OPTIMISATION OF BUNDLE ADJUSTMENTS FOR STEREO PHOTOGRAPHY

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ABSTRACT
Conventional bundle adjustments ignore the invariant geometric relationships that exist between camera pairs in a bundle of stereo photography. Two models for optimising conventional bundle adjustments to take advantage of these relationships are developed. These models are compared with a conventional bundle adjustment. Initial results indicate that both presented models yield improved accuracies when compared to a conventional bundle adjustment.

KEY WORDS: Optimisation, Bundle adjustment, Stereo photography.

INTRODUCTION

In the field of non-topographic photogrammetry bundle adjustments are used in a wide range of applications. Whether implemented as a DLT or self-calibrating model the purpose of the bundle adjustment is to minimise the residuals of all the observations. Each camera thus finds a position and orientation that reflects this minimisation. This approach is well established for single camera imaging geometry as typified by Fraser (1991).

When the imaging is performed by a stereo camera system, inherent in each pair of photographs is the invariant geometrical relationship of the two cameras. If such a bundle of stereopairs were reduced by a conventional bundle adjustment the invariant camera relationships between the two images of each stereopair would be ignored in favour of optimising each camera's position and orientation based on the observations and their random errors.

This paper reports on two methods that have been developed to optimise a conventional bundle adjustment for use with stereo photography so that the invariant relationships are retained for all stereopairs. The mathematical models are developed and the results of a trial comparing the two models with a conventional bundle adjustment are presented.

THE BUNDLE ADJUSTMENT

Slama (1980, Ch2) gives the generally accepted model of the observation equations for a conventional bundle adjustment. These are of the following form:

$$\bar{V} + \bar{B}\Delta = \bar{C} \tag{1}$$

$$\bar{V} = \begin{bmatrix} V \\ \dot{V} \\ \ddot{V} \end{bmatrix}; \quad \bar{B} = \begin{bmatrix} \dot{B} & \ddot{B} \\ -I & 0 \\ 0 & -I \end{bmatrix}; \quad \Delta = \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta} \end{bmatrix}; \quad \bar{C} = \begin{bmatrix} \epsilon \\ \dot{C} \\ \ddot{C} \end{bmatrix}.$$

V = *vector of plate observation residuals;*
$\dot{V}$ = *vector of exterior orientation parameter observation residuals;*
$\ddot{V}$ = *vector of object coordinate observation residuals;*
$\dot{B}$ = *matrix of partial derivatives wrt exterior orientation parameters;*
$\ddot{B}$ = *matrix of partial derivatives wrt object coordinates;*
$\dot{\Delta}$ = *vector of exterior orientation parameter corrections;*
$\ddot{\Delta}$ = *vector of object coordinate corrections;*
ϵ = *vector of plate observation descrepancies;*
$\dot{C}$ = *vector of exterior orientation parameter descrepancies;*
$\ddot{C}$ = *vector of object coordinate discrepancies;*

The structure of the camera parameter portion of the normal equation matrix produced by the least squares solution to this model is shown in figure 1.

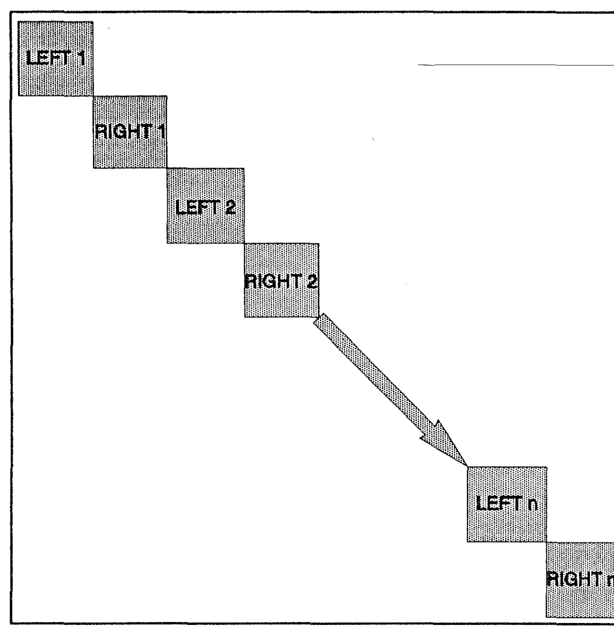

Figure 1 Normal equation structure of the camera parameter portion of a conventional bundle adjustment of stereopairs.

