An Efficient Object Space Algorithm for Spaceborne SAR Image Geocoding

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Abstract

Parametric SAR geocoding algorithms, which make use of a digital elevation model, involve a very computation-intensive resection of terrain points into radar image points, if a *rigorous* transformation of the digital elevation model between map projections is used, and if radar imaging time has to be determined iteratively from *exact* radargrammetric projections. This paper presents alternative methods which are suitable for the transformation of the digital elevation data as well as for the overall map-to-image transformation through warping using an anchor point grid. SAR image geocoding experiments using these fast algorithms have shown a performance improvement by a factor of up to 14. The resulting differences in geocoding accuracy when compared to methods using exact cartographic transformations and rigorous map-to-image resections are negligible.

1 Introduction

The transformation of a digital synthetic aperture radar (SAR) image given in slant range presentation to map projection geometry is called *SAR image geocoding*. Special attention is required if the radar scene covers a land area of non-flat topography. Although in this case the radiometry of the SAR image cannot be corrected from disturbing relief effects, the geometric terrain-induced distortions are eliminated by the utilization of a digital elevation model (DEM) in the geocoding. The processing then comprises the following main steps which are also illustrated in Figure 1:

1. *Map-to-object transformation:* Transformation of a DEM pixel with its elevation H to Cartesian X, Y, Z coordinates (e.g. geocentric) using map projection equations.

2. *Object-to-image transformation:* Resection of Cartesian X, Y, Z coordinates to range and time radar image coordinates using radar *range* and *Doppler* equations as follows (see Curlander 1982, Ref. [1]):

   SAR range equation:
   $$r = |\vec{p} - \vec{s}| \tag{1}$$

   SAR Doppler equation:
   $$\frac{\lambda f_{DC}}{2} = \left(\vec{\dot{p}} - \vec{\dot{s}}\right)\left(\vec{p} - \vec{s}\right)/r \tag{2}$$

   r is the sensor to target slant range, the vectors $\vec{p}$, $\vec{s}$ and $\vec{\dot{p}}$, $\vec{\dot{s}}$ denote target and sensor location and velocity respectively. f_{DC} is the Doppler centroid frequency and λ is the radar wave length.

3. Determination of a grey value within the current pixel cell in the real SAR image by nearest neighbour resampling or by an interpolation algorithm, and assignment of this grey value to that pixel of the ortho image, which geographically corresponds to the position of the input DEM pixel.
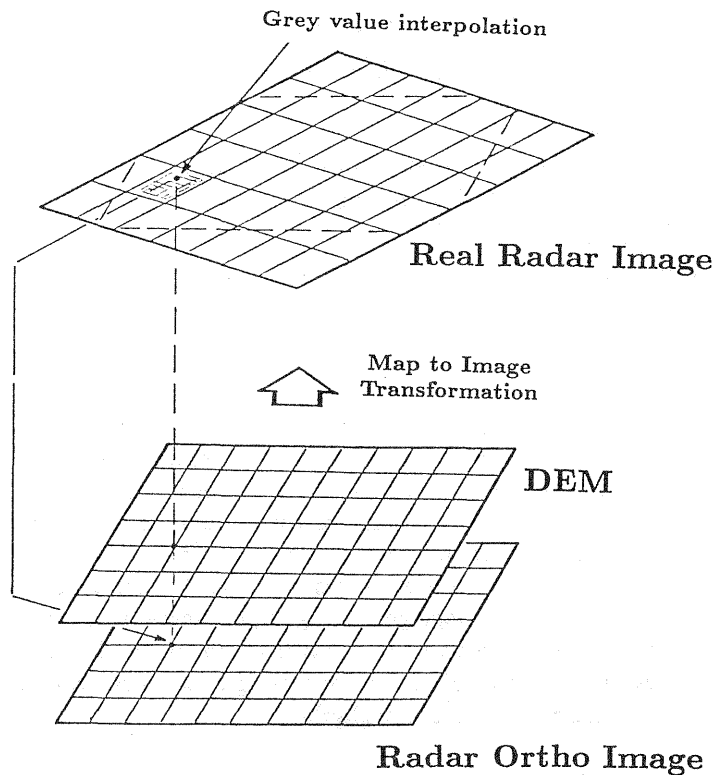
Grey value interpolation

Real Radar Image

Map to Image
Transformation

DEM

Radar Ortho Image

Figure 1: Principle of radar image geocoding.

