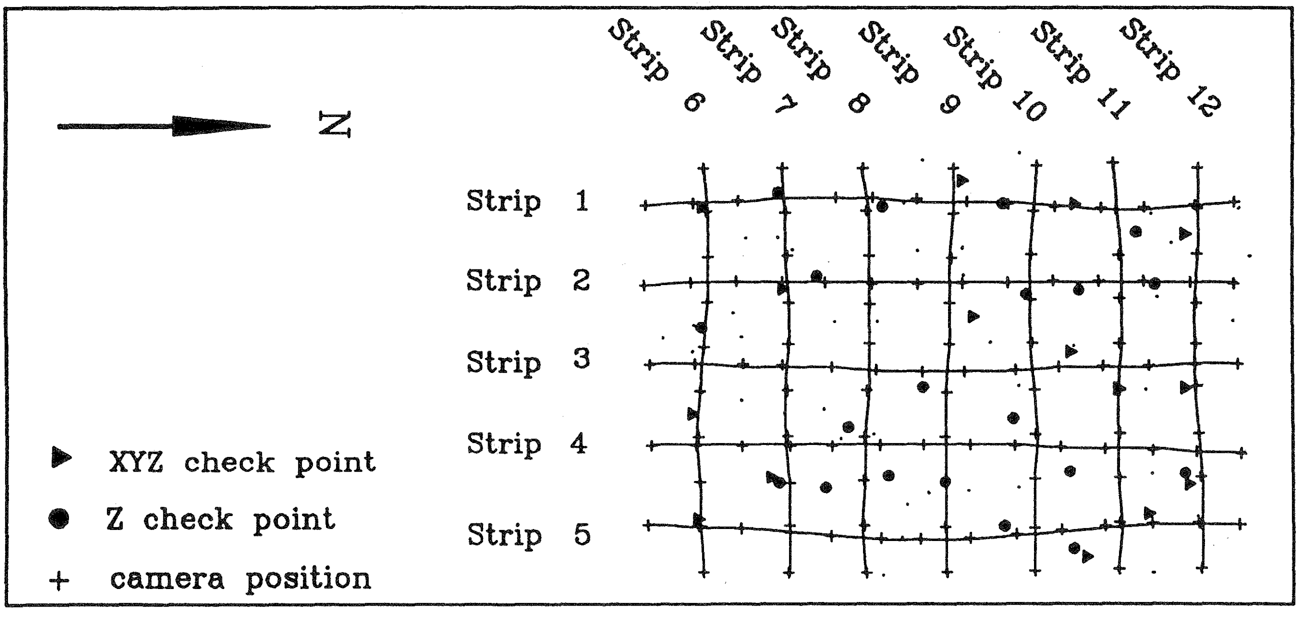

Fig. 3 : GPS test block Glandorf

For each photo strip, the r.m.s values of the differences between the distances at the individual moments of exposure and the arithmetic mean of the distance within the repective strip are presented in fig 4. The r.m.s values vary between 8.6 cm and 37.0 cm. This corresponds to the accuracy of the GPS antenna coordinates after applying constant coordinate shifts per strip. The results firstly indicate that the GPS positions comprise systematic errors, and secondly demonstrate that constant shifts per strip are not sufficient for correcting these errors.

(3.2) The GPS supported block adjustment is analysed by two different block configurations. The first GPS-block (GPS-block Glandorf 5 + 2) comprises the 5 strips 1 to 5 and the two cross strips 6 and 12. The second GPS-block (GPS-block Glandorf 7 + 2) comprises the 7 strips 6 to 12 and the strips 1 and 5 as cross strips (compare fig. 3). In both of the GPS-blocks, only 4 of the signalized terrain points in the corners of the test area are applied as XYZ control points. Thus, two GPS blocks of tpye (b), according to fig. 1 are investigated. The GPS blocks contain 676 tie points, which were marked in the photographs using the Zeiss PM1.