Each camera is represented by a 6x6 symmetric sub-matrix. The total number of camera parameters to be solved for a bundle of **s** stereopairs is 12**s**. An overview of photogrammetric bundle adjustment programs can be found in Karara (1989, Ch6). Bundle adjustments developed in this study were based upon this model.

MODELLING OF CAMERA INVARIANCE

The invariance that exists between the cameras of a stereopair may be divided into two relationships:

a. the relative positions of the cameras (camera base, the exterior orientation position coordinates of each camera pairs perspective centres); and

b. the relative rotations of the cameras (camera convergence, the exterior orientation rotations of each camera pairs coordinate axes).

The type of invariance is dependent upon the type of stereo camera system used. Two stereo camera types considered here are:

a. stereometric camera systems where the invariant camera relationships are precisely known; and

b. non- or semi- metric camera systems where cameras are placed in a fixed but not precisely known relationship for specific projects as typified in Fryer (1990).

By considering type b. camera systems as representing the general case, the numerical values that describe the invariant camera relationships must be solved for as part of the bundle adjustment. Type a. cameras may then be considered as a special case and the known values applied as appropriate.

Two methods were considered in modelling the invariant relationships:

a. by constraint equations between the camera parameters of each stereopair; or

b. by modified collinearity equations relating the camera parameters of each stereopair.

INVARIANCE MODELLED BY CONSTRAINT EQUATIONS

An introduction to the use of constraint equations in analytical photogrammetry in given in Case (1961). Constraint equations are used to express geometric or physical relationships that exist between parameters of an adjustment. In this case the parameters of interest are the exterior orientation parameters of the cameras.

Constraint equations take the following form:

$$v_c + C\Delta = g \qquad (2)$$

v_c = *vector of constraint residuals;*
C = *matrix of constraint parameter coefficients;*
Δ = *vector of parameters used in constraints;*
g = *vector of constraint constants;*

One constraint equation is written for each physical or geometric property that is required to be enforced.

The base constraint

For stereo photography the required constraint is that the computed base distances for all stereopairs be the same. As the camera base is unknown a priori the constraint constant (the camera base distance) must be derived from the adjustment. At each iteration the mean value of all the bases is computed and used as the constraint constant. The base constraint for stereopair i is then:

$$[(X_i^L - X_i^R)^2 + (Y_i^L - Y_i^R)^2 + (Z_i^L - Z_i^R)^2]^{1/2} = B_m \qquad (3)$$

B_m = *mean camera base.*

There is one base constraint equation for each stereopair.

The convergence constraint

The requirement here is that the relative orientations of the camera axes are constant for all stereopairs. This may be achieved by constraining the convergence angles of the three pairs of camera axes (dot products of each pair of X, Y & Z axes) to the mean of the corresponding convergence angles for all stereopairs. As the convergence angles are independent for each axis pair there is one constraint equation for each convergence angle. For stereopair i these are:

$$\begin{aligned} R11_i^L.R11_i^R + R12_i^L.R12_i^R + R13_i^L.R13_i^R &= \gamma X_m \\ R21_i^L.R21_i^R + R22_i^L.R22_i^R + R23_i^L.R23_i^R &= \gamma Y_m \\ R31_i^L.R31_i^R + R32_i^L.R32_i^R + R33_i^L.R33_i^R &= \gamma Z_m \end{aligned} \qquad (4)$$

$R11_i^L..R33_i^R$ = *rotation matrix elements of the left and right cameras of stereopair i;*
$\gamma X_m, \gamma Y_m, \gamma Z_m$ = *mean convergence angles for X, Y and Z axes.*

All constraint equations are non-linear in terms of the camera parameters and must be linearised. Adding the linearised constraint equations to the conventional collinearity equation model changes equation (1) to:

$$\bar{V} + \bar{B}\Delta = \bar{C} \qquad (5)$$

$$\bar{V} = \begin{bmatrix} V \\ \dot{V} \\ \ddot{V} \\ V_c \end{bmatrix}; \quad \bar{B} = \begin{bmatrix} \dot{B} & \ddot{B} \\ -I & 0 \\ 0 & -I \\ C & 0 \end{bmatrix}; \quad \Delta = \begin{bmatrix} \dot{\Delta} \\ \ddot{\Delta} \end{bmatrix}; \quad \bar{C} = \begin{bmatrix} \epsilon \\ \dot{C} \\ \ddot{C} \\ g \end{bmatrix}.$$

C = *matrix of partial derivatives of constraint parameters;*
g = *vector of constraint discrepancies;*

The corresponding normal equation matrix structure is shown in figure 2.