Thus radar image geocoding generally requires the definition of a mathematical (parametric) relation[1] between a point on ground (coordinate triplet in object space) and a corresponding position of the sensor and, consequently, the corresponding point in the SAR image (coordinate pair in image space). Various algorithms for SAR image geocoding, which utilize a DEM and which follow the rigorous method sketched above have been published in the literature. A review of geocoding algorithms can be found in Raggam, Strobl and Triebnig (1986, Ref. [2]).

In particular, the map-to-image transformation (compare Figure 1) comprises CPU-time consuming processing steps since

- costly cartographic mapping equations may have to be applied to each pixel of the DEM in the transformation of DEM raster points to 3D Cartesian coordinates;

- the sensor position (or the SAR image azimuth coordinate, respectively) which corresponds to the imaged object point has to be computed from an iterative solution of radar range and Doppler equation.

More efficient, approximative methods for map-to-image transformation will be presented in this paper. These methods are based on the concept of replacing rigorous equations by approximative and/or interpolative ones. This concept is based on the use of a grid of anchor points and bilinear interpolation. The time-consuming rigorous equations are only evaluated for the anchor points. As will be shown, this method only causes negligible inaccuracies.

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[1] A radargrammetric treatment of *airborne* SAR may neglect earth curvature and rotation as acceptable approximation. Cartesian X, Y, Z then coincide with X_m, Y_m and elevation H, i.e. the map-to-object transformation essential for spaceborne SAR can be skipped.

2 Interpolative Cartographic Transformation of DEM

2.1 Description of Method

The problem of transforming a DEM, or like in this case the transformation of map projection coordinates X_m, Y_m, H to 3D Cartesian coordinates X, Y, Z, is not specific for SAR geocoding but also important in other cartographic applications.

Experiments have been made to study the possibility of using a bilinear interpolation based on a grid of anchor points to speed up this DEM transformation. Their goal was to perform the exact cartographic mapping only for the points of the anchor point grid and to use bilinear interpolation for the points in the grid cells.

For each pixel the final X, Y, Z coordinate values are computed as the sum of the coordinates for zero height (X_0, Y_0, Z_0) and of a proper correction factor for terrain elevation, i.e. the product of the elevation (H) and the elevation coefficient (h_X, h_Y, h_Z):

$$
\begin{aligned}
X &= X_0 + H \cdot h_X \\
Y &= Y_0 + H \cdot h_Y \\
Z &= Z_0 + H \cdot h_Z
\end{aligned}
\tag{3}
$$

By means of bilinear interpolation the variables X_0, Y_0, Z_0 as well as coefficients of elevation h_X, h_Y, h_Z can be interpolated from anchor point information for each pixel to be transformed. Then equations 3 can be used for the determination of the Cartesian X, Y, Z coordinates.

2.2 Test Results

A test program to investigate the performance of this algorithm and to find out its resulting accuracy was implemented. This program computes X, Y, Z Cartesian coordinates for an interpolation grid cell using both bilinear interpolation and the rigorous mapping relations. The interpolation utilizes the rigorously transformed X, Y, Z coordinates of the four corner points of the grid cell and, in addition, the coefficients h_X, h_Y, h_Z, which are computed for these points.

This test program was applied to the Gauss-Krueger map projection (data of a DEM covering a test area around the city of Bonn, FRG). Various assumptions for the mesh sizes in the anchor point grid were made. For each pixel Cartesian X, Y, Z coordinates were determined twice, once in the rigorous way and once using the interpolative method.

Comparing the two resulting coordinate data sets it became obvious that the biggest differences always occured in the center point of the interpolation grid cell. This is due to the earth curvature effect which is neglected by the bilinear interpolation in the grid cell. Table 1 shows a summary of the maximum location errors at the center points. With a grid spacing of 2 km, e.g., the maximum location error is 0.16 meters and with a grid spacing of 4 km it is 0.63 meters.

On a VAX 11/750 computer the procedure under discussion was applied to the whole DEM with an extension of 455 by 480 pixels and a pixel spacing of 50 meters. The result was compared to the rigorously transformed DEM.

According to the experiences gained in the test run, the spacing of the anchor point grid for this test area was set to 4000 meters, which resulted in a sample rate of 80 by 80 DEM pixels. Consequently, the number of anchor points necessary to cover the whole DEM area is 7 by 7 nodes.

