

# ULTRA-PRECISE MEASUREMENT USING AFFINE TRANSFORMATION

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## ABSTRACT

Affine transformation plays a minor role in photogrammetry. However, this transformation shows great effectiveness in the cases where projective transformation is very weak. This paper presents a general orientation theory of two-dimensional affine images and clarifies characteristics of the model construction and the one-to-one correspondence between the model and object spaces. Next, a general free network theory for affine images is constructed with twelve linearly independent vectors. Practical characteristics of the proposed theories are discussed through tests with simulated examples.

## INTRODUCTION

Affine imageries do not exist in reality. For this reason the orientation theory of affine images has probably not been derived until now. However, this orientation theory is of great practical use in many photogrammetric fields such as precise three-dimensional measurement of small objects using a conventional comparator and the analysis of satellite CCD camera imageries. In the latter case the conventional orientation approach of photographs cannot be employed, because the field angle of the CCD camera is very narrow and height differences in the photographed terrain are very small for the flying height of the satellite. Therefore, this paper derives a general orientation theory of two-dimensional affine images and clarifies the geometrical properties of the relative and absolute orientation. Then, a general free network theory for affine images is constructed. The theories proposed here are tested with simulated examples so as to explore the difficulties when applying them to practical cases.

## GENERAL ORIENTATION THEORY OF TWO-DIMENSIONAL AFFINE IMAGES

Let a three-dimensional object space  $(X, Y, Z)$  be projected into a plane based on affine transformation (See Figure-1.). The basic equations relating

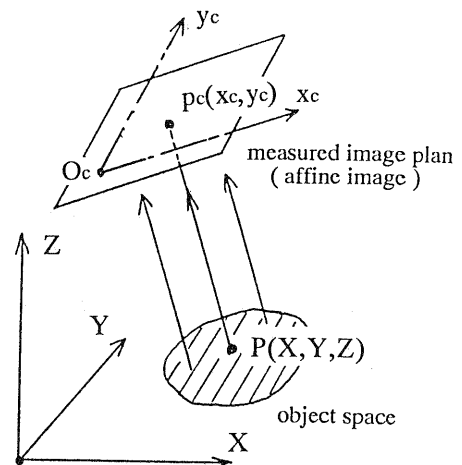


Figure-1 : parallel projection of an object space into the measured plane of the comparator coordinate system .

an object point  $P$  and its measured image point  $p_c(x_c, y_c)$  are described (Okamoto (1989)) as

$$\begin{aligned} x_c &= a_1X + a_2Y + a_3Z + a_4 \\ y_c &= a_5X + a_6Y + a_7Z + a_8 \end{aligned} \quad (1)$$

in which  $a_i$  ( $i=1, \dots, 8$ ) are independent coefficients. Geometrically, the eight orientation parameters of the affine image are considered to be three rotation parameters  $(\omega, \varphi, \kappa)$  of the image, two translation elements  $(X_{oc}, Y_{oc})$  which indicate two of the three-dimensional coordinates of the origin of the measured image coordinate system  $(x_c, y_c)$  with respect to the object space coordinate system  $(X, Y, Z)$ , the image scale  $s$ , and two rotation parameters  $(\alpha, \beta)$  describing the relationship between projected rays and the normal to the image plane. The eight orientation parameters of a single affine image can thus be provided uniquely if four control points are available.

Next, we will consider the orientation problem of a stereopair of two-dimensional affine images (See Figure-2.). The basic equations are written down as

$$x_{c1} = a_{11}X + a_{12}Y + a_{13}Z + a_{14} \quad (2)$$

$$y_{c1} = a_{15}X + a_{16}Y + a_{17}Z + a_{18}$$

for the left image, and in the form

$$x_{c2} = a_{21}X + a_{22}Y + a_{23}Z + a_{24} \quad (3)$$

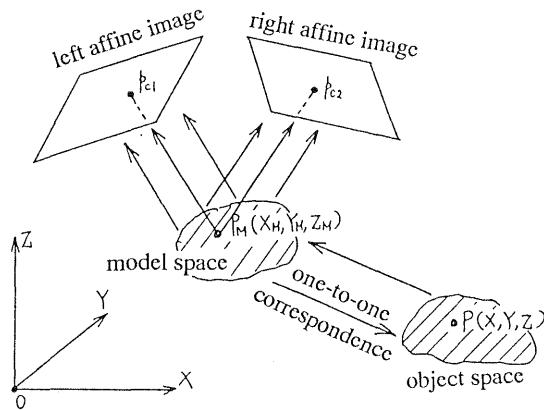


Figure-2 : relative and absolute orientation of a stereopair of affine images

$$y_{c2} = a_{25}X + a_{26}Y + a_{27}Z + a_{28}$$

for the right one, respectively. The condition that Equations 1 and 2 are valid for all object points photographed in common on the left and right images can be formulated as