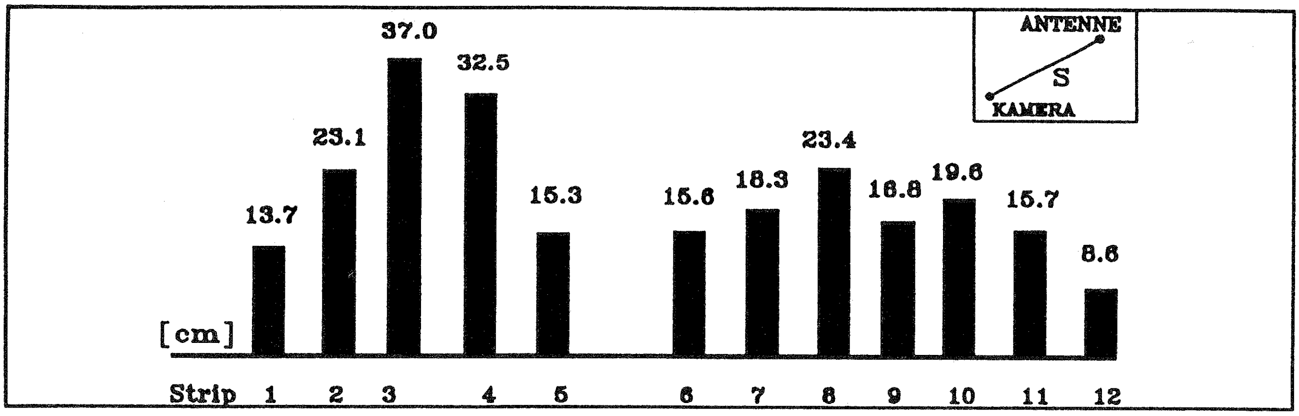

Fig. 4 : RMS differences of S with respect to the arithmetic mean per strip

Fig. 5 represents the corresponding r.m.s values after removing linear drifts per strip, varying between 4.1 cm and 8.4 cm. It must be considered, that these results are based on the comparison of the GPS antenna positions and the camera positions. The attained r.m.s values, therefore, still include effects of the interpolation of the antenna positions as well as the uncertainties of the camera positions used as check values.

The obtained results agree entirely with the expectations. They confirm the findings and the results of the test flight Flevoland (FRIESS 1990B) and demonstrate, that linear drift modelling is sufficient. Stripwise processing of GPS carrier phase data is consequently practicable with regard to linear drift correction per strip.

The adjustment of the two GPS-blocks was carried out with the new version of PATB-RS, which allows the integration of GPS antenna positions as additional observations and the inversion of the normal equation matrix. The a priori standard deviations of the observations are assumed to be 7 μm for the image coordinates, 2 cm for the terrestrial control point coordinates. The standard deviations of the GPS antenna coordinates are assumed to be 8 cm, according to the empirical accuracy analysis. All observations are treated as stochastically independent. The components of the eccentricity of the GPS antenna are introduced as known, constant parameters. Unknown parameters are the coordinates of all terrain points, the parameters of the exterior orientation of each photograph, as well as linear GPS drift parameters per photo strip.

In fig. 6 and 7, the theoretical and the empirical accuracy results of the adjusted coordinates of the terrain points are presented, referring to the two independent GPS supported block adjustments of the GPS-block Glandorf 5 + 2 and of the GPS-block Glandorf 7 +2, respectively. The theoretical accuracy is derived from the inversion of the normal equation system, the empirical accuracy corresponds to the r.m.s. values of the differences of the adjusted coordinates and the given coordinates of the check points.