The application of the rigorous procedure led to a CPU time consumption of about *56 CPU minutes*, while the interpolation approach took about *7 minutes and 55 seconds*. Therefore, CPU time can be reduced by a factor of approximately 7 for this task without any relevant loss of accuracy. Thus the algorithm becomes well suitable to be implemented also on computer workstations or mini-computers.

Table 1: Maximum location errors when using bilinear interpolation instead of map projection equations for the transformation of Gauss-Krueger coordinates to geocentric X, Y, Z coordinates.

Grid size (meters)	Max. location error (meters)
1000	0.04
2000	0.16
3000	0.35
4000	0.63
5000	0.98
6000	1.41

2.3   Relevance for SAR Geocoding

For a spaceborne sensor the maximum location errors of interpolated object points, which are summarized in Table 1, are fully acceptable since geometric capabilities of SAR instruments, i.e. typical SAR processor location accuracies, are in the range of tens to even hundreds of meters.

A high amount of disk space has to be allocated, because each DEM pixel of the original DEM, i.e. 1 integer value, has to be replaced by 3 real values, i.e. the X, Y, Z coordinates of the DEM pixel in the geocentric system, requiring 6 times the disk space of the original DEM in the case of single precision. However, in SAR image geocoding this disk space problem can easily be avoided if the transformation of the DEM pixels is not performed in a preparatory processing step, thereby creating a new data file, but in the course of the geocoding procedure for each pixel to be geolocated *without* creating a new data file for the transformed DEM. This implicit transformation of DEM points can again be done by means of the bilinear interpolation approach utilizing an anchor point grid, or, likewise, the exact transformation of DEM pixels could be implemented within the course of the geocoding procedure. In the latter case a considerable increase of CPU-time requirements would again have to be accepted.

3   Interpolative Correction of Terrain Effects and Geocoding of SAR Images

3.1   Concept of Algorithm

The principle of using exact transformations of a grid of anchor points and of using interpolation to fill the grid cells can also be applied to the correction of terrain effects and geocoding of SAR images.

The idea is to go one step further from the interpolative transformation of the DEM described in the foregoing section and perform the thorough *map-to-image* transformation using this technique. In this algorithm the requirement exists to determine radar image range and time coordinates related to an arbitrary object point by means of interpolation between anchor points containing the following information (see Figure 2):

- azimuth coordinate t_0 and range coordinate r_0 at reference elevation H_0.

- either azimuth and range coordinates (t_1 and r_1) at the reference elevation H_1 or proper elevation correction factors for azimuth and range (t' and r').
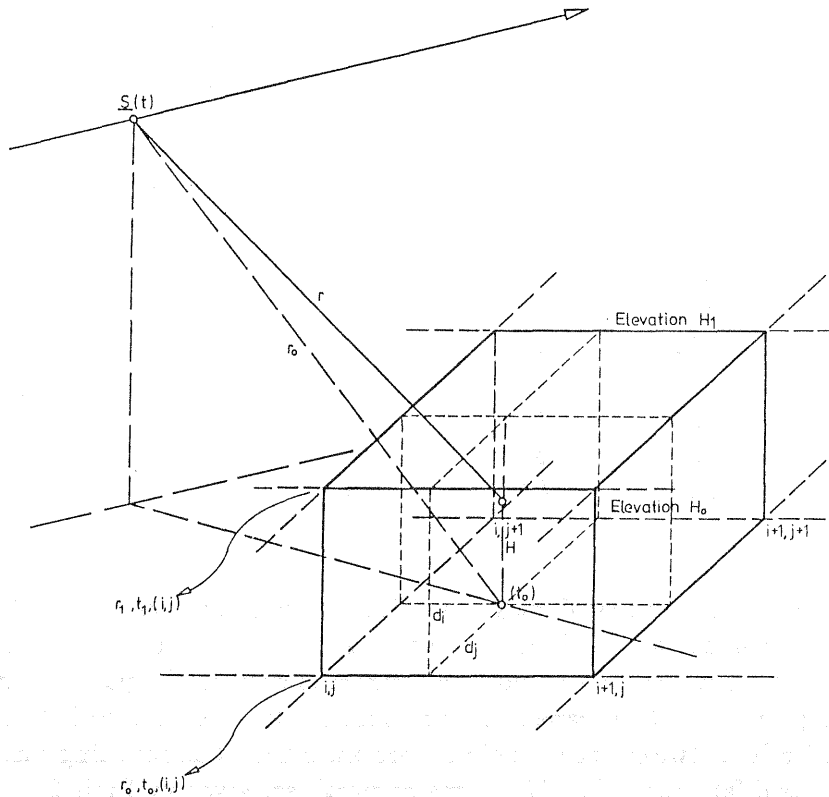
Figure 2: Interpolation of range and time coordinates (r, t) within a grid cell.