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} - x_{c1} \\ a_{15} & a_{16} & a_{17} & a_{18} - y_{c1} \\ a_{21} & a_{22} & a_{23} & a_{24} - x_{c2} \\ a_{25} & a_{26} & a_{27} & a_{28} - y_{c2} \end{vmatrix} = 0 \quad (4)$$

which is equivalent to the coplanarity condition of corresponding rays. Under the condition of Equation 4 we can form a three-dimensional space  $(X_M, Y_M, Z_M)$  which can be transformed into the object space  $(X, Y, Z)$  by a three-dimensional affine transformation having 12 independent coefficients, i.e.,

$$\begin{aligned} X_M &= B_1X + B_2Y + B_3Z + B_4 \\ Y_M &= B_5X + B_6Y + B_7Z + B_8 \\ Z_M &= B_9X + B_{10}Y + B_{11}Z + B_{12} \end{aligned} \quad (5)$$

Also, the space  $(X_M, Y_M, Z_M)$  is equivalent to the model space. From the results obtained above we can find the following characteristics of the orientation problem of overlapped affine images:

- 1) The coplanarity condition of corresponding rays can mathematically provide four orientation parameters among the sixteen ones of the stereopair of affine images, and
- 2) The one-to-one correspondence relating the model and object spaces can be uniquely determined, if four control points are given in the object space.

## GENERAL FREE NETWORK THEORY FOR AFFINE IMAGES

In the simultaneous determination of both orientation parameters of overlapped affine images and coordinates of object points, the basic equations are given in the form of Equation 1 (Okamoto and Akamatu(1991)). These basic equations can be linearized as

$$x_c = \bar{x}_c + X\Delta A_1 + Y\Delta A_2 + Z\Delta A_3 + 1\Delta A_4 + A_1\Delta X + A_2\Delta Y + A_3\Delta Z \quad (6)$$

$$y_c = \bar{y}_c + X\Delta A_5 + Y\Delta A_6 + Z\Delta A_7 + 1\Delta A_8 + A_5\Delta X + A_6\Delta Y + A_7\Delta Z$$

Setting up Equation 6 for all affine images under consideration simultaneously, we have a system of linear equations in a matrix form as

$$A \cdot \Delta x = c \quad (7)$$

in which

**A** : a coefficient matrix of the system of linear equations

$\Delta x$  : a vector of corrections to unknowns

**c** : a vector of constants.

Without object space controls the matrix **A** is singular and its rank deficiency is twelve. In the general free network theory for affine images we have therefore twelve linearly independent vectors satisfying the following relationship:

$$A G = 0 \quad (8)$$

where **G** is a matrix constructed from the 12 linearly independent vectors, i.e.,

$$G = (g_1, g_2, \dots, g_{12}) \quad (9)$$

$$\begin{aligned} g_1^T &= (A_1, 0, 0, 0, A_5, 0, 0, 0, -X, 0, 0) \\ g_2^T &= (0, A_1, 0, 0, 0, A_5, 0, 0, -Y, 0, 0) \\ g_3^T &= (0, 0, A_1, 0, 0, 0, A_5, 0, -Z, 0, 0) \\ g_4^T &= (0, 0, 0, A_1, 0, 0, 0, A_5, 1, 0, 0) \\ g_5^T &= (A_2, 0, 0, 0, A_6, 0, 0, 0, 0, -X, 0) \\ g_6^T &= (0, A_2, 0, 0, 0, A_6, 0, 0, 0, -Y, 0) \\ g_7^T &= (0, 0, A_2, 0, 0, 0, A_6, 0, 0, -Z, 0) \\ g_8^T &= (0, 0, 0, A_2, 0, 0, 0, A_6, 0, 1, 0) \\ g_9^T &= (A_3, 0, 0, 0, A_7, 0, 0, 0, 0, 0, -X) \\ g_{10}^T &= (0, A_3, 0, 0, 0, A_7, 0, 0, 0, 0, -Y) \\ g_{11}^T &= (0, 0, A_3, 0, 0, 0, A_7, 0, 0, 0, -Z) \\ g_{12}^T &= (0, 0, 0, A_3, 0, 0, 0, A_7, 0, 0, 1) \end{aligned} \quad (10)$$

These 12 vectors can easily be found by linearizing the three-dimensional affine transformation (Equation 5) and expressed in the form of Equation 10. Seven of the twelve vectors are related to the three-dimensional similarity transformation and the last five vectors pertain to a model deformation.