Each stereopair is represented by a 12x12 symmetric sub-matrix as a result of the constraint equations. The number of camera parameters is unaltered from the conventional model however there are 4**s** additional constraint equations which will improve this model's degrees of freedom over the conventional model.

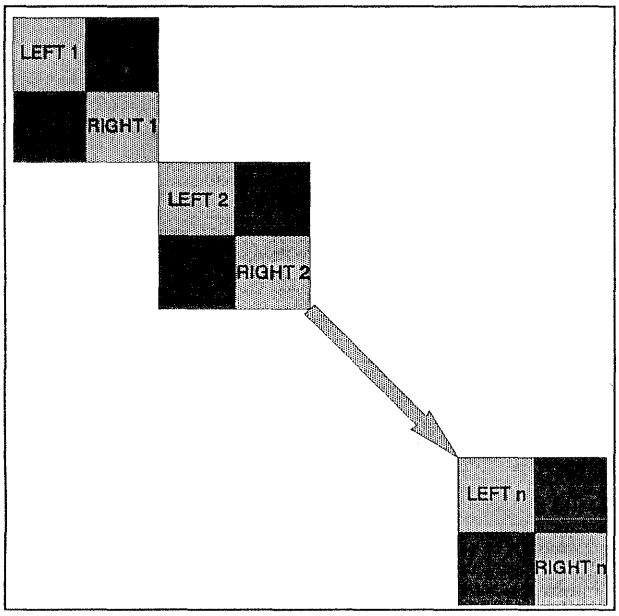

Figure 2 Normal equation structure of the camera parameter portion of the constrained bundle adjustment of stereopairs.

INVARIANCE MODELLED BY MODIFIED COLLINEARITY EQUATIONS

This approach sees the left camera of each stereopair being modelled by conventional collinearity equations and the right camera is modelled by modified collinearity equations written in terms of the left camera coordinate system instead of the object coordinate system. Therefore the position and orientation of the right camera is determined with respect to the left camera. The result is to reduce the number of camera parameters from 12**s** to 6**s**+6 for **s** stereopairs.

The set of modified collinearity equations for the right camera are developed below. Subscripts define the object (L = left camera, R = right camera, P = object point), superscripts define the coordinate system (none = object space coordinate system, L = left camera coordinate system, R = right camera coordinate system), $\tilde{x}$ = vector of plate coordinates, s = point scale factor, $\tilde{R}$ = camera rotation matrix and $\tilde{X}$ = vector of coordinates.

Points on the Left image in object space coordinates:

$$\tilde{x}^L = s_L\tilde{R}_L(\tilde{X}_P-\tilde{X}_L) \tag{6}$$

Points on the Left image in Left camera coordinates:

$$\tilde{x}^L = s_L\tilde{X}_P^L \tag{7}$$

from which:

$$\tilde{X}_P^L = \tilde{R}_L(\tilde{X}_P-\tilde{X}_L) \tag{8}$$

Points on the Right image in object space coordinates:

$$\tilde{x}^R = s_R\tilde{R}_R(\tilde{X}_P-\tilde{X}_R) \tag{9}$$

Points on the Right image in Left camera coordinates:

$$\tilde{x}^R = s_R\tilde{R}_R^L(\tilde{X}_P-\tilde{X}_R^L) \tag{10}$$

and so plate coordinates of the right hand camera expressed in terms of the left hand camera's coordinate system:

$$\tilde{x}^R = s_R\tilde{R}_R^L[\tilde{R}_L(\tilde{X}_P-\tilde{X}_R)-\tilde{X}_R^L] \tag{11}$$

Expressed in the more conventional form:

$$\begin{aligned} x_r &= -c_r\frac{r'_{11}(P-X')+r'_{12}(Q-Y')+r'_{13}(R-Z')}{r'_{31}(P-X')+r'_{32}(Q-Y')+r'_{33}(R-Z')} \\ y_r &= -c_r\frac{r'_{21}(P-X')+r'_{22}(Q-Y')+r'_{23}(R-Z')}{r'_{31}(P-X')+r'_{32}(Q-Y')+r'_{33}(R-Z')} \end{aligned} \tag{12}$$

c_r = *principal distance of right camera*;
$r'_{11}...r'_{33}$ = *rotation matrix elements of right camera in left camera coordinates*;
X',Y',Z' = *X,Y,&Z coordinates of right camera perspective centre in left camera coordinates*;
$P = r_{11}(X_P-X_L)+r_{12}(Y_P-Y_L)+r_{13}(Z_P-Z_L)$;
$Q = r_{12}(X_P-X_L)+r_{22}(Y_P-Y_L)+r_{23}(Z_P-Z_L)$;
$R = r_{31}(X_P-X_L)+r_{32}(X_P-X_L)+r_{33}(Z_P-Z_L)$;
$r_{11}...r_{33}$ = *rotation matrix elements of left camera in object coordinates*;
X_P,Y_P,Z_P = *object coordinates of imaged point*;
X_L,Y_L,Z_L = *object coordinates of left camera perspective centre.*