The entities t_0, r_0 and t_1, r_1 (or t', r' respectively) are again calculated in a rigorous preparatory procedure applied to the anchor points. Corresponding to equations 3 the algorithm assumes that meaningful radar coordinates can be calculated as the sum of interpolated coordinates at reference elevation H_0 and coordinate correction due to terrain elevation H of the actual output pixel. The map-to-image transformation of an arbitrary output pixel is then performed with the following equations:

$$
\begin{aligned}
t &= t_0 + H \cdot t' \\
&= t_0 + H \cdot (t_1 - t_0)/(H_1 - H_0) \quad\quad (4) \\
r &= r_0 + H \cdot r' \\
&= r_0 + H \cdot (r_1 - r_0)/(H_1 - H_0) \quad\quad (5)
\end{aligned}
$$

Here t_0, r_0, t', r' are interpolated bilinearly from the corresponding entities of the anchor points. Thus, the procedure is equivalent to a threedimensional (trilinear) interpolation between the squares at the reference elevations H_0 and H_1 based on the coordinate increments di, dj and elevation H (compare Figure 2).

Within a grid cell non-linearities of the SAR image coordinates as well as earth curvature effects are neglected. In order to be more accurate, one would have to consider proper corrections like discussed in section 3.3.

It is obvious, that once the anchor point information is available, map projection equations or radar projection equations are no longer necessary in this algorithm. Equations 4 and 5 could be applied to other imaging geometries if the appropriate anchor point information is provided.

3.2 Geocoding Accuracy

The replacement of rigorous mapping equations by the linear interpolation method described above usually results in some degradation of geocoding accuracy. This is due to neglected earth

Table 2: Maximum errors of interpolated slant ranges at reference elevation H_0 in near, mid and far range. A 100 kilometer swath has been assumed.

Grid size (meters)	Max. range error (meters)		
	Near range	Mid range	Far range
1000	0.1	0.1	0.1
2000	0.5	0.5	0.5
3000	1.2	1.1	1.0
4000	2.1	1.9	1.8
5000	3.2	3.0	2.8
6000	4.6	4.4	4.1
7000	6.3	5.9	5.5
8000	8.3	7.8	7.2

curvature effects and non-linearities of image coordinates within an "anchor point box".

Therefore, a test program was implemented which determines the differences of slant ranges at the reference elevation H_0, calculated both in a rigorous way and by means of interpolation. A simple imaging geometry with parameters representative for SEASAT or ERS-1 and various widths for the grid cells between anchor points were assumed. The resulting maximum range errors in near, mid and far range of a 100 kilometer swath are given in Table 2.

The experiment showed, that the maximum errors always occur at the center point of a grid cell. The maximum range error at the reference elevation, e.g., is around 3 meters for a mesh size of 5000 meters. It can be seen from Table 2, that the amount of maximum range errors is about the same across the whole SAR swath; differences between near and far range are up to about 1 meter for a cell size of 8000 meters.

With reference to azimuth coordinate calculation the situation is more complex since the Doppler frequency is a function of the geographic location on earth. A practical application of this algorithm to the Bonn test area with a terrain variation of only some 500 meters showed, that for this data set equation 4 leads to sufficiently accurate azimuth coordinates.

3.3 Higher Order Correction Terms

As can be seen from Table 2 the errors of SAR image range coordinates calculated from equation 5 can be kept sufficiently small if a properly small mesh size for the anchor point grid is selected. If the wish exists to be even more accurate one would have to add additional correction factors like

- a factor compensating the non-linearity of range in the across-track direction and earth curvature effects. The maximum errors caused by these phenomena correspond to the values given in Table 2 (about 3 meters for a grid size of 5000 meters).

- a factor compensating the non-linearity of range in elevation direction. For the consideration of this error source one should use a second order elevation correction factor (r'') which might be considered constant for one grid cell or even for the overall scene and causes no additional interpolation.