If measured image coordinates  $(x_c, y_c)$  have random errors, the system of linearized observation equations can be described as

$$v = A\Delta\hat{x} - L \quad (11)$$

where  $v$  is a vector of residuals to the observations. The free network adjustment is then carried out in a following manner (Ebner (1974))

$$v^T P v \rightarrow \min$$

under the condition  $G^T \Delta\hat{x} = 0$  (12)

where  $P$  is a weight matrix of the observations.

### TESTS WITH SIMULATION MODELS

#### ULTRA-PRECISE THREE-DIMENSIONAL MEASUREMENT OF SMALL OBJECTS USING A CONVENTIONAL COMPARATOR

The derived orientation theory of overlapped affine images was tested with simulated examples. For the construction of simulation models a stereopair of affine image planes of small objects measured by a comparator were assumed to be employed and the image coordinates of 25 object points were calculated by means of the convention-

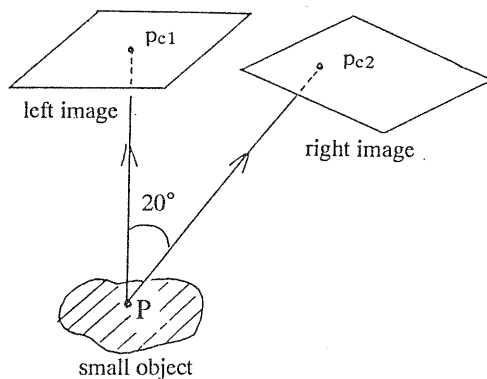


Figure-3 : a stereopair of affine images employed for the ultra-precise measurement of small objects using a comparator

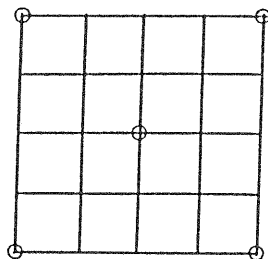


Figure-4 : arrangement of check and control points

al three-dimensional coordinate transformation equations under the following conditions (See Figures-3 and 4.):

- size of small objects :  $100 \times 100 \times 50$  mm ,  
 $10 \times 10 \times 0.15$  mm
- convergent angles : 0deg., 20deg.
- number of control points : 5
- number of check points : 20

Then, the perturbed image coordinates were provided in which the perturbation consisted of random normal deviates having standard deviations of 5 micrometers for the first object and 1 micrometer for the second very small and thin object, respectively.

The obtained results regarding the average internal error at the 20 check points and the average external error are shown in Table-1. We can find in Table-1 the following characteristics of the orientation problem of overlapped affine images:

- 1) Ultra-precise three-dimensional measurement of small objects can really be performed by a conventional comparator using the proposed orientation theory, and
- 2) The second object is very small and thin. However, the obtained external accuracy is rather high. This means that affine transformation can be effectively applied to the analysis of satellite CCD camera imageries, because the satellite CCD camera has a very narrow field angle and because height differences in the photographed terrains are small in comparison with the flying height of the platform.

	small object	very small and thin object
average internal error	7.4 $\mu$ m	1.4 $\mu$ m
average external error	11.2 $\mu$ m	1.9 $\mu$ m

Table-1 : the measurement results of small objects using a conventional comparator

### FREE NETWORK ADJUSTMENT

In order to explore the practical characteristics of the proposed free network theory for affine images, two simulation models were constructed under the following conditions (See Figures-5 and 6.):

#### Simulation Model I

- size of object :  $100 \times 100 \times 50$  mm
- number of overlapped affine images : 2
- convergent angles : -30deg., +30deg.
- standard error of measured image coordinates : 5micrometers
- number of control points : 5
- number of check points : 20

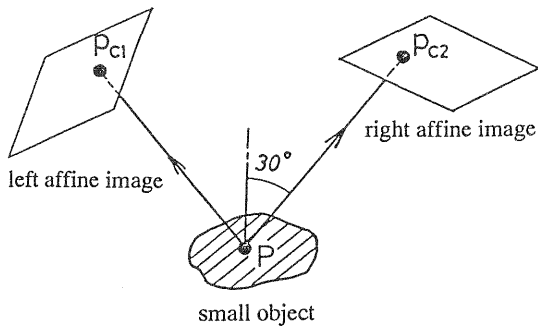


Figure-5 : a stereopair of affine images employed for the free network adjustment

