RECENT DEVELOPMENTS IN CAMERA CALIBRATION FOR CLOSE–RANGE APPLICATIONS

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ABSTRACT

A review of calibration techniques for small format cameras is presented. A model for camera and lens calibration is assessed, parameter by parameter. The importance of each of the parameters for lens and camera calibrations is reviewed, in conjunction with their contribution to systematic errors. Methods for reducing these error sources are indicated.

The effects of fiducial marks and film unflatness on camera and lens calibration are reviewed. Recent research results are referenced throughout the discussion and hints are provided to new users of close–range photogrammetry.

KEYWORDS: Small Format; Camera Calibration; Lens Distortions; Film Unflatness.

1. INTRODUCTION

The basic formulae for analytical photogrammetry can be expressed in several forms. The model used for self–calibration is a convenient one to introduce the recent developments in techniques and research reported in this paper. Explicitly, the self calibration form can be written as

$$x_{ij} - x_p + \frac{(x_{ij} - x_p)}{r}\,\delta r + \Delta x$$

$$= c_x \frac{(X_j - X_{0i})\,m_{11} + (Y_j - Y_{0i})\,m_{12} + (Z_j - Z_{0i})\,m_{13}}{(X_j - X_{0i})\,m_{31} + (Y_j - Y_{0i})\,m_{32} + (Z_j - Z_{0i})\,m_{33}} + dx_{ap} \tag{1}$$

$$y_{ij} - y_p + \frac{(y_{ij} - y_p)}{r}\,\delta r + \Delta y$$

$$= c_y \frac{(X_j - X_{0i})\,m_{21} + (Y_j - Y_{0i})\,m_{22} + (Z_j - Z_{0i})\,m_{23}}{(X_j - X_{0i})\,m_{31} + (Y_j - Y_{0i})\,m_{32} + (Z_j - Z_{0i})\,m_{33}} + dy_{ap} \tag{2}$$

where the subscripts indicate

0: the perspective centre
i: the ith photograph
j: the jth object point

and c_x and c_y are the principal distances derived from the use of the observed image x_{ij} and y_{ij} co–ordinates respectively. Usually these are simplified to a common value c. Also, δr is the radial distortion described by equation (3) and Δx and Δy are the decentering distortions described by equations (5) and (6). The terms $m_{11}, ..., m_{33}$ are the elements of an orthogonal matrix which contains the direction cosines of the three rotational angles; X_{0i}, Y_{0i} and Z_{0i} are the object space co–ordinates of the perspective centre for the photograph i; and X_j, Y_j and Z_j are the co–ordinates of the object points. The radial distortion can be expressed as

$$\delta r = K_1 r^3 + K_2 r^5 + K^3 r^7 + ... \tag{3}$$

where the K's are the coefficients of radial distortion corresponding to infinity forces, δr is in micrometres, r in millimetres and

$$r^2 = (x_{ij} - x_p)^2 + (y_{ij} - y_p)^2 \tag{4}$$

The decentering distortion equations are

$$\Delta x = P_1\left[r^2 + 2(x_{ij} - x_p)^2\right] + 2P_2(x_{ij} - x_p)(y_{ij} - y_p) \tag{5}$$

$$\Delta y = P_2\left[r^2 + 2(y_{ij} - y_p)^2\right] + 2P_1(x_{ij} - x_p)(y_{ij} - y_p) \tag{6}$$

where the P's are the values at infinity focus of the parameters of decentering distortion.

Note that in this brief introduction, the variations of the distortions with focussing and within the depth of field have been ignored and readers desirous of a fuller exposition are referred to Karara (Chapter 5, pp. 56–59, 1989).

The terms dx_{ap} and dy_{ap} have been included in equations (1) and (2) to indicate the use of additional parameters (AP's) which are commonly incorporated in bundle adjustments. A great range and variety of sets of AP's have been proposed by photogrammetrists. Kilpelä (1980) details eight sets of AP's, and Ziemann and El–Hakim (1982) and Murai *et al.*, (1984) have evaluated their effectiveness under different conditions. Although the exact physical meaning of many of the AP terms is unclear, there can be no disputing the effective way by which the systematic errors which remain in image and model co–ordinates after a conventional bundle adjustment can be reduced.