The resulting set of observation equations is:

$$\bar{V}+\bar{B}\Delta = \bar{C} \tag{13}$$

$$\bar{V} = \begin{bmatrix} V \\ \dot{V}_L \\ \dot{V}_R \\ \ddot{V} \end{bmatrix};\quad \bar{B} = \begin{bmatrix} \dot{B}_L & \dot{B}_R & \ddot{B} \\ -I & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & -I \end{bmatrix};\quad \Delta = \begin{bmatrix} \dot{\Delta}_L \\ \dot{\Delta}_R \\ \ddot{\Delta} \end{bmatrix};\quad \bar{C} = \begin{bmatrix} \epsilon \\ \dot{C}_L \\ \dot{C}_R \\ \ddot{C} \end{bmatrix}.$$

V = *vector of plate observation residuals*;
$\dot{V}_L$ = *vector of left camera exterior orientation observation residuals*;

$\dot{V}_R$ = *vector of right camera exterior orientation observation residuals;*

$\ddot{V}$ = *vector of object coordinate observation residuals;*

$\dot{B}_L$ = *matrix of left camera exterior orientation partial derivatives;*

$\dot{B}_R$ = *matrix of right camera exterior orientation partial derivatives;*

$\ddot{B}$ = *matrix of point coordinate partial derivatives;*

$\dot{\Delta}_L$ = *vector of left camera exterior orientation parameter corrections;*

$\dot{\Delta}_R$ = *vector of right camera exterior orientation parameter corrections;*

$\ddot{\Delta}$ = *vector of point coordinate corrections;*

ϵ = *discrepancy vector of plate observations;*

$\dot{C}_L$ = *discrepancy vector of left camera exterior orientation parameters;*

$\dot{C}_R$ = *discrepancy vector of right camera exterior orientation parameters;*

$\ddot{C}$ = *discrepancy vector of object coordinates.*

The normal equation matrix structure for this model is shown in figure 3.

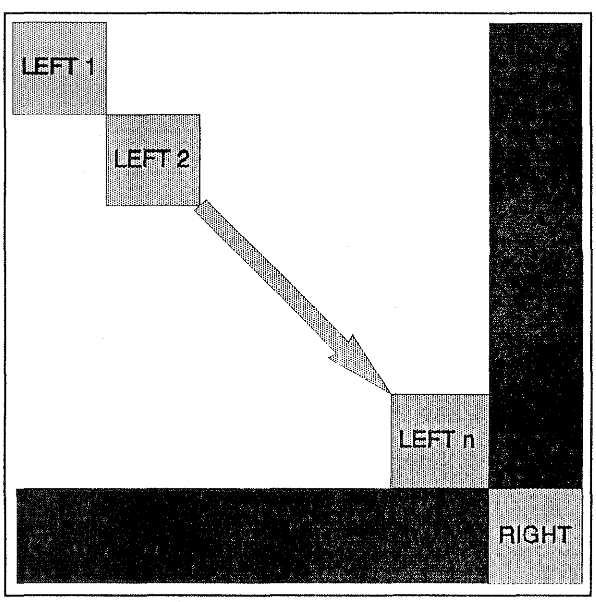

Figure 3 Normal equation structure of the camera parameter portion of the modified collinearity equation bundle adjustment of stereopairs.

Each left camera of a stereopair is represented by a 6x6 symmetric sub-matrix generated by the conventional collinearity equations. The relationship between the left and right cameras is represented by one 6x6 symmetric sub-matrix and two 6**s**x6 sub-matrix generated by the modified collinearity equations for the right cameras. The number of camera parameters is reduced to 6s+6 compared to the conventional model however this model's degrees of freedom is the same as the conventional model.