Then slant range coordinates can be calculated as follows:

$$r = r_0 + \Delta r_0 + r' \cdot H + r'' \cdot H^2 \tag{6}$$

Table 3: CPU time requirements (min:sec) for geocoding the Bonn test area on a VAX 11/750 computer. Different algorithms for map-to-image transformation have been assumed.

Algorithm	Map-to-object	Object-to-image	CPU time requirement
A	rigorous — rigorous		71 : 17
B	rigorous	rigorous	72 : 09
C	interpolative — rigorous		23 : 31
D	interpolative	rigorous	24 : 31
E	interpolative		05 : 09

Equivalent corrections also could be applied to the SAR image azimuth coordinate.

4   Practical Application of the Described Algorithms

The Bonn test area was used for practical experiments to geocode a SAR image using the various algorithms discussed in the foregoing sections. Due to the results obtained in the course of the accuracy studies, which are summarized in Tables 1 and 2 a spacing of 4000 meters was selected for the anchor point grids to be used in the interpolation-based algorithms. Thus, for this test area the sample rate of the anchor point grid was 80 by 80 DEM pixels with a pixel resolution of 50 meters.

Table 3 shows the CPU times required for geocoding the Bonn test area on a VAX 11/750 computer using different geocoding algorithms as follows:

Algorithm A: Rigorous map-to-image transformation using map projection equations and radar projection equations; implicit map-to-object transformation in the process of geocoding each individual DEM pixel.

Algorithm B: Rigorous map-to-image transformation using map projection equations and radar projection equations; external map-to-object transformation in a preparatory processing step, creating a separate data file.

Algorithm C: Implicit bilinear interpolation of Cartesian X, Y, Z coordinates using an anchor point grid; rigorous object-to-image resection using radargrammetric equations.

Algorithm D: External map-to-object transformation based on the anchor point grid, creating a separate data file; rigorous object-to-image resection using radargrammetric equations.

Algorithm E: Bilinear interpolation of radar range and time coordinates using the anchor point grid.

It can be concluded from Table 3 that the map-to-object transformation accounts for the major part of the CPU time required for the overall map-to-image transformation, if it is performed in the rigorous way. This especially is valid for the costly map projection equations of, in the present case, the Gauss-Krueger transformation.

The table shows an obvious decrease of CPU time consumption when proceeding from algorithm A to algorithm E. On the other hand, as could be shown in the foregoing sections, the inaccuracies caused by the use of interpolative methods in parametric SAR image geocoding are either inherently rather small or can be kept within well-defined limits by selecting proper parameters as, e.g., raster sizes of the anchor point grids used. Thus, for the test area used for these experiments a performance improvement by a factor of up to 14 could be achieved.

5   Conclusions

The presented algorithms for SAR image geocoding, which use an anchor point grid and bilinear interpolation instead of rigorous mapping equations proved to be very useful alternatives to rigorous methods. Their main advantages are transparency, modularity and speed. In a preparatory processing step the rigorous projection equations only have to be applied to the anchor points, which usually represent a very small subset of the data. Thereby, considerable CPU time can be saved without producing serious inaccuracies in the image coordinates of the interpolated points. As could be shown, the accuracy loss in comparison to the rigorous geocoding algorithm is only, at the worst, about 3 meters for grid cell sizes of even up to 5000 meters. This number is reduced by selecting a smaller mesh size for the anchor point grid or by adding proper correction factors to the calculation of the radar image coordinates, primarily for the radar range coordinate.

Since no sensor-specific projection equations are used, the algorithm using bilinear interpolation as a substitute for the overall map-to-image transformation procedure can also be applied to any other type of imagery, like e.g. images from optical sensors. Instead, a proper anchor point grid must only be provided by a separate processing, where the particular sensor's imaging equations are employed. The key parameters to be available for the grid points are the along-track coordinate, across-track coordinate and terrain correction factor for along- and across-track coordinate.

No calculations in the $3D$ object space are further necessary. This makes the algorithm more suitable for array processor implementation.

References

[1] J. Curlander. Location of spaceborne SAR imagery. *I.E.E.E. Trans. Geoscience and Remote Sensing*, GE-20(3):359–364, July 1982.

[2] J. Raggam, D. Strobl, and G. Triebnig. *The Rectification of SAR Image Data using a Digital Elevation Model and Image Simulation Techniques. Phase-B Study for ERS-1 Processing and Archiving Facility.* ESA Contract Report 6292/85/HGE-I, Technical Note 17, Graz Research Center, Austria, August 1986.