There always exists a temptation to successively add more and more AP's in order to reduce the size of the residuals of the image co–ordinates. Fraser (1982) has shown that for the case of minimally constrained multi–station adjustment, the use of higher degrees of AP's can lead to a serious deterioration in the accuracy of object co–ordinates. Perhaps this phenomenon was best summarised by P.B. Jones (personal communication) who in 1975 stated that *"... given enough mathematical terms, you can describe an elephant!"*.

The AP model shown here in equations (7) and (8) should be sufficient to remove the majority of systematic effects from most close–range adjustments

$$dx_{ap} = a_1 \bar{x}\bar{y} + a_2 \bar{y}^2 + a_3 \bar{x}^2 y + a_4 \bar{x}\bar{y}^2 \tag{7}$$

$$dy_{ap} = b_1 \bar{x} + b_2 \bar{y} + b_3 \bar{x}\bar{y} + b_4 \bar{x}^2 + b_5 \bar{x}^2\bar{y} + b_6 \bar{x}\bar{y}^2 \tag{8}$$

where

$$\bar{x} = x_{ij} - x_p$$

and

$$\bar{y} = y_{ij} - y_p$$

and

$a_1, ..., a_4$ and $b_1, ..., b_6$ are the coefficients of the AP's.

2. LENS DISTORTION

There are a variety of ways by which the parameters of lens distortion may be determined. These methods range from laboratory calibrations using optical arrangements of collimators or precise test ranges bristling with targets, to calibrations made "on–the–job" with the photography of control frames or "self–calibration" techniques which utilise a multi–station approach and a bundle adjustment to extract the details of the camera's lens characteristics.

In industrial applications where the situation usually demands photography from a multi–station situation, the self–calibration technique has proven to be most popular in the last decade. The other technique worthy of further discussion is the analytical plumb–line method which, unlike other methods, only solves for the parameters of lens distortion and not for principal distance nor offsets of the principal point from the intersection of the fiducial axes.

2.1 Self–Calibration

The technique of self–calibration has been briefly described above, but mention should be made of some of the features of this technique. In the "purest" form of self–calibration one would expect approximately four camera stations to be convergently arranged around an object to which 50 to 500 targets may be affixed. Two or more photographs would be taken from each station and it is desirable to roll the camera through 90° between photographs in order to successfully recover x_p and y_p. If convergent photography is not employed, then the object must be non–planar.

Equations (1) and (2) are solved simultaneously to extract the camera calibration data.

In reality, it may be necessary to alter the focus setting of the camera between camera stations and this complication will lead to the photogrammetrist nominating which parameters should be constant (or *block invariant*) or variable from station to station (*block variant*).

A most important feature of a self–calibrating bundle adjustment is that the photogrammetrist need not know any values for object–space control. In fact object space control is only required for the actual object evaluation with the classical minimum of two horizontal and three vertical control points being known.

2.2 Analytical Plumb–Line

The analytical plumb–line method (Brown, 1971) was developed as a rapid practical way to compute lens distortion parameters at a range of focussed magnifications from 5X to 20X. The principle of this technique is the axiom that straight lines in object space should project through a perfect lens as straight lines onto the image plane. Any variations from linearity is attributed to radial or decentering distortion. Two photographs from one camera station are always taken: one referred to as "horizontal" and the other as "vertical" since the camera is rolled through 90° between shots.

The plumb–line technique is presently applied by many researchers as their method for estimating the parameters of lens distortion. It is a technique well suited to automation as line–following is a relatively simple procedure. The Autoset automatic monocomparator developed by Geodetic Services Inc in Florida (e.g., Fraser, 1986) and the small format Adam Technology MPS–2 analytical stereoplotter (Elfick, 1986) have both been programmed to utilise this technique. Fryer and Mason (1989) used the method for the close–range calibration of a video camera with the data capture being via a frame–grabber into a personal computer.