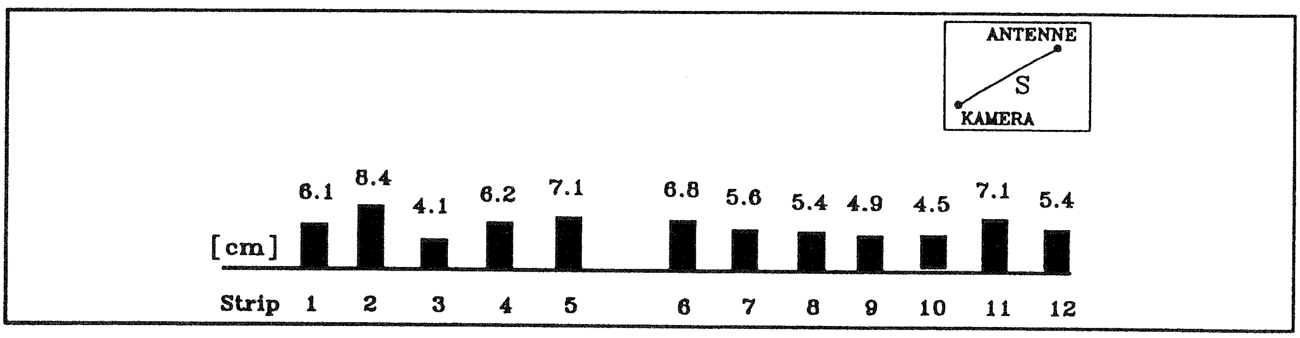

Fig. 5 : RMS differences of S with respect to linear regressions per strip

For the GPS-block Glandorf 5 + 2, the empirical accuracy amounts to 6.3 cm in planimetry and to 8.5 cm in height. For the GPS-block Glandorf 7 + 2 an empirical accuracy of 7.8 cm in planimetry and of 10.8 cm in height is attained. The empirical accuracy of the GPS-block Glandorf 7 + 2 is slightly worse than the accuracy of the block Glandorf 5 + 2. This can be explained by the poor quality of the photographs of the strips 7 and 11, which were taken at the beginning of the flight mission under unfavourable weather conditinns (low clouds).

These results demonstrate the efficiency of GPS-supported aerial triangulation. With 4 XYZ ground control points in the corners of the block and two cross strips at each of the block ends (fig. 1), a block accuracy is empirically attained, which could be reached by conventional block adjustment only with dense ground control.

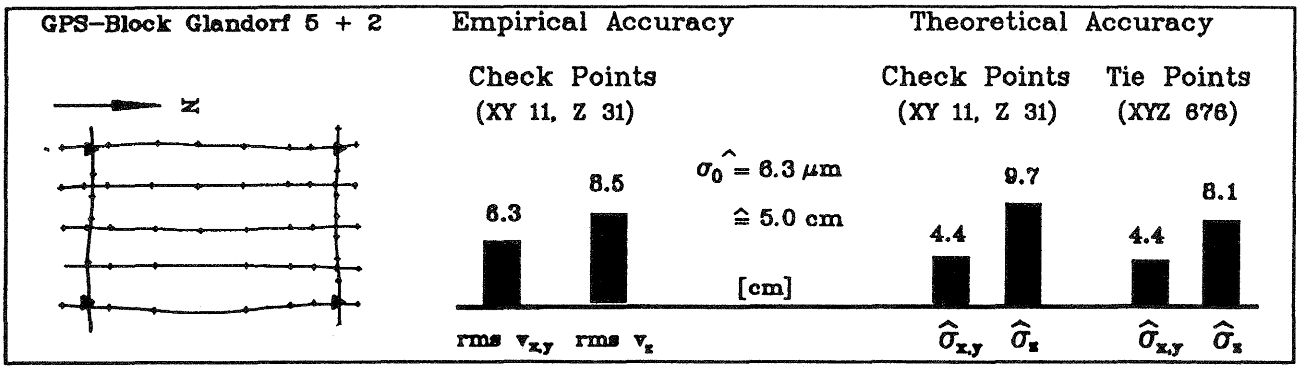

Fig. 6 : Accuracy of the GPS-supported block adjustment of the block Glandorf 5 + 2

Empirical and theoretical accuracy of the adjusted check point coordinates closely agree. For the GPS-block 5 + 2, the theoretial accurcay results in 4.4 cm in planimetry and 9.7 cm in height, the corresponding results of the GPS-block 7 + 2 are 4.3 cm and 9.2 cm, respectively.