Object space orientation and position of the right cameras of each stereopair can be computed from the following relationships by back substitution after the adjustment is completed from:

$$\begin{aligned} \tilde{X}_R &= \tilde{R}_R \Delta \tilde{X}_P^R \\ &= \tilde{R}'_R \tilde{R}_L \Delta \tilde{X}_P^R \\ \therefore \tilde{R}_R &= \tilde{R}'_R \tilde{R}_L \end{aligned} \tag{14}$$

$\tilde{X}_R$ = *vector of right camera coordinates;*

$\tilde{R}_R$ = *rotation matrix of right camera in object coordinates;*

$\Delta \tilde{X}_P^R$ = *vector of object/right camera coordinate differences in right camera coordinates;*

$\tilde{R}'_R$ = *rotation matrix of right camera in left camera coordinates*

$$\begin{aligned} \tilde{X}_L &= \tilde{R}_L \tilde{X} \\ \therefore \tilde{X} &= \tilde{R}_L^{-1} \tilde{X}_L \\ &= \tilde{R}_L^T \tilde{X}_L \end{aligned} \tag{15}$$

$\tilde{X}_L$ = *vector of left camera coordinates*

$\tilde{X}$ = *vector of object coordinates*

$\tilde{R}_L$ = *rotation matrix of left camera in object coordinates*

TEST DATA

Four stereopairs were taken of a simple object. All points on the object were coordinated by theodolite intersection to a precision of ±0.1mm in each of the three coordinate axes. Two stereopairs had horizontal camera bases and two vertical. A total of 19 points were observed on all images with an ADAM Technology MPS-2 Micro Photogrammetric System.

The photographs were taken by two Canon AE-1 Program cameras mounted on a bar with a nominal base of 0.200m. The object filled the image frame at a distance of approximately 1 metre giving a base:height of 1:5. The MPS-2 and both cameras were calibrated by the ADAM software prior to the observations being made. Approximations of the camera parameters for each stereopair were obtained from the ADAM software. Refined approximations using all stereopairs were obtained from a simple bundle adjustment using unweighted plate observations. A self-calibrating bundle adjustment (Fraser 1982) showed that gross image correction parameters Δf (camera principal distance correction) and K_1 (first radial lens distortion coefficient) were statistically not significant in the plate observations.

This data was processed by three adjustment models:

a. a CONVENTIONAL bundle adjustment;
b. the CONSTRAINED bundle adjustment and;
c. the MODIFIED collinearity equation model.

Termination of each adjustment was controlled by either the change in the reference variance or magnitude of parameter corrections reaching specified limits. Tables 1 and 2 give the results of the three adjustments with the object points treated as control (Table 1) and as unknown (Table 2). The precision of the quoted results reflects the magnitude of the parameter correction convergence criteria.

RESULTS

Basis of comparison are:

1. model redundancy;
2. camera axis convergences and base;
3. reference variance;
4. plate observation RMS residuals;
5. object coordinate RMS errors;
6. object coordinate RMS standard error ellipsoid:
7. camera position RMS standard error ellipsoid;

1. The degrees of freedom is increased in the CONSTRAINED solution over the other two due to the nature of the constraint equations.

2. The results for the camera base and orientation are the same for each treatment by the CONSTRAINED model but differs for the other two. At this stage it is not clear whether this is due to the action of the constraint equations or is merely coincidental. The MODIFIED model has larger base and convergence than the CONVENTIONAL model's mean values. The CONSTRAINED model has smaller base and convergence angles than the CONVENTIONAL model's mean values.

3. The reference variance increases from the CONVENTIONAL to the CONSTRAINED to the MODIFIED models indicating progressively less flexible modelling. This should be expected as the CONSTRAINED and MODIFIED models effectively force the cameras into a fixed relationship whereas the CONVENTIONAL model allows the cameras to find optimum positions. Any constraining of the cameras will result in an increase in the plate residuals (see point 4.) and thus the reference variance.

4. Plate observation RMS residuals of the CONVENTIONAL model reduced from the control to the unknown treatment as the points are free to find optimum locations. There was an expected increase in the RMS values for the other two models, however there was no change between the RMS values of the two treatments. An increase over the CONVENTIONAL model is to be expected as the cameras are not free to find optimum individual positions.

5. Using the RMS point coordinate residuals as an indicator of accuracy the CONSTRAINED and MODIFIED models yield better results for both treatments than the CONVENTIONAL. Of the two new models presented the CONSTRAINED model producing the better results. For the CONVENTIONAL model there was an average of 560% increase in the RMS values from the control to the unknown treatments compared to only a 270% increase for the other two models.