Typically, eight to ten plumb–lines are photographed and, say, 50 points on each line on each photograph are digitised. Approximately 1000 points will then be provided as data observations and only the five parameters of radial and decentering distortion must be solved for in a least squares solution, along with two parameters to define the spatial location of each line.

The extrapolation of the plumb–line technique from a laboratory situation at close–focus to the photography of man–made straight objects such as long glass panels in multi–storey buildings has been reported (Fryer, 1987). The further extrapolation to the use of linear features such as railway–lines for the calibration of aerial cameras has also been demonstrated (Fryer and Goodwin, 1989). Also interesting to note is the extension into aerial camera calibration of the close–range self–calibration technique described in Section 2.1, where convergent aerial photography has been used over a test range which was established on flat terrain (Merchant and Tudhope, 1989).

It is important to realise what the analytical plumb–line technique does not provide: it does not provide a solution for the principal distance c, nor the offsets of the principal point x_p and y_p. The relevance of these parameters is discussed further in the next Section when a comparison of the self–calibration and plumb–line techniques is made.

Finally it should be remembered that the lines which are to be photographed do not have to be "plumb" in any real sense of that word. Straightness is the only criterion, the term "plumb–lines" being derived from the earliest use of the technique when wires with weights attached were used. The vertically of the lines is not a feature of the mathematics involved.

2.3 Comparison of Lens Distortion Techniques

The importance of including an allowance for the parameters of radial and decentering lens distortions has been demonstrated in many studies including those conducted by Karara and Abdel–Aziz (1974) and Murai *et al.,* (1984). These studies showed the effect of radial distortion to be almost an order of magnitude larger than decentering distortion. In both these extensive studies the rms values of the residual plate errors decreased by up to a factor of seven when lens distortions were included. Non–metric cameras were shown to be able to approach the accuracy of metric cameras. The K_1 term of radial distortion is always the most significant, with K_2 and K_3 usually not relevant for lenses in typical small format cameras.

A study by Fryer and Fraser (1986) compared the self–calibration and plumb–line methods of lens calibration on some small format cameras which were to be used both in and out of water and in a watertight housing with a plane glass port. The tests were interesting because of the large range of radial distortion present in the situations examined. For example, a 50 mm Distagon f–4 lens fitted to a Rolleiflex SLX reseau camera in an Aquamarin WKD–SLX/6006 submarine housing produced radial distortions in air of +273.2 µm at r = 30 mm and –1551.0 µm under water. These extremely large ranges of distortions were recovered by each of the two methods with an average difference along the radial distortion profiles of 2 µm and a maximum difference of 4.5 µm at r = 30 mm. Similar close agreement occurred with the testing of a 35 mm Nikonos V camera fitted with a Nikkor 28 mm f–3.5 lens.

The magnitude of the decentering distortion profile for the case of the Rolleiflex camera in the underwater housing was the largest experienced by this author in over ten years of examining photogrammetric cameras. The value of 91 µm at r = 30 mm was undoubtedly a consequence of the 12 mm thick plane glass port in the underwater housing not aligning perpendicularly to the optical axis of the camera. The discrepancy in this value of the decentering distortion profile was only 2.1 µm between methods, and since each technique had a one–sigma error bound of slightly over 1 µm, the result was most satisfying.

When the decentering distortion profiles from the Nikonos camera were examined, a large discrepancy at a radial distance r = 18 mm, of 3 µm from the plumb–line technique versus 37 µm from the self–calibrating bundle adjustment was noted. The explanation lies in the high correlation between the values for the parameters P_1, P_2 and x_p, y_p. In situations where a high degree of projective coupling exists between parameters, the self–calibration model can be thought of as over–parameterised in that either set of parameters can adequately describe this component of the systematic error signal.

To prove this hypothesis of high correlation, another bundle adjustment was run where P_1 and P_2 were constrained to their values computed from the plumb–line technique. The result was large changes of 0.14 mm and 0.33 mm in x_p and y_p and no alteration to the object point co–ordinates. The important conclusion is that camera calibration must be viewed not as an end in itself, but rather as a step towards achieving the goal of obtaining the best possible object point co–ordinates. Similarly, the actual values of the parameters K_1, K_2, K_3, P_1 and P_2 may appear to differ from one determination to another, but the shapes of the profiles of radial and decentering distortion must be examined to see if there is any significant difference.