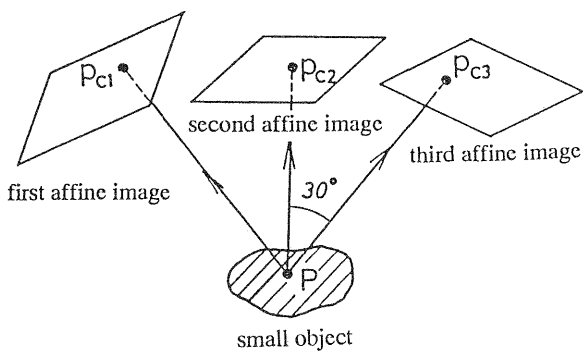


Figure-6 : three overlapped affine images employed for the free network adjustment

### Simulation Model II

size of object :  $100 \times 100 \times 50$  mm  
 number of overlapped affine images : 3  
 convergent angles :  $-30\text{deg.}, 0\text{deg.}, +30\text{deg.}$   
 standard error of measured image coordinates : 5micrometers  
 number of control points : 5  
 number of check points : 20

The free network theory is essentially a linear theory. Thus, we must have fairly good approximations for unknowns, because the basic equations (Equation 1) are non-linear with respect to the orientation parameters and object space coordinates. Further, the free network solution needs an iterative approach in order to obtain a very high external accuracy. This comes from the fact that the solutions obtained in the free network adjustment are not unbiased estimates of both the orientation parameters at the exposure instants and the real object space coordinates but depend mathematically on the given approximations. In this method, the first approximations were calculated by contaminating the control point coordinates with random errors having a standard deviation of 15 microme-

ters. Then, the iterative calculation of the free network adjustment was performed by regarding the solutions obtained in the (i-1)th step as the approximations in the i-th step and replacing only the approximations of the control point coordinates by the true values.

The obtained results regarding the standard error of unit weight, the average internal error at the 20 check points and the average external error are shown in Tables-2 and 3. From these tables the following characteristics may be extracted for the free network adjustment of overlapped affine images:

- 1) The geometry of the stereopair of affine images in Simulation Model I is rather weak. Thus, the average internal and external errors of the particular solutions with fixed control points are about twice as large as the theoretic-

	particular solution	free network solution
standard error of unit weight	5.3 $\mu\text{m}$	5.3 $\mu\text{m}$
average internal error	13.3 $\mu\text{m}$	7.7 $\mu\text{m}$
average external error	11.2 $\mu\text{m}$	9.7 $\mu\text{m}$

Table-2 : the obtained results for Simulation Model I

	particular solution	free network solution
standard error of unit weight	4.7 $\mu\text{m}$	4.7 $\mu\text{m}$
average internal error	7.1 $\mu\text{m}$	4.1 $\mu\text{m}$
average external error	6.2 $\mu\text{m}$	5.8 $\mu\text{m}$

Table-3 : the obtained results for Simulation Model II

cal ones. However, employing three overlapped affine images, the obtained average external error of the particular solutions is almost identical to the theoretical one.

- 2) Applying the free network adjustment, great improvements are recognized in the internal precision. On the other hand, the improvements in the external precision are 13 percents for the weak geometry of the stereopair of affine images and only six percents for the strong geometry.
- 3) The solution sometimes diverged when very high weights are given on the free network constraints(Equation 12).

## CONCLUDING REMARKS

In this paper a general orientation theory of two-dimensional affine imageries has been derived for the ultra-precise three-dimensional measurement by a conventional comparator and for the analysis of satellite CCD camera imageries having a very narrow field angle and very small height differences in the photographed terrains. Also, a general free network theory for affine imageries has been constructed. Tests with simulated examples clarified that the proposed theories were mathematically sound and useful for the analysis of two-dimensional affine imageries.

## REFERENCES

- /1/ Ebner. H.: Analysis of Covariance Matrices. Deutsche Geodaetische Kommission, Series B, No.214, (1974).
- /2/ Okamoto. A.: Orientation Theory for Affine Imageries(in Japanese). In Proceedings of the Fall Congress of the Japan Society of Photogrammetry and Remote Sensing, (1989), pp.73-76.
- /3/ Okamoto. A., Akamatu. S.: Free Network Theory for Affine Imageries(in Japanese). In Proceedings of the Fall Congress of Japan Society of Photogrammetry and Remote Sensing, (1991), pp.37-40.