6. Using the RMS point standard error ellipsoids as an indicator of PRECISION the CONSTRAINED and MODIFIED models are similar but worse than the CONVENTIONAL model for both treatments. The CONSTRAINED model is slightly more precise than the MODIFIED model.

NOTE: these values are generated from the inverted normal equation matrix multiplied by the reference variance. Any reduction in the reference variance of each model by better weight selection may correspondingly reduce these figures. The weights used for the CONVENTIONAL model were kept for the other models.

7. There is an increase in the camera position RMS standard error ellipsoid for each model with the MODIFIED model having the highest values. In the MODIFIED model the right ellipsoid is relative to the left ellipsoid.

CONCLUSIONS

Two bundle adjustment models optimised for use with stereo photography have been developed. Initial testing of these models indicate that they both yield a greater accuracy than a conventional bundle adjustment using the accuracy of the object coordinates as that indicator. Using the error ellipsoids of the object points as an indicator of precision the two models presented do not compare as favourably with the conventional model.

The constrained model has a higher degree of freedom than the conventional model due to the addition of the constraint equations. It also produces the same stereo camera invariant parameters whether the object points are treated as control or unknown.

The modified model requires fewer parameters to be solved for than the conventional model giving a savings on computational power.

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TABLE 1
CONVENTIONAL, CONSTRAINED and MODIFIED BUNDLE ADJUSTMENT RESULTS
POINTS TREATED AS CONTROL
CONVENTIONAL and MODIFIED DEGREES of FREEDOM = 361
CONSTRAINED DEGREES of FREEDOM = 377

ADJUSTMENT TYPE		CONVENTIONAL	CONSTRAINED	MODIFIED	
BASE (mm)		Mean 198.4	197.4	204.1	
X AXIS CONVERGENCE (° ' ")		Mean 11 07 07.4	11 02 58.4	11 26 46.1	
Y AXIS CONVERGENCE (° ' ")		Mean 0 20 09.8	0 19 41.7	0 25 47.2	
Z AXIS CONVERGENCE (° ' ")		Mean 11 06 48.2	11 02 44.3	11 26 29.2	
s_o^2		1.021	2.317	2.697	
RMS PLATE	x	0.021	0.039	0.037	
RESIDUALS (mm)	y	0.021	0.027	0.034	
RMS POINT COORDINATE	X	0.398	0.262	0.329	
RESIDUALS (mm)	Y	0.489	0.218	0.386	
	Z	0.312	0.283	0.297	
RMS POINT STANDARD	X'	0.223	0.308	0.358	
ERROR ELLIPSOID (mm)	Y'	0.324	0.483	0.523	
	Z'	0.216	0.314	0.352	
RMS CAMERA STANDARD	X'	1.320	1.702	L 1.966	R 1.014
ERROR ELLIPSOID (mm)	Y'	1.083	1.036	1.396	0.837
	Z'	1.517	1.958	2.180	0.630

TABLE 2
CONVENTIONAL, CONSTRAINED and MODIFIED BUNDLE ADJUSTMENT RESULTS
POINTS TREATED AS UNKNOWN
CONVENTIONAL and MODIFIED DEGREES of FREEDOM = 304
CONSTRAINED DEGREES of FREEDOM = 320

ADJUSTMENT TYPE		CONVENTIONAL	CONSTRAINED	MODIFIED	
BASE (mm)		Mean 201.4	197.4	205.7	
X AXIS CONVERGENCE (° ' ")		Mean 11 24 15.4	11 02 58.4	11 34 56.2	
Y AXIS CONVERGENCE (° ' ")		Mean 0 24 54.1	0 19 37.0	0 26 23.6	
Z AXIS CONVERGENCE (° ' ")		Mean 11 23 51.2	11 02 44.3	11 34 39.6	
s_o^2		0.7000	2.668	2.966	
RMS PLATE	x	0.017	0.039	0.037	
RESIDUALS (mm)	y	0.016	0.027	0.032	
RMS POINT COORDINATE	X	1.278	0.356	0.885	
RESIDUALS (mm)	Y	3.796	1.234	1.933	
	Z	0.887	0.339	0.469	
RMS POINT STANDARD	X'	9.845	17.860	22.155	
ERROR ELLIPSOID (mm)	Y'	9.645	17.908	19.430	
	Z'	9.398	17.896	20.184	
RMS CAMERA STANDARD	X'	10.269	20.026	L 23.778	R 14.676
ERROR ELLIPSOID (mm)	Y'	26.585	17.851	69.700	0.886
	Z'	10.180	19.865	23.453	0.669