2.4 Radial Distortion and Stereophotogrammetry

The effects of lens distortions, especially radial, has been acknowledged for decades by mapmakers using aerial cameras (for example, Ekelund, 1956). A textbook on photogrammetry published in 1960 (Hallert, 1960, p. 60), describes the stereoscopic photography of a plane surface and how *"...from measurements of the deformations of the surface, the systematic errors which caused the deformations of the bundle of rays can be determined numerically"*.

In the case of aerial photogrammetry, the radial distortion includes the combined effects of earth curvature and refraction as well as lens distortion. In the close–range situation only the latter is relevant and this discussion has been included in this paper to alert researchers and practitioners involved with close–range stereophotogrammetric situations such as archaelogical, architectural, medical, etc., to a potential error source.

It has been mathematically demonstrated (Fryer and Mitchell, 1987) that a relative orientation may be made on photographs incorporating radial distortion and all y–parallaxes can be removed. No residual x–parallaxes will be present in the corners of the overlap region but an appreciable amount will left undetected in the central region of the model. In fact this amount was shown to be of magnitude $1.25\ b^3\ K_1$, where b is the base distance (in mm) between the left and right hand principal points and K_1 is the first term of radial distortion. Unresolved x–parallaxes are equivalent to a height difference and for a typical 70 mm camera, a heighting error of 14 mm for a camera–object distance of 2 m has been reported.

The physical appearance of this effect for a flat surface, such as a building facade, is to have a "hump" in the middle of the stereomodel. Most check points in relative and absolute orientations are placed near the periphery of stereomodels and this effect therefore will pass undetected. On objects such as a building facade the effect will be detected visually but cannot be eliminated by any amount of repeating the orientation process. A specific radial distortion correction must be applied to the analytical stereoplotter's camera calibration files. If the object under examination is itself a curved surface, for example a human back or an archaelogical artefact, then the effect may not be detected.

This Section was specifically included in this paper to highlight the need for the increasing numbers of non–metric camera users, who may not have a complete understanding of the uncompensated systematic errors which may be present in stereophotogrammetry. to proceed with caution in their use of analytical and digital plotting equipment.

3. COMMENTS ON THE OFFSETS OF THE PRINCIPAL POINTS

The importance, or otherwise, of an exact knowledge of the offsets of the principal point, x_p and y_p, from the intersection of the fiducial axes are examined in this Section. In Section 2.3, the high correlation between decentering distortion and x_p, y_p was demonstrated. In the study described, it was observed on each iteration of a self–calibrating bundle adjustment that the values of P_1, P_2 as opposed to x_p, y_p would alternatively increase and decrease in proportion. The values for the co–ordinates of the object points remained unaltered during this process. When the values of P_1, P_2 were constrained to their values as determined by the plumb–line techniques, x_p and y_p altered by up to 0.33 mm but again the object co–ordinates were unaltered.

Perhaps an important feature of these tests was that neither camera had "proper" fiducial marks, but rather the edges of the format were used to establish pseudo fiducial corners. Film stretch and unflatness have been shown to cause up to 100 µm of difference in distance between corners on 35 mm frames (Donnelly, 1988), so an exact knowledge of x_p, y_p can be purely "academic" and not really essential to achieving accurate co–ordinates on the object.

Some recent (1991) adjustments of photography taken with a 125 mm by 125 mm image format camera (a 1943 F–24 reconnaissance camera which has been refurbished with a 90 mm Nikkor lens) has provided some further discussion on this topic. This camera is fitted with a glass reseau and therefore has fiducial marks. Probably due to the refurbishment procedure, it was noted that the decentering distortion parameters P_1, P_2 were larger than usually expected. The bundle adjustment was re–run with P_1, P_2 constrained and the values for x_p, y_p were computed as –0.42 mm and +0.14 mm respectively. More interestingly, the rms values for the residuals on the six station, twelve photograph solution reduced from 5 µm to 4 µm. The object photographed was a large water storage dam which is almost planar in shape.