Proceeding on the assumption that a photogrammetric block at a scale of 1 : 8000 is used for mapping in 1 : 1000, the accuracy in planimetry of the adjusted block has to be 10 cm. The accuracy in height has to reach 12 cm, corresponding to 0.1 ‰ of the flying height. The obtained empirical block accuracy of 6.3 cm and 7.8 cm in planimetry and 8.5 cm and 10.8 cm in height for the two GPS-supported blocks of the test Glandorf consequently meet the accuracy requirements.

This is confirmed by the comparison of the coordinates of all terrain points resulting from a conventional adjustment of the complete test block Glandorf (12 strips) supported by all available control points with the coordinates obtained from the two independently adjusted GPS-blocks Glandorf 5 + 2 and Glandorf 7 + 2. The r.m.s values of the differences between the coordinates of the GPS-block 5 + 2 and the conventional block result to 6.5 cm (1.3 $\sigma_o \cdot s$) in planimetry and to 7.9 cm (1.6 $\sigma_o \cdot s$) in height. The corresponding r.m.s. values for the comparison of the GPS-block 7 + 2 with the conventional block amount to 6.8 cm (1.4 $\sigma_o \cdot s$) and 10.8 cm (2.2 $\sigma_o \cdot s$).

Finally, it is emphasized that the empirically obtained accuracies of the GPS supported block adjustment of the test blocks Glandorf (1.3 $\sigma_o \cdot s$, 1.6 $\sigma_o \cdot s$ in planimetry, 1.7 $\sigma_o \cdot s$, 2.2 $\sigma_o \cdot s$ in height) are in agreement to the theoretical expectations concerning the simulation studies (fig. 2)

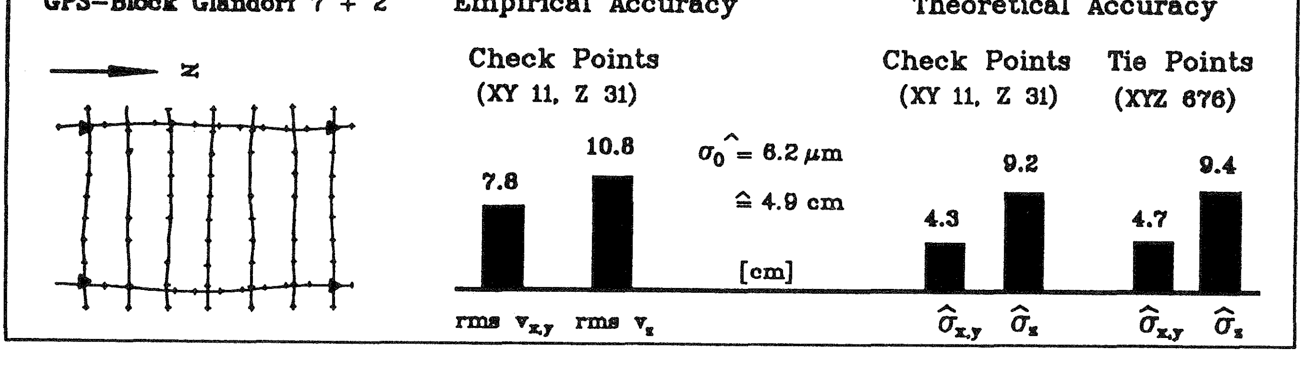

Fig. 7 : Accuracy of the GPS-supported block adjustment of the block Glandorf 7 + 2

5. Conclusions

It can be stated in conclusion that GPS-supported aerial triangulation is ready for practical application. Software for GPS kinematic positioning (SKIP) as well as software for the combined block adjustment (PAT-B, PAT-M) are available. Several geodetic GPS receivers are on the market. If the recommended ground control configurations are applied in addition to stripwise linear modelling of GPS drift errors in the combined block adjustment, kinematic relative camera positioning for aerial triangulation is a highly operational, robust and most economic method which can change thoroughly aerial photogrammetry within a short time.

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