Although the precision of the object co–ordinates on the dam wall did not significantly improve, this experience has tempted the author to offer the following tentative advice. For small format non–metric photogrammetric exercises of low to medium accuracy, say < 1:5000, there appears little benefit in incorporating x_p, y_p in bundle adjustments. This is especially so if no fiducial marks are present and the frame edges are used as pseudo fiducials. The decentering distortion parameters P_1, P_2 appear to suffice. On the other hand, for more accurate tasks with medium–sized camera formats and with cameras possessing fiducial marks, the determination of x_p, y_p and their application in conjunction with P_1, P_2, rather than the sole use of P_1, P_2, is recommended. If the values of x_p, y_p approach or exceed 0.5 mm, then their application is also recommended rather than reliance on P_1, P_2 alone.

4. FILM UNFLATNESS AND STRETCH

4.1 Film Deformation From Planar

The mathematics of all analytical photogrammetry is based on the assumption that the image points are co–planar. This implies that the film in the image plane must be flat during exposure. Non–metric cameras usually do not possess a film flattening mechanism and the shape which film takes has been studied by several researchers, notably Fraser (1982) for 70 mm cameras and Donnelly (1988) for 35 mm cameras.

Fraser (1982) used the set of AP's described by equations (7) and (8) to study the film unflatness in a 500 ELM Hasselblad used to photograph a "cube" of targets from four exposure stations arranged for convergent imaging. As increasing numbers of AP's were used, the rms values of the residuals for the plate co–ordinates were reduced, but the rms error of the object point co–ordinates increased. The dilemma of reducing internal precision at the expense of causing a deterioration in object point accuracy due to over–parameterisation was clearly demonstrated.

Donnelly (1988) examined film unflatness in 35 mm cameras, the Canon AE–1 Program camera in particular. He found the film to bulge away from the image plane by approximately 0.6 mm in the centre and assume a shape which was reasonably constant frame to frame throughout the length of the film.

For most 35 mm cameras, the film transport mechanism is similar and uses a system of guide and support rails to constrain the film longitudinally. There are no specific lateral constraints at either end of the frame, although one end is held by the slot in the film cassette and the other by the wind–on transport sprocket.

Donnelly removed the camera back and took comparative photographs on glass plates of a computer drafted grid of 19 vertical and 13 horizontal lines. Comparisons were made between the positions of the grid intersections on the film with those on the glass plates. The vectors of difference in position were approximately radial from the principal point and up to 60 µm at the edges. In other words, the extent of the film which was exposed was approximately 100 µm longer, and 70 µm wider, than the area of the image format.

A further study by Fryer, Kniest and Donnelly (1990) explored the hypothesis "are the radial effects of film unflatness absorbed by the parameters for radial distortion?". The plumb–line method was used to extract radial distortion profiles from both the glass plates and the unflattened film frames. Up to $r = 18$ mm, the difference between the profiles did not exceed 1.5 µm, well within the error budget, and to the initial surprise of the researchers. (At $r = 18$ mm, δr was –183 µm).

Two reasons have been proposed for the closeness of the radial distortion profiles. Firstly, the radial distortion formula, equation (3), is quite insensitive to small changes in radial distance. Even towards the edge of the format area, δr was only changing by 1.5 µm for every 50 µm of radial distance r. Secondly, in the setting up phase, or the interior orientation, the edges of the frame were used as pseudo fiducial marks and an affine transformation performed. Since the vectors showing the difference in position between the unflattened film and glass plates were basically radial, the affine transformation removed the majority of the effect as it would apply in the determination of radial lens distortion. Quite clearly, the hypothesis was refuted and the effects of film unflatness and the parameters of radial lens distortion must be viewed as independent in terms of camera calibration parameters.

4.2 Film Unflatness and AP's

The model for AP's in equation (7) and (8) have been used by Fraser (1982) and also this author to attempt to improve the internal precision of a self–calibrating bundle adjustment and also the accuracy of object point co–ordinates. Fraser found that the coefficients b_1 and b_2 in equation (8) were the most significant in improving the reliability of the adjustment and the object co–ordinates. These AP's are linear terms in x and y which refer to non–orthogonality and affinity. He found that the other AP's were either not statistically significant (that is, did not effectively remove any systematic error signal) or did reduce overall accuracy. Fraser's model was only minimally provided with control points, a situation which is common in many non–metric close–range situations.

In addition to the AP terms used by Fraser, this author has found the term a_1 in equation (7) to be most useful in a variety of adjustments. This is a second order term, hyperbolic in nature, and it is not difficult to visualise its relevance to unflattened film.

4.3 Attempts at Film Flattening

A wide variety of attempts to flatten film in close–range cameras have been made over the last decade. These have ranged from the professional and demonstrably very successful vacuum systems in Geodetic Services Inc's CRC–1 camera (Brown, 1984) to commercially available backs for 35 mm to 70 mm cameras such as the Pentax 645 or the Contax RTS III (Fryer, Kniest and Donnelly, 1992) to experimental vacuum systems such as that described by Donnelly (1988). The alternative to vacuum back systems is the addition of a reseau grid, with or without a pressure plate. One difficulty with reseau systems is the focussing problem caused by the addition of the glass pane between the lens and the image plane. A thin reseau pane is essential, but its location adjacent to the focal plane camera shutter system can pose engineering problems.

The commercial pricing of film flattening devices for small format cameras probably reflects the low level of acceptance and use of these devices by the "amateur" photogrammetric community. The cost of vacuum backs, or reseau plates, seems exorbitant to this author, often more than doubling the price of the original camera and lens combination. Surely this is a reflection of the numbers of units sold on a worldwide basis, each unit representing an almost individual order.

There can be no doubting the improvement in overall accuracy provided by a film flattening device. The references noted earlier in this section show improvements two–fold or better, with the ultra–specific flat vacuum back of the CRC–1 camera coupled with automatic image co–ordinate measurements achieving accuracies up to one part in a million (Fraser, 1992).

Vacuum back system have demonstrated a capability to produce more accurate results than reseau systems. Apart from the difficulties which were noted earlier, another difficulty which has been observed with reseau systems includes lack of contact between the film and the glass plate, probably caused by trapped air. This is identifiable in gross cases since not all reseau crosses will appear sharp (Chandler, Cooper and Robson, 1989). Another problem can arise in the case of backing paper with 120 roll film when air becomes trapped between the paper and film. Lack of flatness of both the reseau plate and the pressure plate has also been identified as a source of error, along with a lack of parallelism between those surfaces.

There is some evidence that film unflatness can cause "larger than expected" errors in object point co–ordinates in configurations with weak geometry, especially in stereophotogrammetric situations (Robson, 1992). In other situations, often where the majority of control and object points lie near the centre of the frame, the effect of out–of–plane deformation is not large. Attempts to model the shape of the deformed film surface, and correct all observed image co–ordinates accordingly, have not yet proved to be successful.

5. PRINCIPAL DISTANCE

In close–range photogrammetric situations, the distinction between focal length and principal distance becomes an important consideration. The convention is that principal distance is the perpendicular distance from the perspective centre of the lens system to the image plane. Focal length is that value of the principal distance which corresponds with infinity focus. The constant c is used in the earlier equations to describe the principal distance at any setting.

The principal distance may be evaluated in a number of ways, with an accuracy of 10 µm to 20 µm being attainable without too much difficulty. A very simple method is to photograph two points spaced equi–distant either side of the camera's axis and distant from camera at the required focus setting. Two targets on a straight fence when the whole set–up is on level ground are ideal. The targets and the fiducial centre must lie on a straight line if no corrections for camera axis tilt is to be applied. The principal distance can then be calculated from the simple geometry of similar triangles once the distances between the imaged targets, the targets themselves and the camera to target have been measured. The only correction which must be applied is for lens distortion and one presumes this has been previously computed by a method such as the plumb–line technique.

If the camera and lens are being calibrated using the self–calibration technique, then from a 3–D target array and/or a convergent multi–station configuration, a value of c will be produced from the adjustment.

The effect of not considering radial distortion when attempting to solve for c has been demonstrated by Webb (1987). With a simple non–metric "point–and–shoot" Canon AF35M camera he reduced the rms value for the plate residuals and the uncertainty in c by a factor of two by considering only the K_1 term. When radial distortion is ignored, the image locations of points in the control field will all be affected by varying amounts of radial distortion and the derived value for c will, consequently, be in error or, at least, have a poor precision.

For much work in close–range photogrammetry, an accurate knowledge of the principal distance is not warranted. Given accurate 3–D control and an estimate of c, the bundle adjustments with AP's will derive the "best" value for that configuration. In situations where stereophotogrammetry is being employed on objects such as building facades which are essentially planar, control points around the periphery will be used to scale the model for the object space and an *a priori* value for c will not be significant. As discussed earlier though, an allowance for lens distortion must be applied in this case.

6. FIDUCIAL MARKS

One of the distinguishing features of non–metric cameras is, often, their lack of fiducial marks. A commonly used technique to overcome this shortcoming is to compute pseudo fiducial corners of the image frame by digitising some points along the edges of the frame during the interior orientation procedure. Lines describing the frame edges are calculated and their intersections computed. The corners of the frame are not directly digitised as they often appear "furry" in nature. Analytical stereoplotters which cater for the small format market possess software to aid this process (for example, the ADAM Technology MPS–2).

The next stage of the interior orientation is, usually, to calculate an affine transformation to define an image co–ordinate system. In this paper, the use of the affine transformation is questioned, in light of several recent tests undertaken by the author and also the experience of Robson (1992). Lens distortions, and film unflatness effects have their largest impact at the frame edges. To "blindly" apply an affine transformation to frame corners which have not been observed, but only calculated, is to transfer and distribute spurious image corrections across the entire frame.

A much more satisfactory approach is to apply a conformal transformation which does not make the assumptions which are inherent in an affine. Recent tests on non–metric camera data have shown improvements up to two–fold in the final values for object co–ordinates, even in situations with four slightly convergent camera stations and eight photographs. Robson (*ibid*) similarly shows the affine transformation to produce inferior results, especially in situations of weak geometry.

The case against the use of the affine transformation with non–metric cameras can be argued on the grounds of "over–parameterisation". The level of redundancy arising from four fiducials is too low, given the assumptions made in the derivation of those corner fiducials which may not have been observed but computed. With tests done on metric cameras with well–defined fiducial marks, no such problems have arisen. The misuse of the affine transformation with non–metric cameras is one more error source which awaits the inexperienced user of close–range photogrammetry.

Several authors report the addition of fiducial marks to non–metric cameras. Warner and Carson (1991) detail the addition of V–shaped notches to the edge of the frame of a Pentax 645 camera and also along the small cylindrical rollers at the edge of the format area. A weakness in their system was the projection of the V–shaped fiducials a distance of 0.7 mm from the roller to the image plane. The authors concluded that fiducial marks cut into the frame edge were more precise and suited to affine transformations. However the fundamental difficulties which can arise from the affine transformation were noted and they suggested the incorporation of a reseau plate would allow for the determination of film deformation over the entire format area.

Chandler, Cooper and Robson (1989) experimented with, respectively, local bi–linear and second order polynomial techniques for interpolation from either the surrounding four reseau crosses or across the entire format. The best improvement in object point co–ordinates was achieved by the use of an interior orientation comprised of a local bi–linear computation based on the surrounding four reseau points. In this way, local out–of–plane film deformations were constrained and systematic errors not introduced into the remainder of the image co–ordinates.

7. CONCLUSIONS

The formulae pertinent to the close–range calibration of cameras and lenses has been detailed. The techniques of self–calibration and plumb–lines has been discussed and their strengths and weaknesses evaluated. The influence of lens distortions on a range of close–range photogrammetric camera–object configurations has been explored and advice presented to new users of photogrammetry as to how these error sources may be eliminated or otherwise recognised.

Experimental results which indicate the relevance or importance of parameters such as the offsets of the principal points, the role of fiducials and methods of interior orientation were discussed. The influence of film deformations, including unflatness, were examined and the weaknesses caused by over–parameterisation when the affine transformation was used for interior orientation without fiducial marks explained.

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